Problem #5

February 20, 2012

Difficulty: medium
Prerequisite: High School algebra and complex numbers

(a) By completing the square derive that the two solutions of

$$ax^2 + bx + c = 0, \ a \neq 0$$

are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

(b) Show that any polynomial of degree three has at least one root in $\mathbb{R}$. That is, the equation

$$a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0, \quad a_i \in \mathbb{R}, \ a_3 \neq 0,$$

has at least one solution in $\mathbb{R}$.