Department of Mathematical Sciences

College of Science

United Arab Emirates University

Numerical Analysis 1 Spring2010 Section: 51

Final Exam Thursday, June 10, 2010 Instructor: Dr. Hani Siyyam

Student Name:……………………………………………………………………………….I.D.NO.:………………………..

Question One: Let $f(x) = x^3 - 2x^2 - 5.$

(a) With $x_0 = 1, x_1 = 4.$ Find x_2 using the Secant method.

(b) With $x_0 = 2.$ Find x_1 using the Newton method.

Question Two: For a function f, the forward divided differences are given by

$x_0 = 0.2$	$f[x_0] = 4$		
		$f[x_0, x_1] = 5$	
$x_1 = 0.4$	$f[x_1] = ?$		$f[x_0, x_1, x_2] = ?$
		$f[x_1, x_2] = ?$	
$x_2 = 0.6$	$f[x_2] = 6$		

Determine the missing entries in the table.

Question Three: Use the most accurate three-point formula to determine each missing entry in the following table

x	$f(x)$	$f'(x)$
8.1	16.94410	?
8.3	17.56492	?
8.5	18.19056	?
8.7	18.82091	?

Question Four:

(a) Find the constants c_0, c_1 and x_1 so that the quadrature formula

$$\int_0^1 f(x)dx = c_0 f(0) + c_1 f(x_1)$$

has the highest possible degree of precision.

(b) Let $A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}$. Find the spectral radius of A.

Question Five: For the system

$$3x_1 + x_2 + x_3 = 2$$
$$2x_1 + 4x_2 + x_3 = 1$$
$$x_1 + x_2 + 3x_3 = -2$$

(a) Find the first iteration of the Jacobi method for the system $Ax = b$, where $x^{(0)} = \begin{Bmatrix} 2 \\ 1 \\ 0 \end{Bmatrix}$.

(b) Find the first iteration of the Gauss-Seidel method for the system $Ax = b$, where $x^{(0)} = \begin{Bmatrix} 2 \\ 1 \\ 0 \end{Bmatrix}$.

(c) Find $\|T_j\|_\infty$.

(d) Determine whether A is strictly diagonally dominant matrix or not.

(e) Find the number of iterations needed to apply the Jacobi method to obtain an approximation to the solution of $Ax = b$ accurate to within 10^{-6}. Use $\|.\|_\infty$ and $x^{(0)} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ where needed.

Question Six: Use Gaussian elimination with partial pivoting and three-digit chopping arithmetic to solve the following system

$$58.9x_1 + 0.03x_2 = 59.2$$
$$-6.10x_1 + 5.31x_2 = 47.0$$

Question Seven: Assume that the LU-decomposition of A is stored in the matrix $B = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 3 & 5 \\ -1 & 2 & 4 \end{bmatrix}$. Find the matrix A so that A=LU, then solve the system $Ax = \begin{bmatrix} 3 \\ 5 \\ -11 \end{bmatrix}$.

Question Eight: Obtain a factorization of the form $A = P^T LU$ for the following matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 2 & 5 & 3 \end{bmatrix}.$$

<u>Question Nine:</u> Let $A = \begin{bmatrix} 4 & 2 \\ 2 & 10 \end{bmatrix}$.

(a) Show that A is positive definite.

(b) Find the Choleski factorization of A.

GOOD LUCK