# Solution to Problem #6

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(a) Notice that $\|u+v\|^2 = (u+v)\cdot(u+v) = u\cdot u + 2u\cdot v + v\cdot v = \|u\|^2 + 2u\cdot v + \|v\|^2$. Similarly, $\|u-v\|^2 = \|u\|^2 - 2u\cdot v + \|v\|^2$. Subtracting the latter equation from the former yields $\|u+v\|^2 - \|u-v\|^2 = 4u\cdot v$, so

$$u \cdot v = \frac{1}{4}(\|u+v\|^2 - \|u-v\|^2).$$

(b) Adding both equations in part (a) gives instead

$$\|u+v\|^2 + \|u-v\|^2 = 2(\|u\|^2 + \|v\|^2),$$

as desired.

*Notes:*

- *The last equation is known as the* parallelogram law. *It generalizes Pythagorean Theorem: if $u$ and $v$ are perpendicular, then $\|u+v\| = \|u-v\|$, yielding*

$$\|u+v\|^2 = \|u\|^2 + \|v\|^2.$$