Solution to Problem #2

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A circle of center (a, b) and radius r has equation $(x-a)^2 + (y-b)^2 = r^2$. Substituting $(1,0)$, $(-1,2)$, and $(3,1)$ in (x, y) yields, respectively,

$$\begin{aligned} a^2+b^2-2a+1 &= r^2 && (1)\\ a^2+b^2+2a-4b+5 &= r^2 && (2)\\ a^2+b^2-6a-2b+10 &= r^2 && (3) \end{aligned}$$

The quadratic factors are eliminated by subtracting (2) to (1) and then (3) to (2). It yields, respectively,

$$\begin{aligned} b-a &= 1 && (4)\\ 8a-2b &= 5 && (5) \end{aligned}$$

This last system of linear equations is easily solved: $a = 7/6$, $b = 13/6$ and, substituting these last values in (1), $r^2 = 85/18$. Therefore, the desired equation is $(x-(7/6))^2 + (y-(13/6))^2 = 85/18$.

Notes:

1. *Congratulations to our student Shaikhah Abdulla Al-Yammahi (Mathematical Sciences), who submitted a right solution following the same outline.*

2. *An honorable mention is given to Reem Alomari (Mathematical Sciences), who used the following approach. One may compute the equation of the perpendicular bisectors of the segments joining, respectively,* $(1,0)$ *to* $(-1,2)$ *and* $(-1,2)$ *to* $(3,1)$*. The intersection of these last lines give the center of the circle. The radius is then given by the distance between this center and the given points. This outline is correct, but unfortunately, mistakes were done in the computation.*

3. *As observed by Dr.Ahmed, the equation of the circle may be obtained using linear algebra. The general equation of a circle may be written as* $a(x^2 + y^2) + bx + cy + d = 0$*. Substituting* $(1,0)$*,* $(-1,2)$*, and* $(3,1)$ *in* (x,y) *gives three linear equations in* (a,b,c,d)*, in addition to the general one.*

One may write the resulting four equations in matrix form. The matrix of coefficients is then singular since, multiplied by $(a, b, c, d) \neq 0$*, it gives* 0*. Therefore, the determinant of this matrix is* 0*, yielding an equation in* x *and* y *which is, precisely, the equation of the circle.*