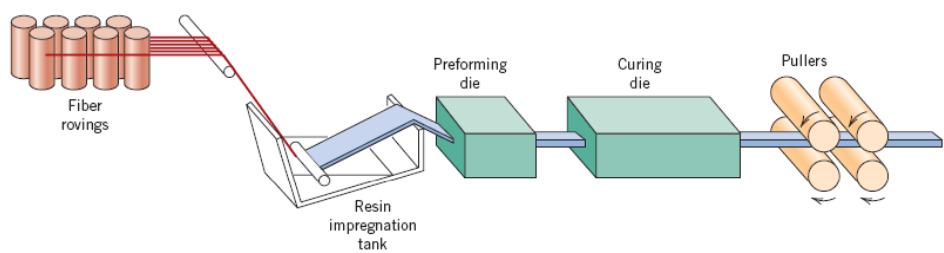
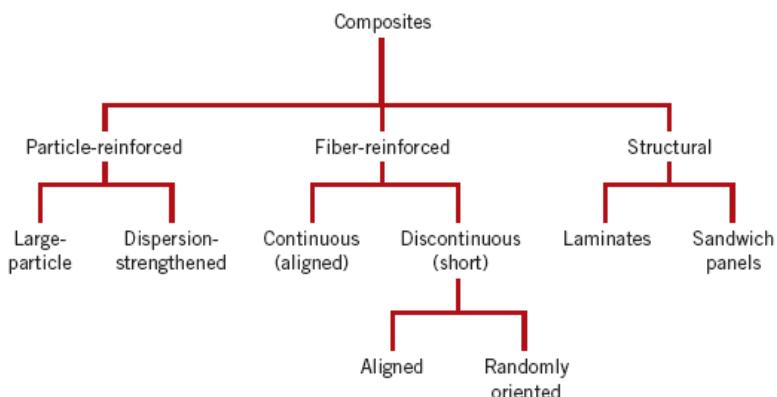


# Composite Materials

## Chapter 16



# Composite Strength: **Longitudinal Loading**

**Continuous fibers** - Estimate fiber-reinforced composite strength for long continuous fibers in a matrix ( $1 = V_m + V_f$ )

- Longitudinal deformation

$$\sigma_c = \sigma_m V_m + \sigma_f V_f \quad \text{but} \quad \varepsilon_c = \varepsilon_m = \varepsilon_f$$

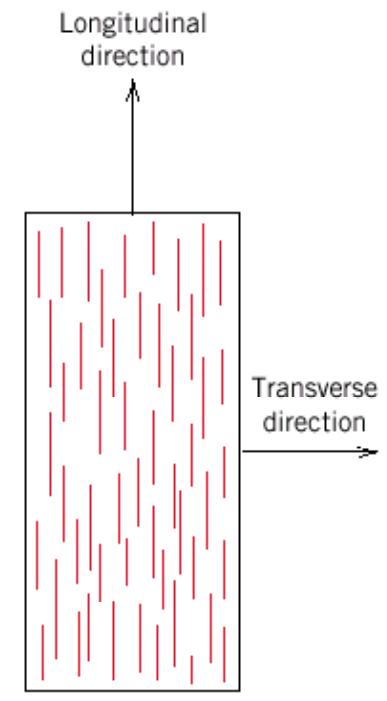
↑                           ↑  
volume fraction              isostrain

∴  $E_c = E_m V_m + E_f V_f$       longitudinal (extensional) modulus

$$\frac{F_f}{F_m} = \frac{E_f V_f}{E_m V_m}$$

*f* = fiber  
*m* = matrix

$$(TS)_c = (TS)_m V_m + (TS)_f V_f$$



# Composite Strength:

## **Transverse Loading**

- In transverse loading the fibers carry less of the load - isostress

$$\sigma_c = \sigma_m = \sigma_f = \sigma$$

$$\varepsilon_c = \varepsilon_m V_m + \varepsilon_f V_f$$

∴

$$\frac{1}{E_{ct}} = \frac{V_m}{E_m} + \frac{V_f}{E_f}$$

transverse modulus

**16.8** A continuous and aligned fiber-reinforced composite is to be produced consisting of 45 vol% aramid fibers and 55 vol% of a polycarbonate matrix; mechanical characteristics of these two materials are as follows:

	<i>Modulus of Elasticity [GPa (psi)]</i>	<i>Tensile Strength [MPa (psi)]</i>
Aramid fiber	131 ( $19 \times 10^6$ )	3600 (520,000)
Polycarbonate	2.4 ( $3.5 \times 10^5$ )	65 (9425)

Also, the stress on the polycarbonate matrix when the aramid fibers fail is 35 MPa (5075 psi).

For this composite, compute

- (a)** the longitudinal tensile strength, and
- (b)** the longitudinal modulus of elasticity

## Problem 16.8

- 45% aramid fiber
- 55% poly-carbonate matrix
- If the Stress of the matrix when fiber fails=35MPa.

Find for the composite:

1. Longitudinal TS
2. Longitudinal E

	E(GPa)	TS(MPa)	
Aramid fiber	131	3600	Maximum and the failure stress of fiber
Polycarbonate	2.4	65	Maximum but not the failure stress of matrix

# Solution:

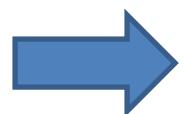
(a) The longitudinal tensile strength is determined using Equation 16.17 as

The failure stress  
of the matrix

$$\sigma_{cl}^* = \sigma_m^* (1 - V_f) + \sigma_f^* V_f$$

Maximum and  
the failure  
stress of fiber

$$= (35 \text{ MPa})(0.55) + (3600)(0.45)$$



$$= 1640 \text{ MPa}$$

Note: Usually composite material is designed that matrix fails before fibers

(b) The longitudinal elastic modulus is computed using Equation 16.10a as

$$E_{cl} = E_m V_m + E_f V_f$$

$$= (2.4 \text{ GPa})(0.55) + (131 \text{ GPa})(0.45)$$



$$= 60.3 \text{ GPa}$$

**16.9** Is it possible to produce a continuous and oriented aramid fiber–epoxy matrix composite having longitudinal and transverse moduli of elasticity of  $35 \text{ GPa}$  ( $5 \times 10^6 \text{ psi}$ ) and  $5.17 \text{ GPa}$  ( $7.5 \times 10^5 \text{ psi}$ ), respectively? Why or why not? Assume that the elastic modulus of the epoxy is  $3.4 \text{ GPa}$  ( $4.93 \times 10^5 \text{ psi}$ ).

## Problem 16.9

Is it possible to produce a continuous and oriented aramid fiber-epoxy matrix composite having longitudinal and transverse moduli of elasticity of 35 GPa and 5.17 GPa, respectively, given that the modulus of elasticity for the epoxy is 3.4 GPa?

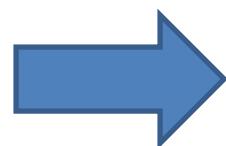
The Idea is if  $V_f$  calculated from longitudinal condition equal  $V_f$  calculated from transverse condition.

- Solution:

For the longitudinal modulus  $E_{cl}$  (using Equation 16.10b),

$$E_{cl} = E_m(1 - V_{fl}) + E_f V_{fl}$$

$$35 \text{ GPa} = (3.4 \text{ GPa})(1 - V_{fl}) + (131 \text{ GPa})V_{fl}$$

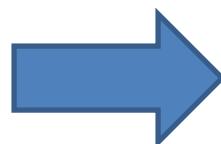


$$V_{fl} = 0.248.$$

Now, repeating this procedure for the transverse modulus  $E_{ct}$  (using Equation 16.16)

$$E_{ct} = \frac{E_m E_f}{(1 - V_{ft}) E_f + V_{ft} E_m}$$

$$5.17 \text{ GPa} = \frac{(3.4 \text{ GPa})(131 \text{ GPa})}{(1 - V_{ft})(131 \text{ GPa}) + V_{ft}(3.4 \text{ GPa})}$$



$$V_{ft} = 0.351.$$

Thus, since  $V_{fl}$  and  $V_{ft}$  are not equal, the proposed composite is *not possible*.

**16.12** In an aligned and continuous carbon fiber-reinforced nylon 6,6 composite, the fibers are to carry 97% of a load applied in the longitudinal direction.

**(a)** Using the data provided, determine the volume fraction of fibers that will be required.

**(b)** What will be the tensile strength of this composite? Assume that the matrix stress at fiber failure is 50 MPa (7250 psi).

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	<i>Modulus of Elasticity [GPa (psi)]</i>	<i>Tensile Strength [MPa (psi)]</i>
Carbon fiber	260 ( $37 \times 10^6$ )	4000 (580,000)
Nylon 6,6	2.8 ( $4.0 \times 10^5$ )	76 (11,000)

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## Problem 16.12

- Carbon fibers carry 97% load
- Nylon 6.6 matrix carry 3%
- If the load is applied in the longitudinal direction.
- Find for the composite:
  1. Fiber volume fraction for fiber.
  2. TS of the composite? Assume stress failure of the matrix is 55 MPa

	E(GPa)	TS(MPa)
Carbon fiber	260	4000
Nylon 6.6	2.8	76

- Solution:
- A)

From Equation 16.11

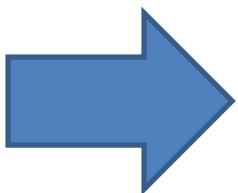
$$\frac{F_f}{F_m} = \frac{E_f V_f}{E_m V_m} = \frac{E_f V_f}{E_m (1 - V_f)}$$

Now, using values for  $F_f$  and  $F_m$  from the problem statement

$$\frac{F_f}{F_m} = \frac{0.97}{0.03} = 32.3$$

And when we substitute the given values for  $E_f$  and  $E_m$  into the first equation leads to

$$\frac{F_f}{F_m} = 32.3 = \frac{(260 \text{ GPa})V_f}{(2.8 \text{ GPa})(1 - V_f)}$$



And, solving for  $V_f$  yields,  $V_f = 0.258$ .

(b) We are now asked for the tensile strength of this composite. From Equation 16.17,

Failure stress of  
the matrix

$$\sigma_{cl}^* = \sigma_m'(1 - V_f) + \sigma_f^* V_f$$

$$= (50 \text{ MPa})(1 - 0.258) + (4000 \text{ MPa})(0.258)$$

$$= 1070 \text{ MPa}$$

Maximum and  
the failure  
stress of fiber

