Foundations of Geometry Fall 2009: Final Exam

Name: ....

ID: ....

Exercise 1. Let P and Q be two points inside the Poincaré disk such that P and Q are not on a diameter. Explain how to construct the hyperbolic line through P and Q.

Exercise 2. For each of the following statements say whether it is true or false.

(1) For any three points P, Q and T in the Poincaré disk, we have $d_H(P,Q) + d_H(Q,T) = d_H(P,T)$. True ☐ False ☐

(2) The 5th postulate in hyperbolic geometry is equivalent to Playfair's Postulate. True ☐ False ☐

(3) Any hyperbolic line has infinitely many limiting parallels. True ☐ False ☐

(4) The hyperbolic circle centered at A and of radius r is the same as the Euclidean circle of center A and radius $|ln(\dfrac{1+r}{1-r})|$. True ☐ False ☐

Exercise 3. Consider the triangle $\triangle OAB$ where $O = (0,0)$, $A = (0,3)$ and $B = (4,0)$. Then, consider the points $A' = (2,0)$, $B' = (0,2)$ and $O' = (\frac{4}{3}, 2)$.

(a) Find the equation of the line through A and B.

(b) Prove that O' belongs to the segment $\overline{AB}$.

(c) Prove that the three segments $\overline{AA'}$, $\overline{BB'}$ and $\overline{OO'}$ meet at one point (without writing the equations of lines).

Exercise 4. Let $\mathcal{C}$ be the circumscribed circle of the triangle $\triangle ABC$. Let O be the center of $\mathcal{C}$ and P the point where the bisector of $\angle ABC$ intersects $\mathcal{C}$.

(a) Prove that $\triangle OPC \cong \triangle OPA$.

(b) Prove that $\overline{OP}$ is perpendicular to $\overline{AC}$.

Exercise 5. Let $A = (1, 1)$, $B = (3, 3)$ and $C = (t, t)$. Determine the measure of the angle $\angle BAC$ for any value of $t \neq 1$.

Exercise 6. **(1)** Is the line of equation $y = -x + 4$ tangent to the circle of equation $x^2 + y^2 = 8$? Justify your answer!!

(2) Prove that in a parallelogram, opposite sides are congruent!

(3) Let $A = (1, 1)$, $B = (5, 5)$ and $C = (3, 6)$. Find the coordinates of the centroid (the intersection point of the three medians) of ABC.