United Arab Emirates University
College Of Science
Department Of Mathematical Sciences
Linear Algebra & Eng.Appl.
Math 2220

Final Exam

January 6th, 2010

Student's name: ……………………………………

ID#:……………………………………………….

Instructor:……………………………… ***Section:***…………………

Exam regulations:

1) *This exam consists of* ***10 questions*** *and* ***8 pages*** *including this cover page.*
2) *The time limit is* ***120 minutes****.*
3) *Only regular scientific calculators are allowed, but may not be shared.*
4) *Show all your work in order to qualify for full credit.*
5) *Total grade is* ***25***

Questions	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7	Q.8	Q.9	Q.10	Total/25
Grades											
Outcomes	2	2	3	4	6	6	8	6,7	7	6,8	

Problem 1 (3 Points):
Solve the following system of linear equations,

$$2x_1 - 3x_2 = 1$$
$$-x_1 + 3x_2 = 0$$
$$x_1 - 4x_2 = 3$$

Problem 2 (2 Points):
For which value of k is the system inconsistent, show all the steps.

$$x + 2y = k$$
$$2x - k\,y = 4$$

1) All numbers k >0.
2) For k= - 4
3) For k = 2 or k=6.
4) For the odd value of k i.e for k=1, 3, 5, 7, 9, 11, 13, ...
5) None of the above.

Problem 3: (3 points)

Prove that A is non-singular and then find A^{-1}.

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 4: (3 points)

Let

$$A = \begin{bmatrix} -3 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(a) Find the eigenvalues of A.
(b) Find the corresponding eigenvectors.
(c) Find a matrix P that diagonalizes A.

Problem 5: (2 points)
Determine the following values:

1) Log (2+2i)

2) $(-1+i)^{2/3}$

Problem 6: (2 points)

(i) Write the following in the form a+ib:
$e^{(\pi/2)i+200\pi i}$

(ii) Write the following complex number in the polar form $z = 8 + i$

Problem 7: (3 points)

Evaluate the integral $\int_{\Gamma} f(z)dz$; if Γ is the half circle oriented counterclockwise, with radius 1, about the origin, from i to -i. and $f(z)=\frac{1}{\bar{z}}$.

Problem 8 (2 Points): True or False?

1) If $z = x + i\,y$ then $\left|e^{z}\right| = e^{x}$ [ ]
2) The complex exponential e^{z} is periodic of period $2\pi i$ [ ]
3) $|\sin(z)| \leq 1$, for all complex numbers z [ ]
4) There is exactly one value of the cube root of $(1+i)$ [ ]

Problem 9: (3 points)

Find u and v so that $f(z) = u(x, y) + iv(x, y)$, determine all points at which the Cauchy-Riemann equations are satisfied, and determine all points at which the function is differentiable

$$f(z) = \overline{z}^{2}$$

Problem 10 (2 Points):

a) Show that

$$\tan 3i = i \tanh 3$$

b) Sketch the curve, determine its initial and terminal points, whether it is closed or open

$$\Gamma(t) = 1 - 2i + 5e^{3it} \quad , \; for \quad 0 \le t \le \frac{\pi}{2}$$