United Arab Emirates University
College of Sciences
Department of Mathematical Sciences

Final Examination

MATH 1120 Calculus II for Engineering

Date: Wednesday, January 6, 2009
Time: 8:00 – 10:00 am (120 Minutes)

Instructor: ______________________ Section: ______________________

ID No: 200 ______________________ Name: ______________________

Instructions

1. This exam consists of 18 questions in 11 pages including this front page.
2. Write your section number, your instructor name, your name and ID above on this page.
3. Read the questions carefully before you start working.
4. Show all your work to get full credit in the multiple steps section.
5. Organize well your work and submit a clean copy as much as possible.
6. No extra sheet is allowed.
7. **NO GRAPHING CALCULATOR!**

Part A			Part B			Part C			
Outcomes	Question No	Points	Outcomes	Question No	Points	Outcomes	Question No	Points	Total
	1			7			14		
	2			8			15		
	3			9			16		
T2 T5	4		T7 T8 T9	10		T4	17		
	5			11			18		
	6			12					
				13					
	Total			Total			Total		

Part A: Multiple Choice Problems (2 Points Each)

1. Let $\boldsymbol{a} = \langle 1,\ 0,\ -2 \rangle$ and $\boldsymbol{b} = \langle 2,\ 1,\ -2 \rangle$. Then, $\text{Proj}_{\boldsymbol{b}}\, \boldsymbol{a}$ is

(A) $\frac{6}{5}\langle 1,\ 0,\ -2 \rangle$ (B) $\frac{5}{6}\langle 1,\ 0,\ -2 \rangle$ (C) $\frac{2}{3}\langle 2,\ 1,\ -2 \rangle$ (D) $\frac{3}{2}\langle 2,\ 1,\ -2 \rangle$

1. ________

2. If $f(x, y) = x^3y^2$, then $f_{xx} + f_{xy}$ is

(A) $12x^2y$ (B) $6x^2y + 6xy^2$ (C) $6xy + 3x^2y$ (D) $12xy^2$

2. ________

3. The equation of the circle $(x-1)^2 + y^2 = 1$ can be expressed in polar coordinates by

(A) $r = \theta$ (B) $r = 1 + \sin\theta$ (C) $r = 2\cos\theta$ (D) $r^2 = 1 + 2r\sin\theta$

3. ________

 Points Earned: ________

4. The area of the region bounded by the circle of radius 2 centered at the origin is calculated by

(A) $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x\,dx dy$ (B) $\int_{0}^{2\pi}\int_{0}^{2} r\,dr d\theta$

(C) $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} dy dx$ (D) $\int_{0}^{2\pi}\int_{-2}^{2} r\,dr d\theta$

4. ___________

5. $r(t) = \langle \sin(2t),\ \cos(2t) \rangle$ is perpendicular to $r'(t)$ when t has the value

(A) $t = 0$ (B) $t = \frac{\pi}{2}$ (C) None (D) Any value

5. ___________

6. The solid whose volume is given by $\int_{0}^{2\pi}\int_{0}^{1}\int_{0}^{\sqrt{1-r^2}} r\,dz dr d\theta$ is

(A) a cylinder (B) a sphere (C) a half sphere (D) a cube

6. ___________

 Points Earned: ___________

Part B: Multiple Steps Problems

Show all your work to get the full credit.

7. (6 points) Consider the vector–valued function $r(t) = \langle\, x(t),\ y(t),\ z(t)\,\rangle$, where $x(t) = 1 + \cos t$, $y(t) = 2$ and $z(t) = 2 + \sin t$.

(1) Find the relation between $x(t)$ and $z(t)$ independently to t.

(2) Sketch the graph of $r(t)$ and describe it.

 Points Earned: ____________

8. (10 points) Consider $f(x, y) = x^3y - xy + x^2$.

(1) Find all critical points of $f(x, y)$.

(2) Classify each critical point as a local maximum, a local minimum, or a saddle point.

Points Earned: __________

9. (7 points) Sketch the region R bounded by $x = 0$, $y = 1$ and $y = \frac{x}{2}$ and compute the double integral $\iint_R \frac{\sin y}{y}\, dA$.

Points Earned: ____________

10. (15 points)

(1) Let Q be the solid bounded by the upper hemisphere $z = \sqrt{4 - x^2 - y^2}$ and the xy–plane. Evaluate the triple integral $\iiint_Q e^{(x^2+y^2+z^2)^{3/2}}\, dV$.

(2) Consider a solid formed by $x^2 + y^2 = 4$ and $z = 1$ and $z = 2$. If the solid has the density $\rho(x, y) = e^{x^2+y^2}$, find its total mass.

11. (6 points) Consider $\boldsymbol{r} = \langle\, x,\ y,\ z\,\rangle$.

(1) Write down the formula of the norm of $\boldsymbol{r}$.

(2) Find the gradient of the norm of $\boldsymbol{r}$.

(3) Evaluate $\nabla\|\boldsymbol{r}\|$ at $(1,-1,0)$.

 Points Earned: __________

12. (12 points)

(1) Let C be the arc parametrically defined by $x(t) = 1$, $y(t) = t$ and $z(t) = t^2$, $0 \le t \le 1$. Compute the line integral $\int_C y\,dx + z\,dy + x^2 y\,dz$.

(2) Let $\boldsymbol{F}(x, y, z) = \langle\, y,\ x^2 y,\ y + 3z \,\rangle$.

i. Find the curl of $\boldsymbol{F}$.

ii. Find the divergence of $\boldsymbol{F}$.

13. (5 points) Let R be the region inside the upper half unit circle on the xy–plane, i.e., the region bounded by $y = 0$ and $y = \sqrt{1 - x^2}$. The region R is heated so that the temperature at a point (x, y) inside and on the boundary of R is given by $T(x, y) = x^2 - x + 2y^2$.

Find the hottest point (i.e., the point at which $T(x, y)$ has the the absolute maximum value) in R and the coldest point (i.e., the point at which $T(x, y)$ has the absolute minimum value) in R.

 Points Earned: ________

Part C: True or False with Justification (1.5 Points Each)

State whether or not each statement is true.

If it is not true, give a counterexample or explain why it is not true.

14. The graph of the vector–valued function $\boldsymbol{r}(t) = 2t\boldsymbol{i} + (1+3t)\,\boldsymbol{j} - 4t\boldsymbol{k}$ in $\mathbb{R}^3$ is a line. Here $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are standard basis vectors for $\mathbb{R}^3$. .... ☐

15. Any two lines in $\mathbb{R}^3$ are either parallel or intersecting. .... ☐

16. The graph of $z + y = 5$ in $\mathbb{R}^3$ is perpendicular to the x–axis. .... ☐

17. For two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, if $\boldsymbol{a} \cdot \boldsymbol{b} = 0$, then either $\boldsymbol{a} = \boldsymbol{0}$ or $\boldsymbol{b} = \boldsymbol{0}$. .... ☐

18. A sphere is the set of all points in $\mathbb{R}^3$ whose distance from a fixed point is constant.
.... ☐

Points Earned: ________