United Arab Emirates University
Department of Mathematical Sciences
Final Exam, Real Analysis (Math 310)
Thursday, June 10^{th}, 2010
Duration: Two hour

Name: **Student Number**:

[Question 1, 2+3 pts] (a) Define a cluster point c of a set A.

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Show that if c is a cluster point of $A \subseteq \mathbb{R}$ and $f(c) \notin f(A)$, then $f(c)$ ia a cluster point of $f(A)$.

[Question 2, 3 pts] Use the definition to show that $Lim_{x \to 2+} \frac{x}{x-2} = \infty$.

[Question 3, 2+3 pts] (a) State the sequential criteria of a nonuniform continuous function f on a set A.

(b) Give an example of a continuous function f on a set A which is NOT uniformly continuous on A.

[Question 4, 3+3 pts] Determine whether the following functions are uniformly continuous on their indicated domains :
(a) $f(x) = \frac{1}{x},\ x \in [a, \infty)$ with $a > 0$?

(b) $f(x) = \sin(\frac{1}{x}),\ x \in (0, 1]$?

[Question 5, 3+3 pts] Which of the following is Riemann integrable on its indicated domain? Justify your answer.

(a) $f : [0,1] \to \mathbb{R}$ **defined by** $f(x) = \begin{cases} 0\,; & x \in \{\frac{1}{n}; n \in \mathbb{N}\} \\ e^x\,; & o.w. \end{cases}$

(b) $g : [0,1] \to \mathbb{R}$ **defined by** $g(x) = \begin{cases} 0\,; & x \in \mathbb{Q} \\ \frac{1}{x}\,; & o.w. \end{cases}$

[Question 6, 4 pts] Use the Mean Value Theorem to prove that

$$1 + x \leq e^x, \ \forall x \in \mathbb{R}.$$

[Question 7, 2+3 pts] (a) Show that if $f \in R[a, b]$, then $f^2 \in R[a, b]$

(b) Show that if $f, g \in R[a, b]$, then $fg \in R[a, b]$

[Question 8, 5 pts] Show that if $f \in R[a, b]$, which is continuous at a point $c \in [a, b]$, then the function

$$F(x) = \int_a^x f, \quad \textbf{for } x \in [a, b]$$

is differentiable at c and $F'(c) = f(c)$.

[Question 9, 5 pts] Show that if f is continuous on $[a, b]$, then there exists $c \in [a, b]$ such that $\int_a^b f = f(c)(b - a)$.
Hint: As f is continuous on $[a, b]$, it has absolute max and min. Then use the I.V.T. to prove the existence of c.

GOOD LUCK