Department of Mathematical Sciences

College of Science

United Arab Emirates University

Numerical Analysis 1 Spring2010 Section: 52

Final Exam Tuesday, June 8, 2010 Instructor: Dr. Hani Siyyam

Student Name:……………………………………………………………………………….I.D.NO.:……………………….

Question One: Let $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9.$

(a) With $x_0 = -1, x_1 = 0.$ Find x_2 using the Secant method.

(b) With $x_0 = -0.5.$ Find x_1 using the Newton method.

Question Two: For a function f, the forward divided differences are given by

$x_0 = 0.0$	$f[x_0] = ?$		
		$f[x_0, x_1] = ?$	
$x_1 = 0.2$	$f[x_1] = ?$		$f[x_0, x_1, x_2] = 8$
		$f[x_1, x_2] = 6$	
$x_2 = 0.4$	$f[x_2] = 5$		

Determine the missing entries in the table.

Question Three: Use the most accurate three-point formula to determine each missing entry in the following table

x	$f(x)$	$f'(x)$
2.0	3.6887983	?
2.1	3.6905701	?
2.2	3.6688192	?
2.3	3.6245909	?

Question Four: Let $h = \frac{b-a}{3}, x_0 = \frac{b+2a}{3}, x_1 = \frac{2b+a}{3}$. Find the degree of the precision of the quadrature formula

$$\int_a^b f(x)dx = \frac{3h}{2}[f(x_0) + f(x_1)].$$

Question Five: Assume that A is split into $A = M - N$, where

$$M = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}, N = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) Find the first iteration of the Jacobi method for the system $Ax = b$, where $b = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$ and $x^{(0)} = \begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix}$.

(b) Find the first iteration of the Gauss-Seidel method for the system $Ax = b$, where $b = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$ and $x^{(0)} = \begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix}$.

(c) Find $\rho(T_j)$.

(d) Find $\|T_j\|_\infty$.

(e) Determine whether A is strictly diagonally dominant matrix or not.

(f) Find the number of iterations needed to apply the Jacobi method to obtain an approximation to the solution of $Ax = b$ accurate to within 10^{-6}. Use $\|\cdot\|_\infty$ and $x^{(0)} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ where needed.

Question Six: Use Gaussian elimination with scaled partial pivoting and three-digit chopping arithmetic to solve the following system

$$\begin{aligned} 0.03x_1 + 58.9x_2 &= 59.2 \\ 5.31x_1 - 6.10x_2 &= 47.0 \end{aligned}$$

Question Seven: Assume that the LU-decomposition of A is stored in the matrix $B = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 5 \end{bmatrix}$. Find the matrix A so that A =LU, then solve the system $Ax = \begin{bmatrix} 2 \\ 2 \\ -5 \end{bmatrix}$.

Question Eight: Obtain a factorization of the form $A = P^T LU$ for the following matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix}.$$

Question Nine: Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 13 \end{bmatrix}$.

(a) Show that A is positive definite.
(b) Find the Choleski factorization of A.

GOOD LUCK