United Arab Emirates University
Department of Mathematical Sciences
Set Theory and Logic (MATH 245)
Section 51
Final Exam
Tuesday June 8th 2010 - 08:00 - 10:00

Name :____________________________

Student Number:____________________________

Exercise	points	Max
1		6
2		3
3		3
4		3
5		6
6		2
7		4
8		9
9		4
Total		

Part 1: Propositions and Logic Proofs (12 points).

Exercise 1(Truth tables and rules - 6 points=2+2+2)
1) Show the following using truth table:

$$P \Rightarrow (Q \wedge R) \equiv (P \Rightarrow Q) \wedge (P \Rightarrow R).$$

2) Show the following by giving only one line of the truth table:

$$(P \wedge Q) \Rightarrow R, P \nvdash R.$$

3) Identify the rule that allows to write the following result:

$$P \vee (Q \vee S), \sim P \vdash Q \vee S.$$

Exercise 2(Paragraph proof- 3 points)
Write a biconditional proof for the following statement:
For any 2 integers a and b we have: a divides b if and only if a divides $b-a$.

Exercise 3(Quantifiers- 3 points=1+2)
1) Negate the following proposition and find a counter-example showing that the statement is not true:
The product of any two integers is even.

2) Label the following statements by **True** or **False**
a- $(\forall x)(\exists y)(f(x)=k)$ defines f as a constant function.
b- $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x=y^2)$.
c- $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(x=y^2)$.
d- The negation of "All the functions are differentiable" is "Some functions are not differentiable".

Part 2: Sets and Mathematical Induction (15 points).

Exercise 4(True or False- 3 points)
Label the following statements as **True** or **False**. Correct the false statements.
1) $\sqrt{2} \in \{x \in \mathbb{Z}; x^2 = 2\}$.
2) $[1,3] \subset \{x \in \mathbb{R}; (x-1)(x-3) < 0\}$.
3) $1 \in P(\mathbb{Z})$.
4) $\{1\} \subset P(\mathbb{Z})$.
5) $(0,1) \subset [0,1]$.
6) $\phi \in P(\phi)$.

Exercise 5(Set Notations- 6 points= 2+2+2)
1) Write in set-builder notation the set of all polynomials with real coefficients and of degree 2.

2) Write as a roster the set $(A \cup B) \setminus (A \cap C)$, where $A = \{0, 2, 4, 6\}$, $B = \{3, 4, 5, 6\}$ and $C = \{0, 1, 2\}$.

3) Draw a Venn Diagramm for $A \cap \overline{B} \cap C$.

Exercise 6(Paragraph proof- 2 points)
Show that for any 2 sets A and B we have:
$A \setminus (A \cap B) = A \setminus B$.

Exercise 7(Mathematical Induction- 4 points)
Show that for any $n \geq 1$, we have:

$$1 + 3 + 5 + ... + (2n - 1) = n^2.$$

Part 3: Relations and Functions (13 points).

Exercise 8(Functions- 9 points =3+4+2)
1) Consider the function $p : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ defined by $p(x, y) = x$.
a- Is p a one-to-one-function?
b- Is p an onto function?
c- Is p an invertible function?
Justify your answers.

2) Consider the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = x^2$.
a-Find $f^{-1}([0, 1])$.
b-Find $f(\mathbb{R})$.

3) Given two odd functions f and g. Is the function fg odd or even? Justiy your answer.

Exercise 9(Equivalence relation- 4 points)
1) Show that the relation defined on $\mathbb{Z}$ by:

$$aRb \Longleftrightarrow a - b = 3p$$

is an equivalence relation.
2) Identify $[a]$ induced by R for any element a in $\mathbb{Z}$.