

## Problem #9

May 5, 2012

*Difficulty:* medium

*Prerequisite:* Calculus I, induction principle (Introduction to Analysis)

For  $x > 0$ , let  $F_1(x) = x$ ,  $F_2(x) = x^x$ ,  $F_3(x) = x^{(x^x)}$ ,  $F_4(x) = x^{(x^{(x^x)})}$ , etc.

(a) Compute  $F_2'(x)$  and  $F_3'(x)$ .

(b) Prove that  $F_n'(1) = 1$  for all  $n = 1, 2, 3, \dots$