United Arab Emirates University

Department of Mathematical Sciences

Calculus I

Time: 120 minutes | Final Exam | January 2, 2010

Textbooks or notes may **not** be used.

Name:

ID:

Section:

Instructor:

1. **(8 Points.)**
Find the following limits.

$$(a) \qquad \lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$$

$$(b) \qquad \lim_{x \to 1} \frac{\sqrt{x} - x^2}{1 - \sqrt{x}}$$

2. **(9 Point.)**
(a) Find the constant c that makes the function f continuous on $(-\infty, \infty)$:

$$f(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4, \\ cx + 20 & \text{if } x \geq 4. \end{cases}$$

(b) If $\sqrt{x+y} = 1 + x^2y^2$, find the derivative $\frac{dy}{dx}$.

3. **(9 Points.)**

(a) Find an equation for the tangent line to the curve

$$y = \int_4^{x^2} \sqrt{t^2 + 2}\, dt$$

at $x = 2$.

(b) Use *Newton's method* in two steps with $x_0 = 1$ to approximate $\sqrt[3]{7}$.

4. **(8 Points.)**
Find the absolute maximum and absolute minimum of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on the interval $[-2, 1]$.

5. **(8 Points.)**
A box with no top is to be built by taking a 6"-by-10" sheet of cardboard and cutting squares of equal size x out of each corner and folding up the sides. Find the value of x that maximizes the volume of the box.

6. **(12 Points.)**
(a) Find the horizontal and the vertical asymptotes for the function

$$f(x) = \frac{2x - 2}{x^2 - x}.$$

Let $g(x) = x^4 + 4x^3$.
b) Find the open intervals where g is increasing and those where g is decreasing.
c) Find local maxima and minima for g.
d) Find the open intervals where g is concave up and those where g is concave down.
e) Find all inflection points of g.
f) Sketch the graph of g.

7. **(8 Points.)**
(a) Determine the position function if the velocity function is $v(t) = 3e^{-t} - 2$ and the initial position is $s(0) = 0$.

(b) Compute the integral

$$\int_1^2 \frac{3x}{x^2 + 3}\, dx.$$

8. **(8 Point.)**
a) Without solving the inverse, find the derivative of *the inverse function* of

$$f(x) = 2x + \cos x \quad \text{at } x = 1.$$

b) Simplify the expression

$$\sec(\tan^{-1} x).$$