Problem #1

November 25, 2011

Difficulty: medium
Prerequisite: graph sketching (Calculus I)

a) Show that, if the curves $y = a^x$ and $y = x^a$ meet without crossing (where $a > 0$ and $x \geqslant 0$), then $a = \mathrm{e}$.

b) Without using a derivative, deduce that $y = x^x$ reaches its minimum at $x = 1/\mathrm{e}$ (where $x \geqslant 0$).