United Arab Emirates University

Department of Mathematical Sciences

Calculus I

Time: 120 minutes | Final Exam | June 5, 2010

Textbooks, Calculators, or Notes may **not** be used.

Name:

ID:

Section:

Instructor:

Show all your work.

1. **(5 Points: 2, 1.5, 1.5)**
   Find the following limits.

$$(a) \quad \lim_{x \to 2^+} \frac{x^2 - 4}{(x-2)^2}$$

$$(b) \quad \lim_{x \to 0} \frac{x^3}{\sin x - x}$$

$$(c) \quad \lim_{x \to \infty} \frac{x - 1}{\sqrt{4x^2 + 1}}$$

2. **(5 Points: 2, 1.5, 1.5)**

(a) Find $f'(x)$ if $f(x) = \int_1^{x^3} e^{t^2}\, dt$.

(b) If $\sin x + \cos y = \sin x \cos y$, find the derivative $\frac{dy}{dx}$.

(c) Use *the definition of the derivative* to find $f'(x)$ if $f(x) = \frac{1}{x^2}$.

3. **(5 Points: 2.5 for each part)**
(a) Use linear approximations to estimate $\sin 1$.

(b) Find the absolute maximum and absolute minimum of $f(x) = x^3 - 6x^2 + 9x + 2$ on the interval $[-1, 4]$.

4. **(5 Points.)**
Find the point on the parabola $y = 9 - x^2$ closest to the point $(-3, 9)$.

5. **(5 Points: 1 point for each part.)**
Let $f(x) = \frac{x-1}{x^2}$.
(a) Find the domain of $f(x)$.
(b) On which open intervals is f is increasing? decreasing? Find the coordinates (both x and y) of the local maxima and minima for f if there exist any.
(c) On which open intervals is f concave up? concave down? Find the coordinates (both x and y) of the inflection points of f if there exist any.
(d) Find all asymptotes, both vertical and horizontal ones.
(e) Sketch the graph of g.

6. **(5 Points: 1.5, 1.5, 2)**
Find the following integrals:

$$(a) \quad \int_0^1 \frac{e^{2x} - 1}{e^x}\, dx.$$

$$(b) \quad \int_0^1 \frac{(\sqrt{x} + 2)^3}{\sqrt{x}}\, dx.$$

(c) Find the area between the curves $y = x^2 - 1$ and $y = 1 - x$ on the interval $0 \leq x \leq 2$.

7. **(5 Point: 2.5 for each part.)**
a) Without solving the inverse, find the derivative of *the inverse function* of

$$f(x) = x^3 + 2x + 1 \quad \text{at } x = 1.$$

b) Simplify the expression

$$\cos(\tan^{-1} x).$$