

Dept. of Mathematical Sciences, UAEU

ABSTRACT ALGEBRA (Math340), Section 01, Spring 2010

Final Exam

Dr. Jianhua Gong, Wednesday, June 9, 2010

Name _____ ID _____ Grade ___/40

1. (7 marks): Consider the cyclic group \mathbb{Z}_{60} .
 - (a) Give the order of 12.
 - (b) Compute 50^{-2} .
 - (c) The cyclic group \mathbb{Z}_{60} has a unique subgroup H of order 6.
What is H ?
 - (d) Find all generators of the subgroup H .

2. (7 marks): Let σ be the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & 3 & 7 & 5 & 6 & 2 & 8 & 4 & 1 \end{pmatrix}.$$

- (a) Write σ as product of disjoint cycles.
- (b) Is σ in the alternating group A_9 ?
- (c) Find $|\langle \sigma \rangle|$.
- (d) Calculate σ^{30} .

3. (6 marks) :

- (a) Consider the product group $U(8) \times U(9)$.
 - i. Compute the order of $|U(8) \times U(9)|$.
 - ii. Give the inverse $(3, 5)^{-1}$.
- (b) Find all abelian groups of order 2010.
- (c) Are the groups $\mathbb{Z}_8 \times \mathbb{Z}_{10} \times \mathbb{Z}_{24}$ and $\mathbb{Z}_4 \times \mathbb{Z}_{12} \times \mathbb{Z}_{40}$ isomorphic? Why or why not?

4. (7 marks): Given the factor group $\mathbb{Z}_{120}/\langle 30 \rangle$.
- (a) Why do the left and right cosets of the subgroup $\langle 30 \rangle$ coincide?
 - (b) What is the identity of this factor group?
 - (c) Find the order of the factor group.
 - (d) Find the order of the element $12 + \langle 30 \rangle$.
 - (e) What is the inverse $(12 + \langle 30 \rangle)^{-1}$?

5. (7 marks): Consider the ring \mathbb{Z}_{10} .

(a) Determine whether or not \mathbb{Z}_{10} is

- i. a division ring;
- ii. an integral domain;
- iii. a field.

(b) Solve the equation $x^2 + 3x + 2 = 0$ in \mathbb{Z}_{10} .

6. (6 marks):

- (a) State the First Isomorphism Theorem.
- (b) Let $\phi : G \longrightarrow G'$ be a homomorphism. Show that $\ker(\phi)$ is a normal subgroup of G .
- (c) Given a ring R , prove that every multiplicative invertible element is not a 0 divisor.