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Linear Programming , Final Exam

Tuesday, January 5, 2010 Fall, Section 01

Name: ...................................I.D..................Grade (40):............

(1) (6 marks) Find a minimum-cost shipping schedule for the transportation costs given in the table. You need only to set up the transportation problem by introducing variables, forming an objective function, and writing the constraint inequalities or equalities.

Supply points	D_1	D_2	D_3	D_4	D_5	Supply
S_1	1	3	12	∞	9	100
S_2	5	4	9	7	10	120
S_3	∞	6	20	10	11	90
Demand	60	80	70	60	40	

(2) (7 marks) The Moscow Industrial Ghetto (MIG) manufactures large and small iron dragons. The large dragons are used for garden decorations and the small dragons are used for automobile ornaments. Each week up to 1000 pounds (lb) of iron can be purchased. MIG employs five workers, each of whom works 30 hours (hr) per week. Their salary is fixed cost that does not depend on productivity. The machinery used is a available for a maximum of 300 hours per week. The following table gives data about dragons

	Small dragon	Large dragon
Iron per dragon (lb)	3	125
Labor per dragon (hr)	2	12
Machine time per dragon (hr)	1.5	9
Selling price per dragon ($)	35	300

(a) Set up the LP problem to determine how many dragons of each size MIG should make to maximize total selling price.

(b) Let u_1, u_2 and u_3 be the prices per pound of iron , per hour of labor and per hour of machine time, respectively. Set up the dual problem to minimize the cost.

(3) (7 marks) Consider the LP problem:

Minimize: $z = 3x_1 + 6x_2$

Subject to:

$$\begin{array}{rcrcl} x_1 & + & 2x_2 & \leq & 6 \\ x_1 & - & x_2 & \geq & 2 \\ x_1 & - & x_2 & \leq & 4 \\ x_1, & x_2 & \geq & 0. & \end{array}$$

(a) Sketch the feasible region of this LP problem.

(b) Determine the optimal solution using the Corner Point Theorem. Also give the reason why you can use the Corner Point Theorem.

(c) Show that the feasible region in (a) is convex.

(4) (6 marks)

(a) Give equivalent LP problem that you can put into canonical form by adding slack variables.

Minimize:

$$z = 2x_1 - 3x_2$$

Subject to:

$$\begin{aligned} &2x_1 - 5x_2 \leq 12 \\ &3x_1 - 2x_2 \geq -15 \\ &x_1 \leq 0, x_2 \text{ unrestricted} \end{aligned}$$

(b) For the following system of linear equations, choose the departing basic variable when the entering basic variable is x_1.

$$\begin{aligned} 3x_1 + 2x_2 - 4x_3 + 6x_4 + x_5 &= 5 \\ 2x_1 - 3x_2 + 2x_3 + 5x_4 + x_6 &= 6 \\ x_1 + x_2 - x_3 + x_4 + x_7 &= 3 \\ 4x_1 + 3x_3 + 2x_4 + x_8 &= 2. \end{aligned}$$

(5) (7 marks) Consider the LP problem:

Maximize: $z = 2x_1 + 3x_2 + 3x_3$

Subject to:

$$\begin{array}{rcrcrcr} 3x_1 & + & 2x_2 & & & \leq & 60 \\ -x_1 & + & x_2 & + & 4x_3 & \leq & 10 \\ 2x_1 & - & 2x_2 & + & 5x_3 & \leq & 50 \\ & & x_1, & x_2, & x_3 & \geq & 0 \end{array}$$

(a) Solve this LP problem using the simplex method.

(b) Form the dual problem of this LP problem.

(c) Give the optimal solution to the dual problem in (b).

(6) (7 marks) Given the initial tableau T of a LP problem and the tableau T^* by several pivoting of the simplex method.

$$T = \begin{bmatrix} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & \\ x_6 & -2 & 0 & 6 & 0 & 2 & 1 & -3 & 20 \\ x_2 & -4 & 1 & 7 & 0 & 1 & 0 & -1 & 10 \\ x_4 & 0 & 0 & -5 & 1 & 3 & 0 & -1 & 60 \\ & 0 & 0 & 13 & 0 & -6 & 0 & 2 & 100 \end{bmatrix}$$

$$T^* = \begin{bmatrix} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & \\ x_1 & & -\frac{1}{3} & & & & \frac{1}{6} & -\frac{1}{6} & \\ x_5 & & -\frac{1}{3} & & & & \frac{2}{3} & -\frac{5}{3} & \\ x_4 & & 1 & & & & -2 & 4 & \\ & & -2 & & & & 4 & -8 & \end{bmatrix}.$$

(a) For T, find the constraint matrix A, the resource matrix b and the cost matrix C.

(b) For T^*, find the inverse of basis matrix B^{-1}, the basic variable matrix X_B and the basic cost matrix C_B.

(c) Use the matrix formulas to complete the tableau T^*.