

Problem #4

January 14, 2012

Difficulty: hard

Prerequisite: induction principle (Introduction to Analysis)

The *Fibonacci sequence* is defined recursively by $F_0 = 0$, $F_1 = 1$, and $F_{n+2} = F_n + F_{n+1}$, so

$$\{F_0, F_1, F_2, \dots\} = \{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \dots\}.$$

Show that

$$F_m F_{n+1} - F_{m+1} F_n = (-1)^n F_{m-n}$$

where $m \geq n \geq 0$.