United Arab Emirates University
Department of Mathematical Sciences
Set Theory and Logic (MATH 245)
Section 52
Final Exam
Wednesday June 9th 2010- 6:00 8:00

Name :_____

Student Number:_____

Exercise	points	Max
1		6
2		3
3		3
4		3
5		6
6		2
7		4
8		9
9		4
Total		

Part 1: Propositions and Logic Proofs (12 points).

Exercise 1(Truth tables and rules - 6 points=2+2+2)
1) Show the following using truth table:

$$(P \wedge Q) \Rightarrow R \equiv (P \wedge \sim R) \Rightarrow \sim Q.$$

2) Show the following by giving only one line of the truth table:

$$P \Rightarrow (Q \vee R), P \nvdash Q.$$

3) Identify the rule that allows to write the following result:

$$(P \Rightarrow Q) \Rightarrow P, P \Rightarrow Q \vdash (P \Rightarrow Q) \Rightarrow Q.$$

Exercise 2(Paragraph proof- 3 points)
Show by cases that:
For any 3 integers a, b and c we have: a divides b or a divides $c \Rightarrow a$ divides bc.

Exercise 3(Quantifiers- 3 points=1+2)
1) Negate the following proposition and find a counter-example showing that the statement is not true:
The sum of any two integers is positive.

2) Label the following statements by **True** or **False**
a- $(\exists a, b, c)(\forall x)(f(x) = ax^2 + bx + c)$ defines f as a quadratic function.
b- $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x = 2y)$.
c- $(\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x = 2y)$.
d- The negation of "Some numbers are multipe of 7" is "Some numbers are not multiple of 7".

Part 2: Sets and Mathematical Induction (15 points).

Exercise 4(True or False- 3 points)
Label the following statements as **True** or **False**. Correct the false statements.
1) $\frac{1}{2} \in \{x \in \mathbb{Z}; 2x = 1\}$.
2) $\{-1, 1\} \subset \{x \in \mathbb{R}; x^2 - 1 < 0\}$.
3) $\sqrt{2} \in P(\mathbb{R})$.
4) $\{sqrt2\} \in P(\mathbb{R})$.
5) $\{0, 3\} \subset (0, 3]$.
6) $\{\phi\} \in P(\phi)$.

Exercise 5(Set Notations- 6 points= 2+2+2)
1) Write in set-builder notation the set of all positive integers that are multiples of 3.

2) Write as a roster the set $A \setminus (B \setminus C)$, where $A = \{0, 2, 4, 6\}$, $B = \{2, 3, 4, 5, 6\}$ and $C = \{0, 1, 2\}$.

3) Draw a Venn Diagramm for $A \cap (\overline{B \cap C})$.

Exercise 6(Paragraph proof- 2 points)
Show that for any 2 sets A and B we have:
$(A \cup B) \setminus C = (A \setminus B) \cup (A \cap C)$.

Exercise 7(Mathematical Induction- 4 points)
Show that for any $n \geq 1$, we have:

$$2 + 4 + 6 + 8 + ... + 2n = n(n + 1).$$

Part 3: Relations and Functions (13 points).

Exercise 8(Functions- 9 points =3+4+2)
1) Consider the function $p : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ defined by $p(x) = (x, y)$.
a- Is p a one-to-one-function?
b- Is p an onto function?
c- Is p an invertible function?
Justify your answers.

2) Consider the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = x^2$.
a-Find $f^{-1}([1, 2])$.
b-Find $f(\mathbb{R})$.

3) Given two odd functions f and g. Is the function $f + g$ odd or even? Justiy your answer.

Exercise 9(Equivalence relation- 4 points)
1) Show that the relation defined on $\mathbb{Z}$ by:
$aRb \Longleftrightarrow a - b = 2p$ for some integer p is an equivalence relation.
2) Identify $[a]$ induced by R for any element a in $\mathbb{Z}$.