MODELING OF MECHANICAL SYSTEMS

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Second Edition

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PREFACE

Modeling of the engineering systems is considered as one of the fundamental issue that needed by the engineers, since understanding the behavior of the engineering systems is an essential matter for the engineers to manufacture, modify as well as to maintain systems. Mechanical system is one of the most know engineering systems, where the systems move and subjected to dynamic loads. The authors would like to present their experiences in the modeling of engineering systems, especially mechanical engineering systems. This humble contribution is the first an anticipated series that will be published one by one to assist students in their study at the college of engineering.

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Chapter 1

1.1 FUNDAMENTALS

Newton's Law

First law: Every object in a state of uniform motion tends to remain in that state of motion (i.e. either is at rest or moves at a constant velocity) unless an external force is applied to it.

Second law: The sum of the forces on an object is equal to the total mass of that object multiplied by the acceleration of the object, i.e., the acceleration of a body is directly proportional to the net force acting on the body, and inversely proportional to its mass.

Thus, $\Sigma F = ma$

Where,

ΣF =is the net force acting on the object.

m= is the mass of the object, and

a =is the acceleration of the object.

Force and acceleration are both vectors (as denoted by the bold type). This means that they have both a magnitude (size) and a direction relative to some reference frame.

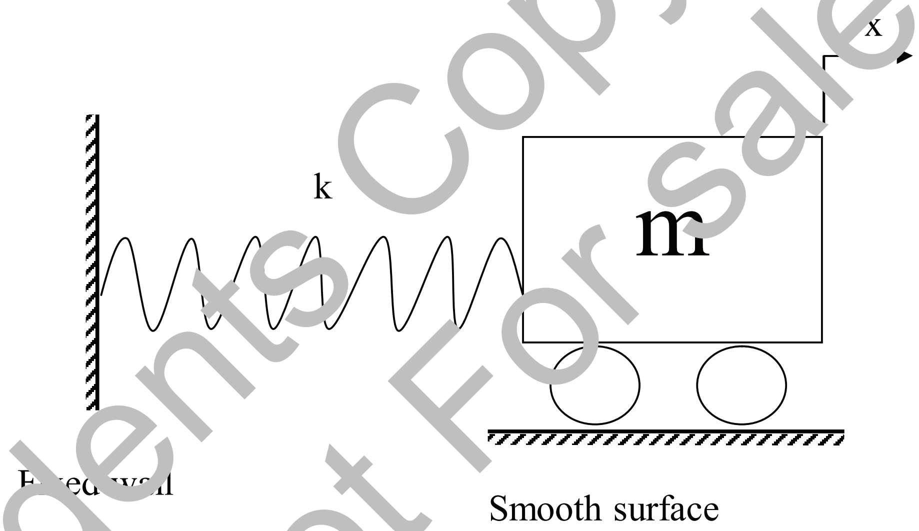
Third law: For every action there is an equal and opposite reaction.

1.2 BASICS OF THE MECHANICAL MODELING

1. Study the system diagram as well as understand the statement given.
2. Identify the elements connected with the mass, like dampers, springs, levers, ropes, etc.
3. Assume the direction of the motion in case is not shown.
4. Draw free body diagram (FBD) of the system.
5. Disconnect (i.e., disassemble, cut) the elements that connected with the mass and replace each one of them with the corresponding reactional forces using Newton’s third law.
6. Apply Newton’s second law of motion.
7. Establish the mathematical model of the system.
8. Organize the system of equations.

1. Problems

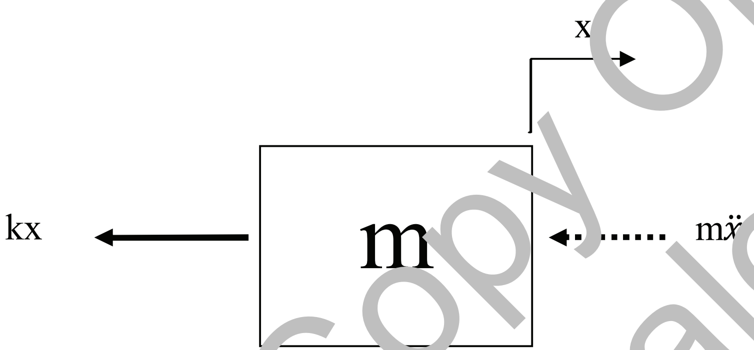
Find the equation of motion of the following system which consist of a mass connected with spring of stiffness k. Knowing that the condition of the contact surface of the mass with the ground is considered as a frictionless.



Solution

Draw the free body diagram of the system by disconnecting the spring from the mass and replacing the element (i.e., Spring) by the corresponding reactional force.

F.B.D



x = displacement (m)

$x' = \frac{dx}{dt}$ = velocity (m / s)

$x'' = \frac{d^2x}{dt^2}$ = acceleration (m / s^2)

Apply Newton's Second Law :

$\overset{+}{\rightarrow} \sum F_x = m\ddot{x}$

$-K\,x = m\ddot{x}$

$m\ddot{x} + kx = 0$ eq. of motion

Knowing that the system of equation shown above is named as single degree of freedom, since there is one mass in the system and moves in one direction. Besides it is considered as undamped system, since no friction or any damping element exist in the system.

1.2. For the problem 1.1, find expression for the natural angular frequency of the same system.

Natural Frequency : w_n (circular)

From previous problem :

$$m\ddot{x} + \ k.x = 0$$

$$\ddot{x} + w^2.x$$

Where natural frequency (circular) :

$w_n = \sqrt{\frac{k}{m}}$ Where ,

k = spring stiffness (N / m)

m = mass (Kg)

w_n depends only on the mass & stiffness of the system , where are properties of the system .

1.3. Suppose that the natural frequency is needed for problem 1.2

w_2 : angular frequency (rad / sec)

f_n : natural frequency (hertz) , (Hz)

Where ,

$$f_n = \frac{wn}{2\pi}$$

So for the case of $mx + k\ddot{x} = 0$ the natural frequency becomes :

$$f_n = \frac{}{2\pi}\sqrt{\frac{k}{m}}$$

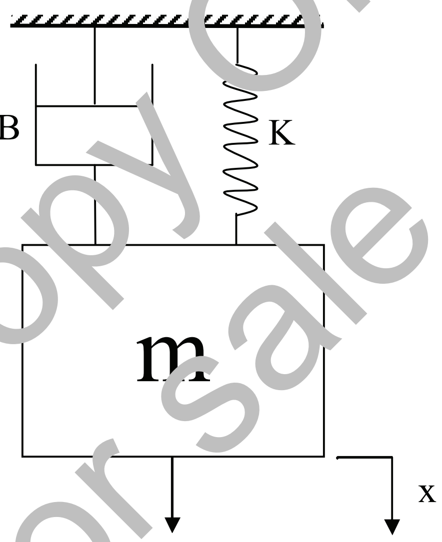
Note : period of oscillation can be defined as τ , where :

$$\tau = \frac{}{f}$$

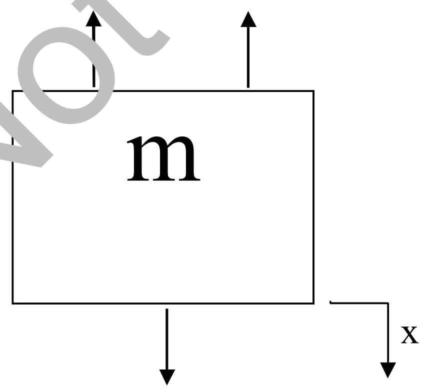
1.4. A vertical hanged mass m is connected by a couple of elements, a spring of stiffness k and a damper of a coefficient B, as shown below. The system itself is subjected to an external force F(t) on the mass and is directed toward down. Find the mathematical model of the system.

B : Damping coefficient

F (t) : External force



The first step of the job is to draw the free diagram of the system.

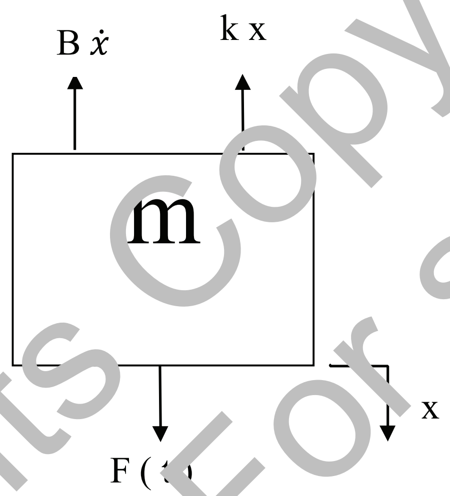


The next step is to apply Newton's second law on the FBD:

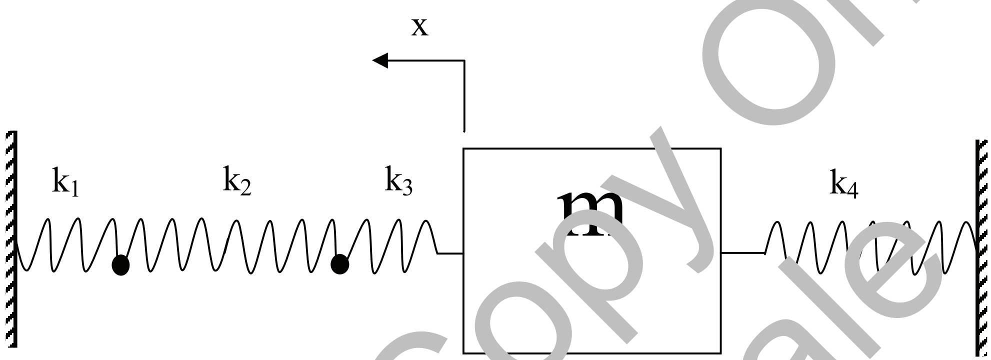
$-B\dot{x} - kx + F(t) = m\ddot{x}$

$m\ddot{x} + B\dot{x} + kx = F(t)$

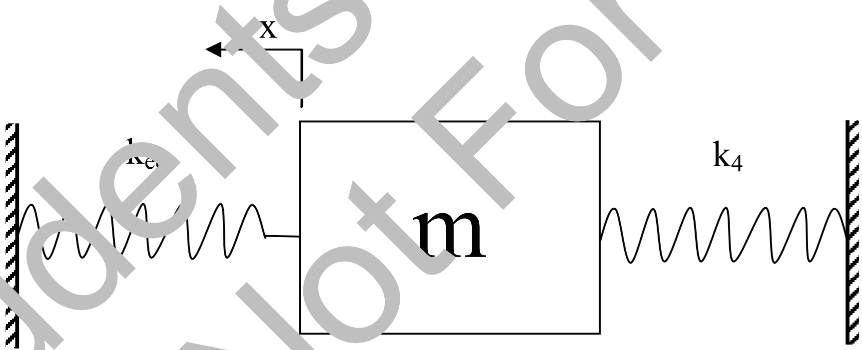
This eq. represents of mathematical model of a single degree of freedom system (SDOF) with viscous damping (forced) .

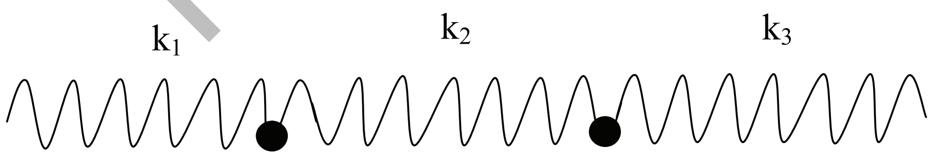


5 A mass m is connected from the right side with a spring of k4, and from the left side with three springs (k1 to k3) in series as illustrated below. Find the equation of motion using Newton's second law.



It is important to find the equivalent stiffness of the springs in series

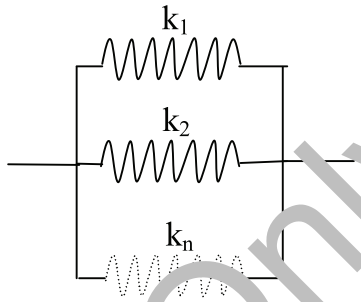




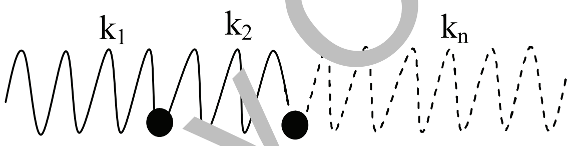
Three springs in series

Spring in parallel

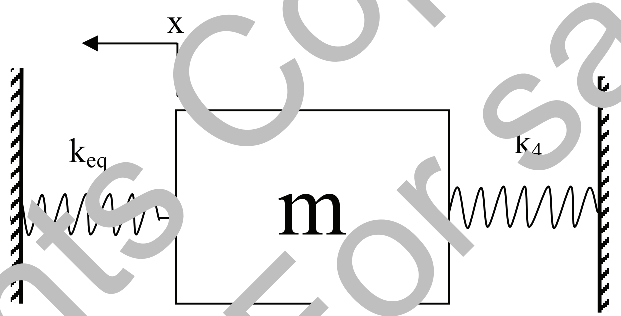
$k_{eq} = k_1 + k_2 + \cdots + k_n$



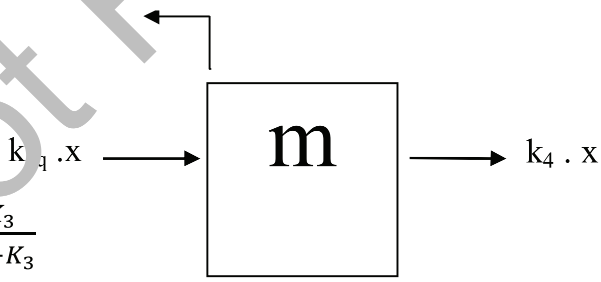
Springs in series



$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n}$$



Drawing F . B . D



Where $keq = \frac{K_1.K_2.K_3}{K_1+K_2+K_3}$

Applaying Newtons second law :

$\overset{+}{\leftarrow} \sum F\, x = m\ddot{x}$

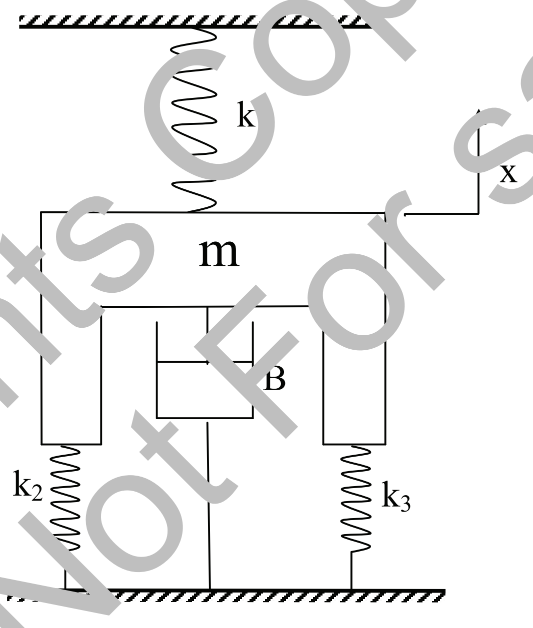
$-keq.x - k_4.x = m\ddot{x}$

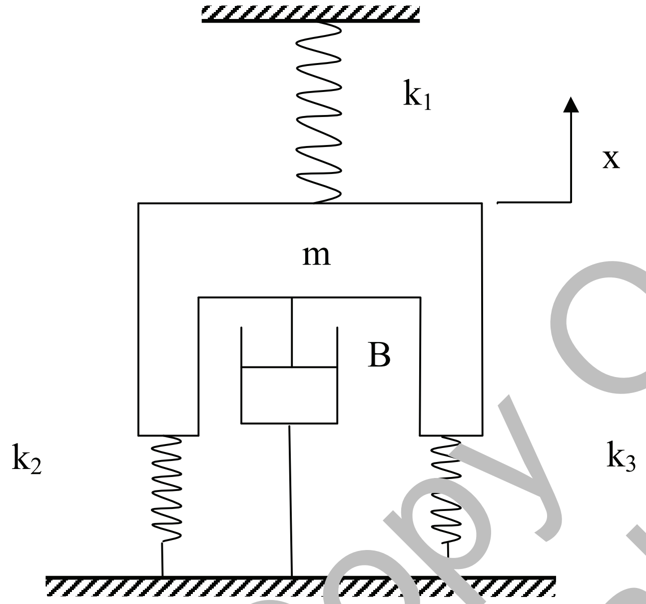
$m\ddot{x} + x(k_4 + keq) = 0$

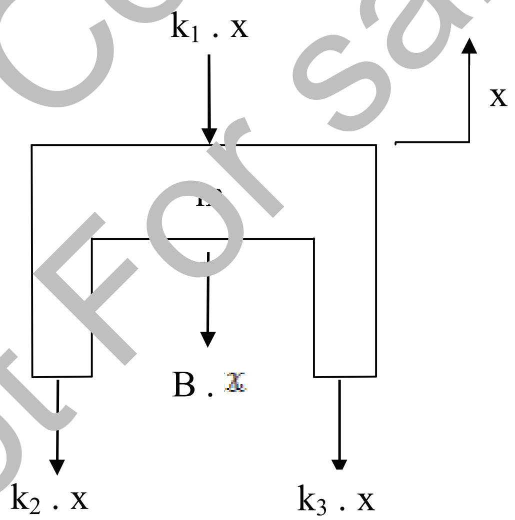
$m\ddot{x} + k_T.x = 0$

Where $k_T = k_4 + k_{eq}$

6 A mass has a shape of inverted U shape and is moving vertically toward up, as shown in the figure below. The mass is connected with two springs and a damper from the lower side, and with one spring from the top. Develop the mathematical model of the system.







Newton's 2nd law of motion :

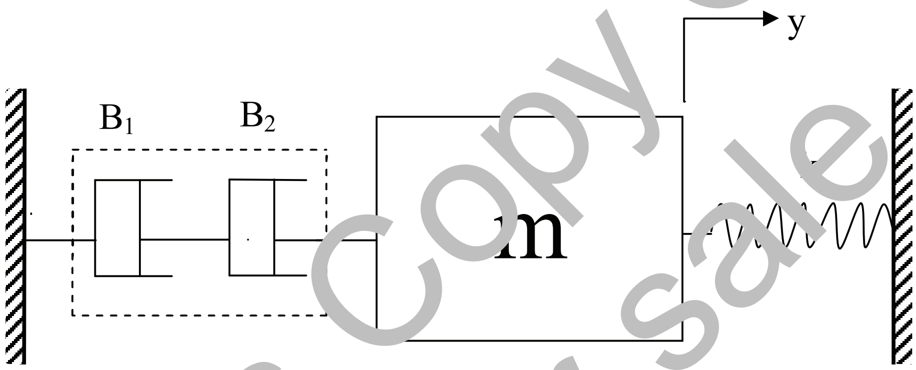
$+\uparrow \sum F\ x = m\dot{x}$

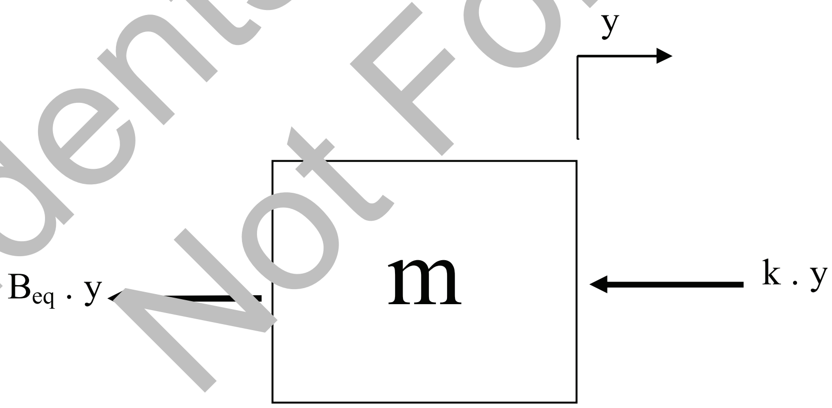
$-k_1.x - k_2.x - k_3.x - B.\dot{x} = m\ddot{x}$

$m\ddot{x} + B\dot{x} + k_{eq}.x = 0$

k_{eq} = ? equivalent stiffness of system

7 Two dampers B_1 and B_2 are in series and connected to the left side of the mass m, as shown in the figure below. The mass moves to the left, and it is connected with a spring of stiffness k from the left side. Draw the FBD of the system and find the mathematical model.





Dampers in parallel : $B_{eq} = B_1 + B_2 + \cdots$

Dampers in series : $\frac{1}{B_{eq}} = \frac{1}{B1} + \frac{1}{B2} + \cdots$

Applying Newton's 2nd law :

$$\overset{+}{\leftarrow} \sum F\ y = m\ddot{y}$$

$$-B_{eq}.y - K.y = m\ddot{y}$$

$$m\ddot{y} + B_{eq}.y = 0, where\ B_{eq} = \frac{B_1.B_2}{B_1+B_2}$$

Chapter 2

Solved Problems

2.1. Determine the natural frequency of the system shown in Figure (2.1)

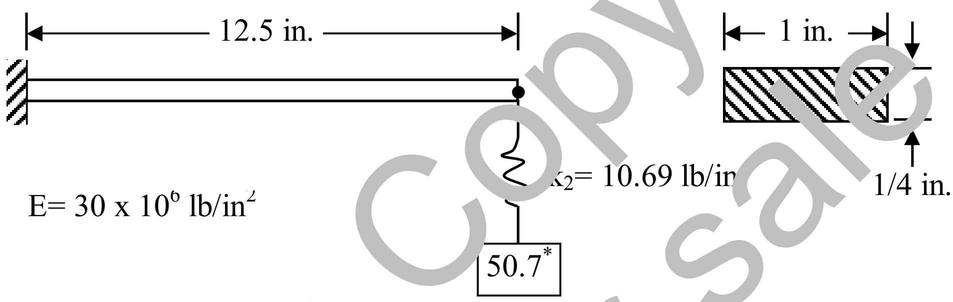


Figure (2.1)

Solution:

From Appendix

The spring constant k_1, which represent the static force required to produce a unit deflection at the free end, is equal to

$$k_1 = \frac{3EI}{L^3} = \frac{3 * 30 * 10^6 * 1 * (0.25)^3 /12}{(12.5)^3} = 60\ lb/in$$

The equivalent spring constant k_e for the system in which k_1 and k_2 are assembled in series is given by

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$= \frac{1}{60} + \frac{1}{10.69}$$

$$\therefore\ k_e = 9.07\ lb/in$$

The natural frequency is then given by

$$\omega = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{k_e}{W/g}}$$

$$= \sqrt{\frac{9.07}{50.7\ /386}} = 8.31\ rad/sec$$

$$\therefore f = \frac{1}{t} = \frac{W}{2\pi}$$

$$= \frac{8.31}{2\pi} = 1.32\ cps.$$

2.2. The rigid steel frame shown in Figure (2.2) is subjected to a horizontal dynamic force as shown. It is required to determine the natural frequency of the frame. Assume the mass of the columns is negligible and the girder is sufficiently rigid to prevent rotation at the tops of the columns.

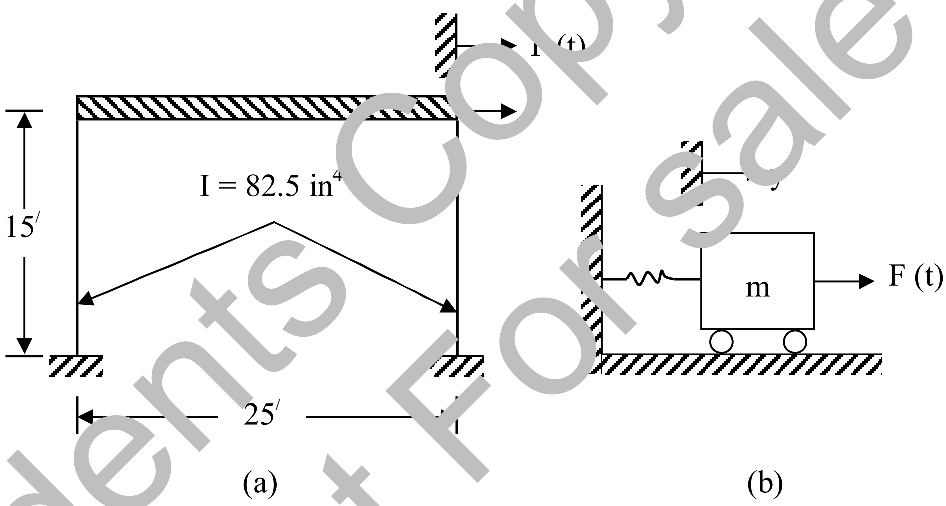


Figure (2.2)

Solution:

The frame may be modeled by the spring- mass system shown in Figure 2.2(b).

From Appendix

$$k = 2 * \frac{12EI}{L^3}$$

$$= 2 * \frac{12 * 30 * 10^6 * 82.5}{(15 * 12)^3} = 10185.2\ lb/in$$

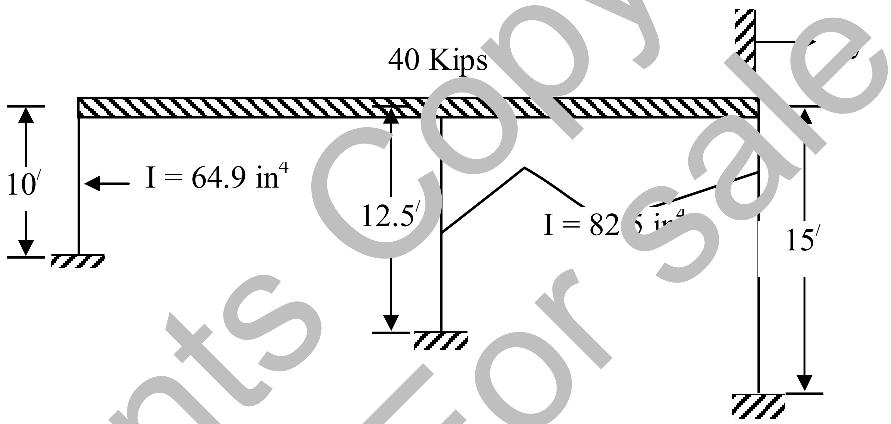
$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{k}{W/g}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{10185.2}{5000.386}} = 4.463\ cps.$$

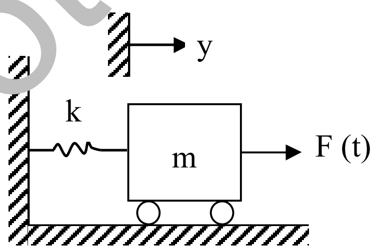
2.3. For the rigid steel frame shown in figure:

(i) Determine the natual frequency in the horizontal model. Assume the mass of the columns is negligible and the horizontal girder is sufficiently rigid to prevent rotation at the tops of the columns.

(ii) If the system has 15% of critical damping, compute the damped natural frequency.

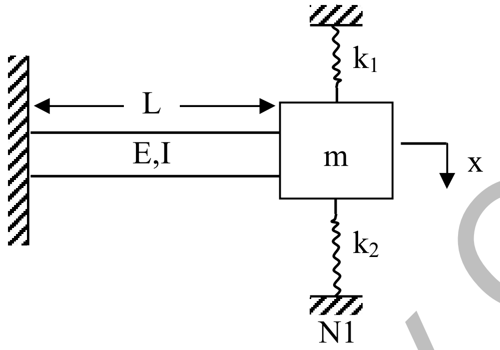


(a)



(b)

2.4. A Cantileaer beam with a mass as shown is connected with two springs. Find the mathematical model of the system.



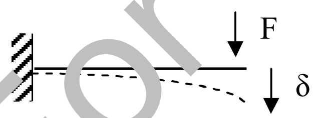
Solution:

The stiffress of the beam can be expressed by:

$$F = k.\Delta \Rightarrow k = \frac{F}{\Delta}$$

Since for cantilever beam, deflection equal:

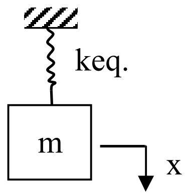
$$\delta = \frac{F.L^3}{3EI}$$



$$\therefore k = \frac{F}{\delta} = \frac{3EI}{L^3}$$

Both K_1 and K_2 have same displacement , series means they are in parallel , whereas wrongly considered in sense based on the figure.

$K_{equivalent} = k + k_1 + k_2$

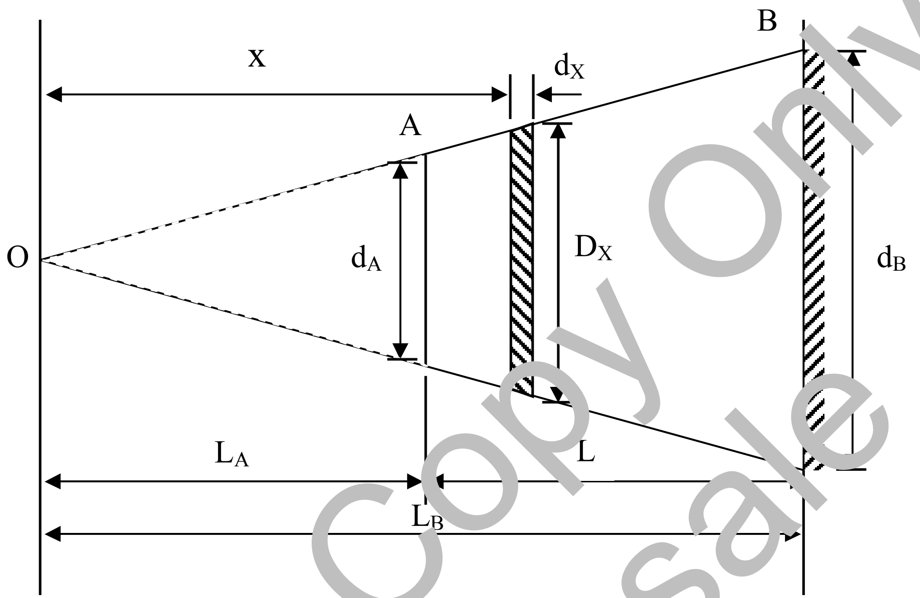


$$+\downarrow \sum F = m\ddot{x}$$

$$-Keq.x = m\ddot{x}$$

$$m\ddot{x} + k_{eq}.x = o \quad (Free\ Vibration)$$

2.5. A tapered bar subjected to axial load F, derive the axial stiffness K of the bar from first principle.



Solution:

The ratio of diameters d_A and d_B with respect to their corresponding lengths, L_A and L_B, can be expressed by:

$$\frac{L_A}{L_B} = \frac{d_A}{d_B}$$

By triangular similarities, a ratio for the diameter D_x can be obtained as well with respect to the origin point o:

$$\frac{D_X}{d_A} = \frac{X}{L_A} \quad \Rightarrow \quad D_X = \frac{d_A . X}{L_A}$$

So the cross sectional area for any distance with respect to the origin point O, is given by:

$$A(X) = \frac{\pi}{4}\ {D_x}^2 = \frac{\pi}{4}\ \frac{{d_A}^2.X^2}{{L_A}^2}$$

Basedon the general formula used to calculate the elongation for an axial bar with continusly varying loads or dimensions:

$$\delta = \int_o^L d\delta = \int_o^L \frac{N(X).d_x}{E.A(X)}$$

N(X) = Internal axial force acting at cross section area A(X)

The obtained expression of A(X) can be substituted in δ equation , which yield

$$\delta = \int \frac{N(X)d_x}{EA(X)} = \int_{L_A}^{L_B} \frac{F d_x(4{L_A}^2)}{E(\pi {d_A}^2 X^2)} = \frac{4.F.{L_A}^2}{\pi E.{d_A}^2} \int_{L_A}^{L_B} \frac{d_x}{x^2}$$

By integration for the limit:

$$\delta = \frac{4.F.{L_A}^2}{\pi E.{d_A}^2} \left[-\frac{1}{x}\right]_{L_A}^{L_B} = \frac{4.F.{L_A}^2}{\pi E.{d_A}^2} \left(\frac{1}{L_A} - \frac{1}{L_B}\right)$$

Which can be simplified to:

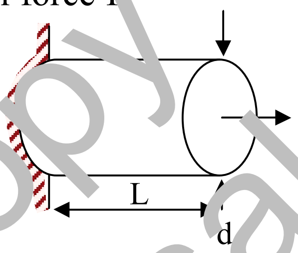
$$\frac{1}{L_A} - \frac{1}{L_B} = \frac{L_B - L_A}{L_A.L_B} = \frac{L}{L_A.L_B}$$

$$\delta = \frac{4.F.L}{\pi E.{d_A}^2}\left(\frac{L_A}{L_B}\right)$$

Eventually, $\frac{L_A}{L_B} = \frac{d_A}{d_B}$, therefore

$\delta = \frac{4.F.L}{\pi E.d_A.d_B}$ For tapered bar with circular cross sectional area.

To verify this formula in case we have a uniform circular bar with dimeter d subjected to axial force F.



$\delta = \frac{4.F.L}{\pi E.d.d} = \frac{F.L}{\frac{\pi}{4}d^2.E} = \frac{F.L}{A.E}$

Which is the classical equation used for estimating δ for axially loaded members

For the stiffness,

$F = k.\,\delta$

$k = \frac{F}{\delta}$, from the driven formula:

$\frac{F}{\delta} = \frac{\pi E.d_A.d_B}{4.L}$

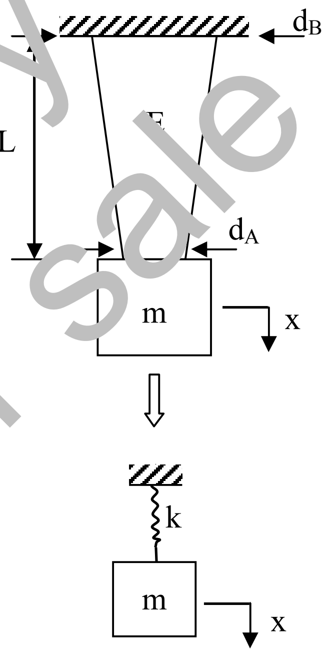
$\therefore K = \left(\frac{\pi E}{4L}\right).d_A.d_B$

The equivalent stiffness of tapered axial bar

2.6. For the system shown, find the mathematical model of the spring mass equivalent.

If the tapered bar has length of 500 cm and 20 cm, 10 cm for the bigger and the simaller diameters respectively and made of steel of E= 200 GPa, estimate the error result from approximating the tapered bar into a circular bar with an average diameter of the tapered diameters.

For tapered bar , the stiffness



$$k = \left(\frac{\pi E}{4L}\right) d_A . d_B$$

$$+\downarrow \sum F = m\ddot{x}$$

$$-kx = m\ddot{x}$$

$$m\ddot{x} + kx = 0$$

For tapered bar

$$k = \frac{\pi \times 200\times10^9 N/m^2 \times 0.2\times0.1 m^2}{4 \times 0.5m}$$

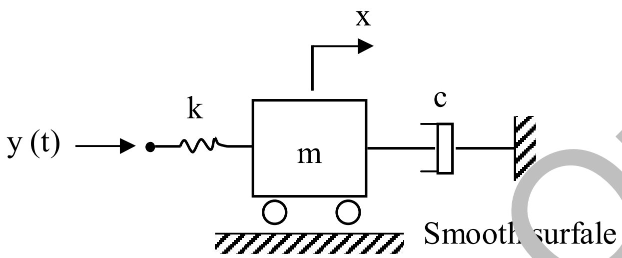
$$k = 6.28\ GN/m$$

Using average diameter which is 15cm

$$k = \frac{\pi \times 0.15^2 m^2 \times 200\times10^9 N/m^2}{4 \times 0.5m} = 7.07\ GN/m$$

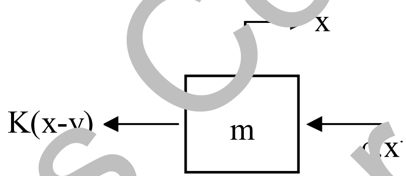
Error % $= \frac{7.07-6.28}{7.07} \times 100\% = 12.6\%$ which is relatively big error

2.7. Formulate the mathematical model of the following system which is subjected to displacement y(t) on the spring.



Solution:

By drawing free body diagrame, it is shown that the spring k is subjected to a couple of displacements y and x , means it will cause a force with respect to the net difference between the two displacements.



Where mass displacement is considered greater than external displacement.

$$\xrightarrow{+} \sum F = m\ddot{x}$$

$$-c.\dot{x} - k(x-y) = m\ddot{x}$$

$$-c.\dot{x} - kx + ky = m\ddot{x}$$

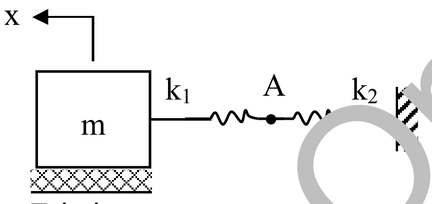
Where ky is considered as an external force.

By simplifying the above equation

$$m\ddot{x} + c.\dot{x} + kx = k.y$$

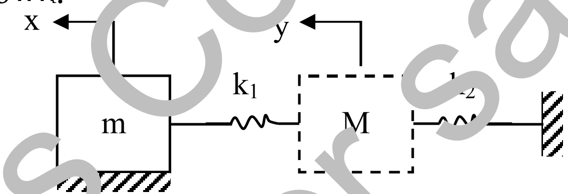
2.8. Amass is connected with two spring in series.

If it is desired to find the displacement of point A, Find an expression for this task.

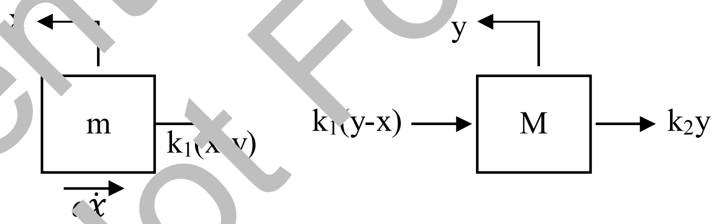


Solution:

In the normal situation , if the equivalent stiffness for k_1 and k_2 considered, finding displacement of point A becomes impossible. In this situation, since target is displacement of a point, a false mass M can be introduced at point A to facilitate this mission as shown.



By drawing free body diagram of the two masses.



For mass m , by applying newton's second law yield:

$$\overset{+}{\leftarrow} \sum F = m\ddot{x}$$

$$-k_1(x-y) - c.\dot{x} -= m\ddot{x}$$

$$m\ddot{x} + c.\dot{x} + k_1(x-y) = o \quad \dots\dots\dots\dots\dots \quad (1)$$

For mass m

$$\overset{+}{\leftarrow} \sum F = m\ddot{y}$$

$$-k_1(y - x) - k_2 y = m\ddot{y} \; \ldots\ldots\ldots\ldots\ldots (2)$$

Since M is false, than eq_2 becomes:

$$k_2 y + k_1(y - x) = o \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

From eq_3 a relation of x with respect to y can be found

$$k_2 y + k_1 y - k_1 x = o$$

$$y = \frac{k_1 . x}{k_1 + k_2} \quad \ldots\ldots\ldots\ldots (4)$$ which can be substituted in eq. (1) to eliminate y:

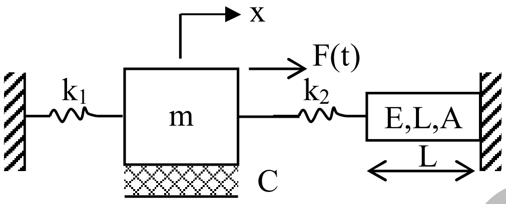
$$m\ddot{x} + c\dot{x} + k_1 x - \frac{{k_1}^2 x}{k_1 + k_2}$$

$$m\ddot{x} + c\dot{x} + x\left(k_1 - \frac{{k_1}^2}{k_1 + k_2}\right) = o$$

$$m\ddot{x} + c\dot{x} + x\left(\frac{k_1 k_2}{k_1 + k_2}\right) = o \qquad - eq_5$$

Where once eq. (5) is solved , the resultsol x can be used to find the values of point y whith represents displacement of point A.

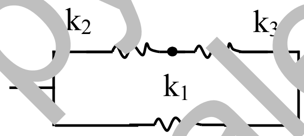
2.9. Establish mathematical model of the following system



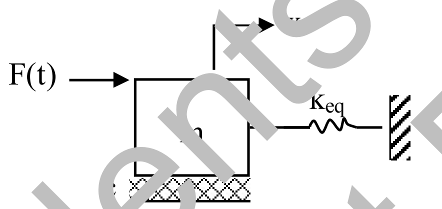
$k_3 = \frac{A.E}{L}$

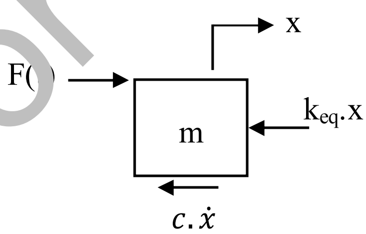
k_2 and k_3 in series with k_1 in parallel

$k_{eq} = k_1 + \frac{k_2.k_3}{k_2+k_3}$



Free body diagram



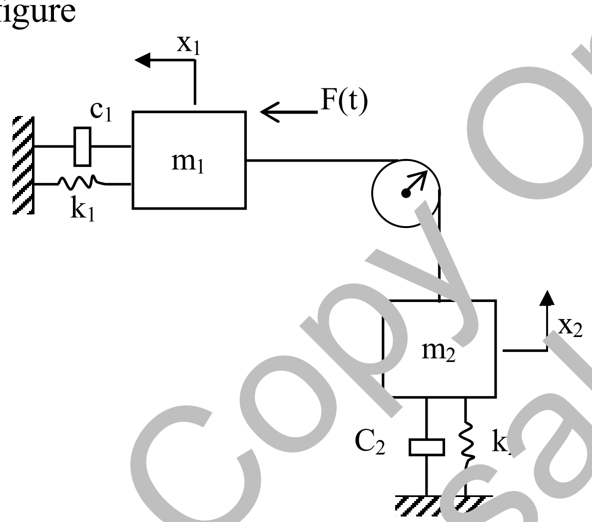


$\overset{+}{\rightarrow}$ $\sum F = m\ddot{x}$

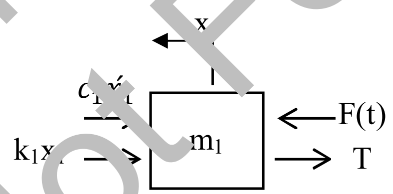
$F(t) - k_{eq}.x - c.\dot{x} = m\ddot{x}$

$m\ddot{x} + c.\acute{x} + k_{eq}.x = F(t)$ (Forced vibration)

2.10. Develop mathematical model of two masses m_1 and m_2 connected by a rob with high stiffness The rob passes over a drum which can be considered as negligible inertia, as shown in figure



By drawing free body diagram of mass one through disconnecting elements as well as disconnect the rob.

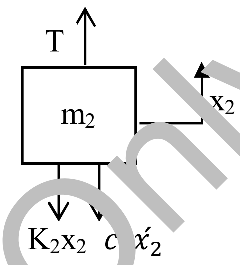


$\overset{+}{\leftarrow} \sum F = m_1 x_1$

$F(t) - c_1\dot{x}_1 - k_1 x_1 - T = m_1\ddot{x}_1$

$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 + T = F(t) \quad - eq_1$

For mass two, the same procedure is followed for free body diagram



$+\uparrow \sum F = m_2\ddot{x}_2$

$T - k_2x_2 - c_2\acute{x}_2 = m_2\ddot{x}_2$

$m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 - T = o \quad - eq_2$

Since the rob with high stiffness , means

$x_1 \approx x_2 \Rightarrow \dot{x}_1 \approx \dot{x}_2 \Rightarrow \ddot{x}_1 \approx \ddot{x}_2 \quad - eq_3$

From eq_2 , the tension in the rob is:

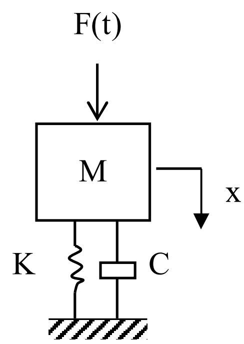
$T = m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 \quad - eq_4$

Subistitute eq_3 and eq_4 in eq_1 gives:

$\ddot{x}_1(m_1 + m_2) + \dot{x}_1(c_1 + c_2) + x(k_1 + k_2) = F(t)$

$M\ddot{x}_1 + \dot{x}_1c + xk = F(t)$

Where



$M = m_1 + m_2$

$C = c_1 + c_2$

$K = k_1 + k_2$

And the system can be represented by the figure shown.

Problems

3.1. If the weight W of the system shown in figure P3.1 has an initial displacement of y_o=1.0 and an initial velocity $\dot{y}_o$ = 20 in /sec., determine the displacement and the velocity 1 sec later. Assume W=3000 lb, EI= 10^8 lb-in^2 and k = 2000 lb / in.

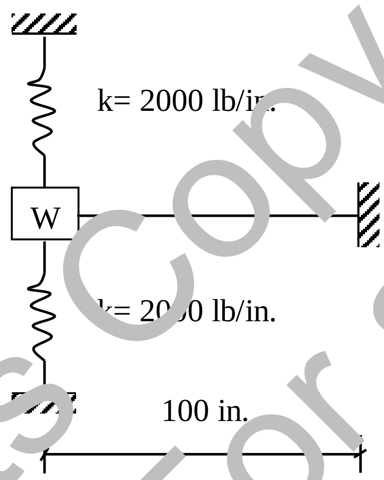


Figure P3.1

3.2 A vertical pole 100 in. long and fixed at the base supports a concentrated weight of 1000 lb. at its upper end as shown in figure P3.2 neglecting the mass of the vertical pole. If the modulus of elasticity E = 30 x 10^6 psi. and the moment of inertia I = 30 in.^{4} find the natural period and the natural frequency. Assume that the effect of gravity is small and non linear effects may be neglected.

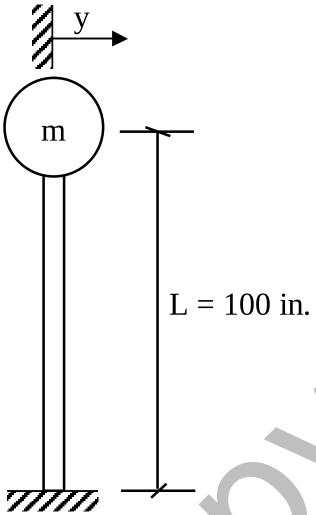


Figure P3.2

3.3. A fixed beam of span L flexural rigidity EI is carrying a concentrated weight W at the centre of the span. Determine the natural period and natural frequency. Neglect the mass of the beam.

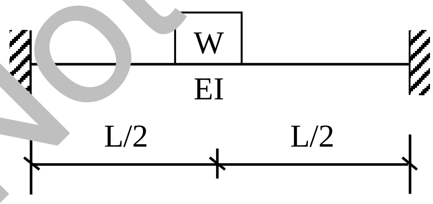


Figure P3.3

3.4. A mass of 36 lb is held by three springs as shown in Figure P3.4. neglecting the rolling friction in the floor as are the inertial effects of the rollers, determine the natural frequency.

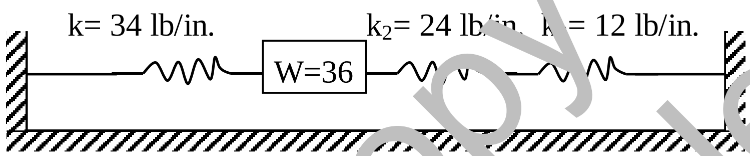


Figure P3.4

3.5. Assume single degree of freedom in the horizontal direction determine the natural frequency for horizontal motion in the plane of each of the steel frames shown in Figure P3.5. Assume the horizontal girder to be infinitly rigid and neglect the mass of the columns.

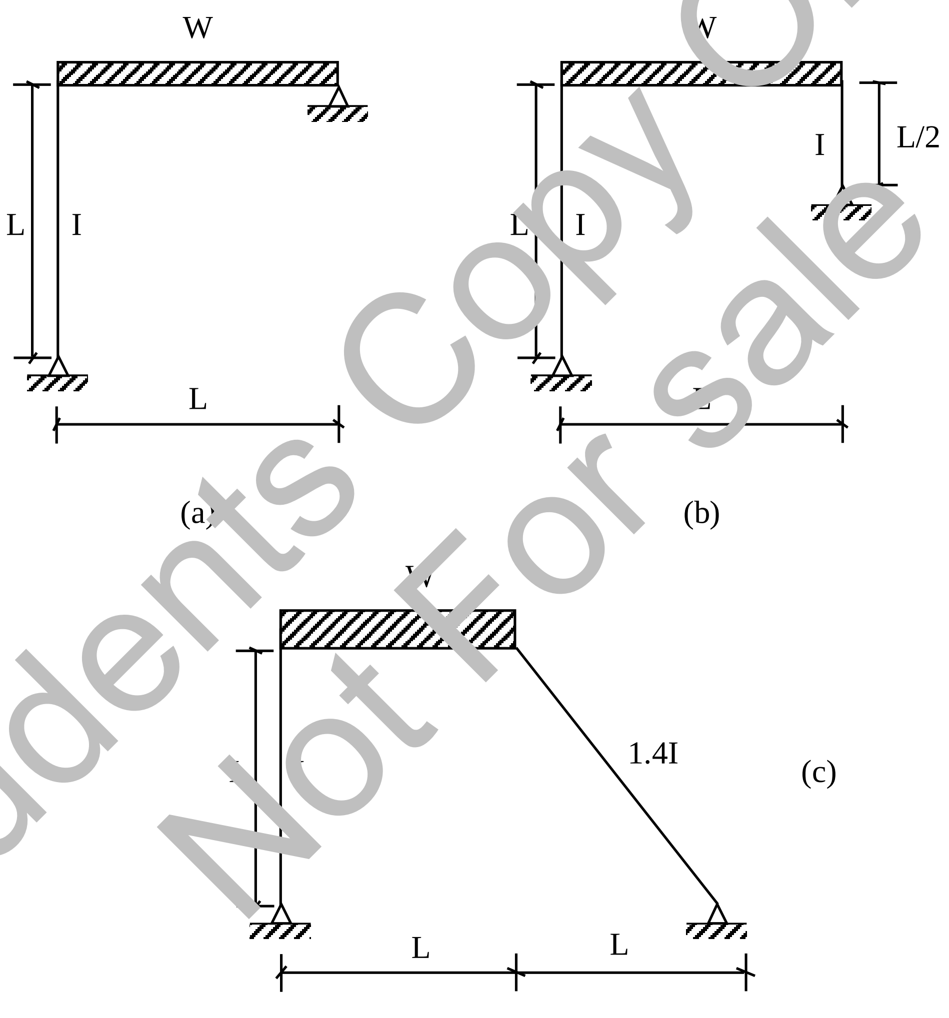


Figure P3.5

3.6. Assume a single degree of freedom in vertical direction determin the natural period of the system shown in Figure P3.6, Assume E is equal to unity.

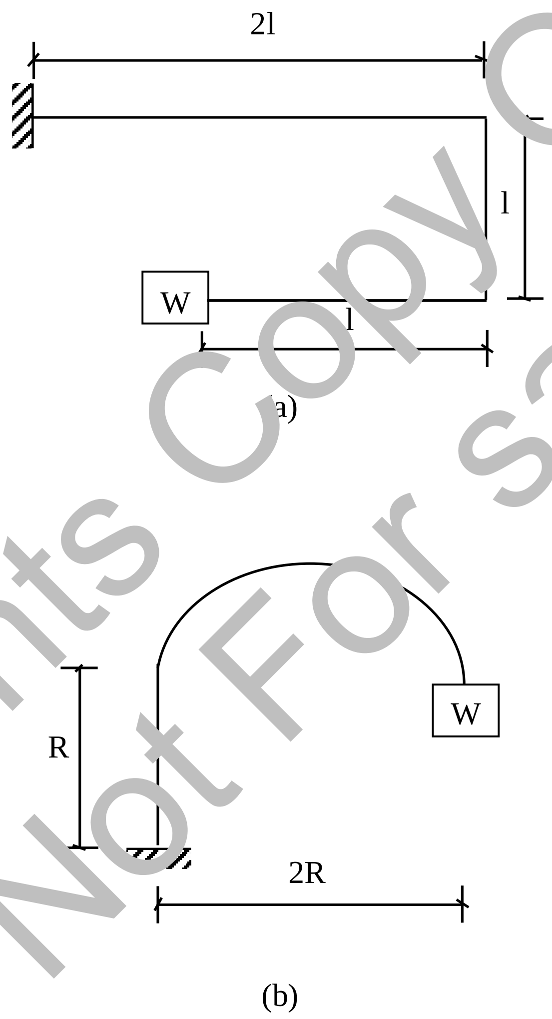


Figure P3.6

3.7. Determine the natural period of the propped cantilever shown in Figure P3.7

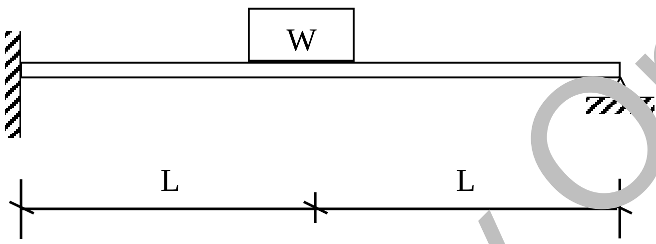


Figure P3.7

3.8. Develop the mathematical model of the following cases through drawing free body diagram.

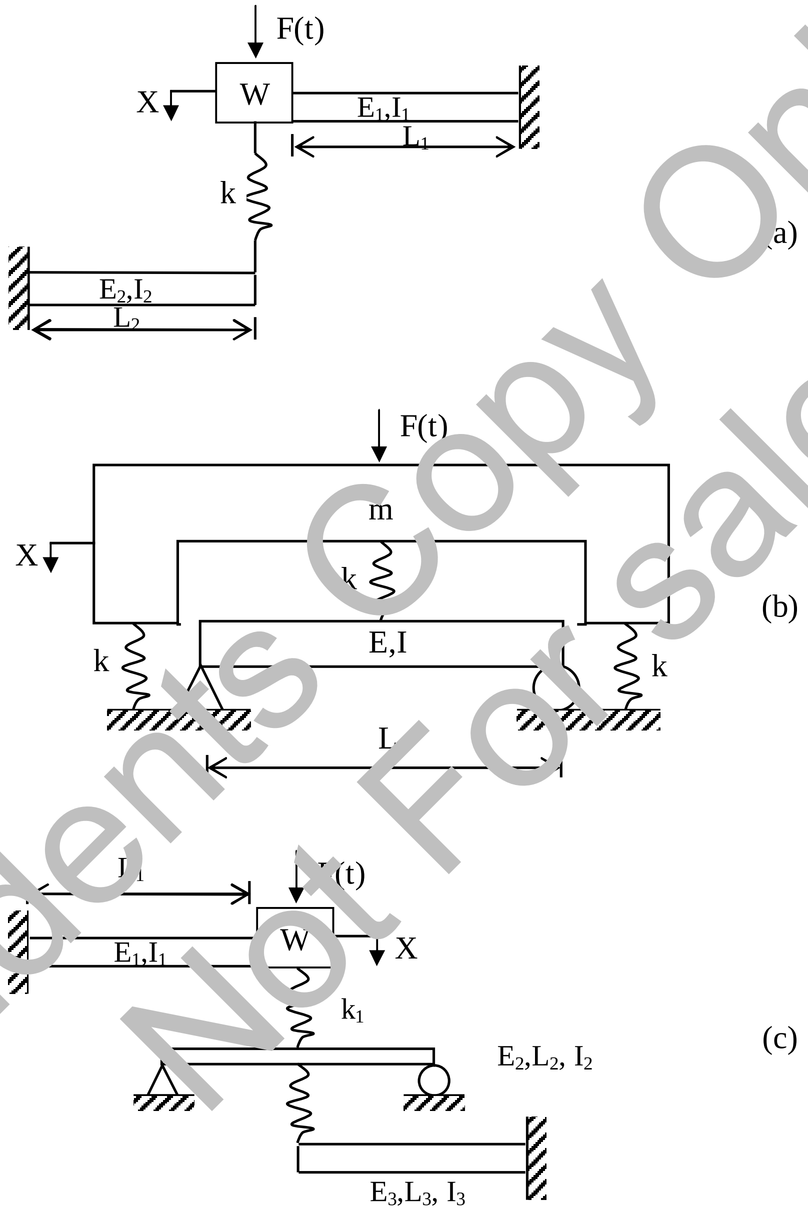


Figure P3.8

3.9. Establish mathematical model of the following cases:

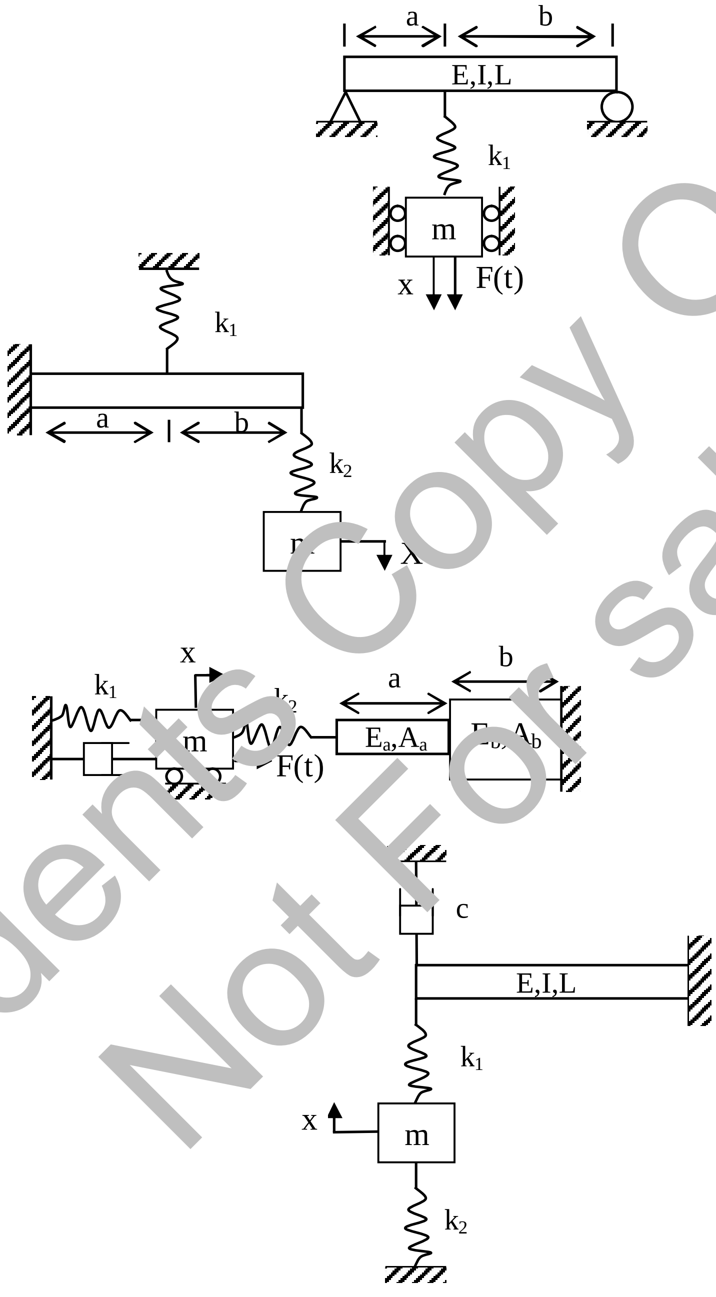
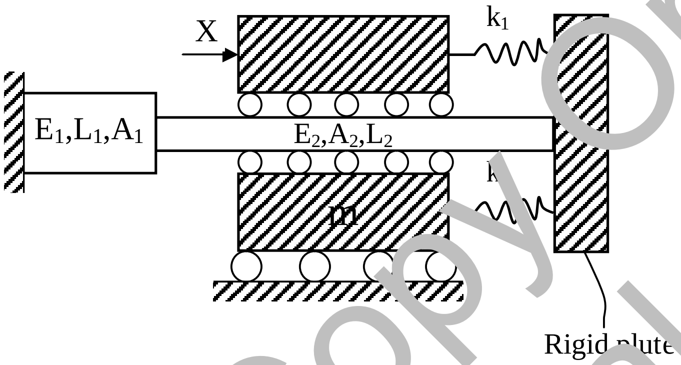


Figure P3.9

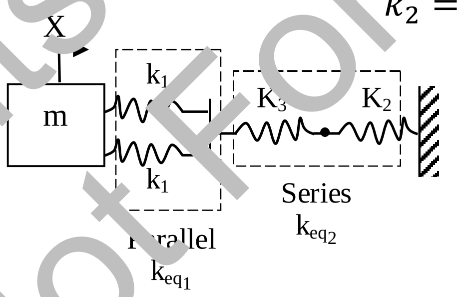
3.10. A hollow cylindrical mass connected with two spring which are connected with rigid plate. A rod is passing through the mass and connected with rigid plate as shown in Figure. Find the equivalent stiffness of the system.



Solution:

The system can be represented in term of springs.

$$k_2 = \frac{E_1.A_1}{L_1}\ , k_3 = \frac{E_2.A_2}{L_2}$$



$$\frac{1}{k_{eq}} = \frac{1}{k_{eq1}} + \frac{1}{k_{eq2}}$$

$$k_{eq1} = k_1 + k_1$$

$$k_{eq2} = \frac{k_2.k_3}{k_2 + k_3}$$

Chapter 4

Using ODE45 MATLAB to solve differential equations

MATLAB's standard solver for ordinary differential equations (ODEs) is the function ode45. This function implements a Runge-Kutta method with a variable time step for efficient computation. ode45 is designed to handle the following general problem:

$$\frac{dy}{dt} = f(t, y) \qquad y(t_o) = y_o \qquad \ldots (1)$$

where t is the independent variable (time, position, volume) and **y** is a vector of dependent variables (displacement, velocity, temperature, position, concentrations,) to be found. The mathematical problem is specified when the vector of functions on the right-hand side of eq. (1), **f(t,y)**, is set and the initial conditions, **y = yo** at time

to are specified. The notes here apply to versions of MATLAB above 5.0 and cover the basics of using the function ode45.

Syntax for ode45

ode45 may be invoked from the command line via

[t,y] = ode45(fname, tspan, y0)

where

fname: name of a <u>function</u> **Mfile**, an inline function object or an anonymous function used to evaluate the right-hand-side function in eq. (1) at a given value of the independent variable and dependent variable(s). If an **Mfile** is used, the function definition line usually has the form

function dydt = *fname***(t,y)**

and the file is stored as fname.m. The output variable (**dydt**) must be a vector with the same size as y. Note that the independent variable (**t**) must be included in the input argument list even if it does not explicitly appear in the expressions used to generate **dydt**. The variable **fname** can contain the name of the Mfile or can be a function handle generated by an inline or anonymous function.

Tspan: two -element vector defining the range of integration (**[to tf]**) or can be a vector of values for which the solution is desired.

y_o vector of initial conditions for the dependent variable. There should be as many initial conditions as there are dependent variables.

t: Value of the independent variable at which the solution array (**y**) is calculated. Note that by default this will not be a uniformly distributed set of values.

y: Values of the solution to the problem (array). Each column of y is a different dependent variable. The size of the array is **length(t)**-by-**length(y0)**

Example1:

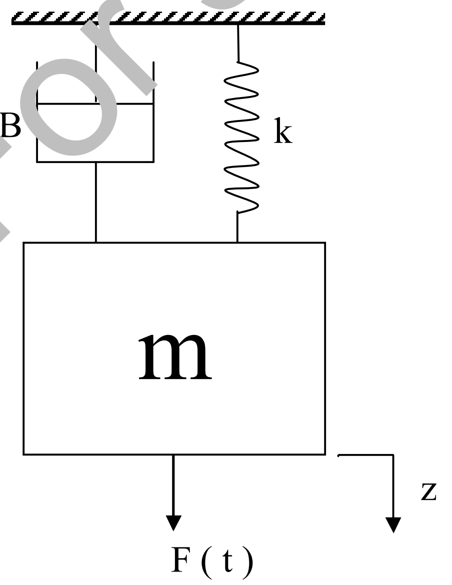
Write Matlab code for the system shown in the figure to show displacement versus time (0<t<[illegible]) for the following cases:

a) Free response F(t)=0

b) Forced response F(t)= 50*sin(2t)

m=1 kg, B= 2 N.s/m, k=100 N/m, X_o= 0.5 m, V_o= 0.2 m/s

$$m\ddot{z} + B\dot{z} + kz = f_a(t)$$



Solution:

Express the equation of motion in a state-space form

$x_1 = z \Rightarrow$ displacement

$x_2 = \dot{z} \Rightarrow$ velocity

$\dot{x}_1 = \dot{z} = x_2$

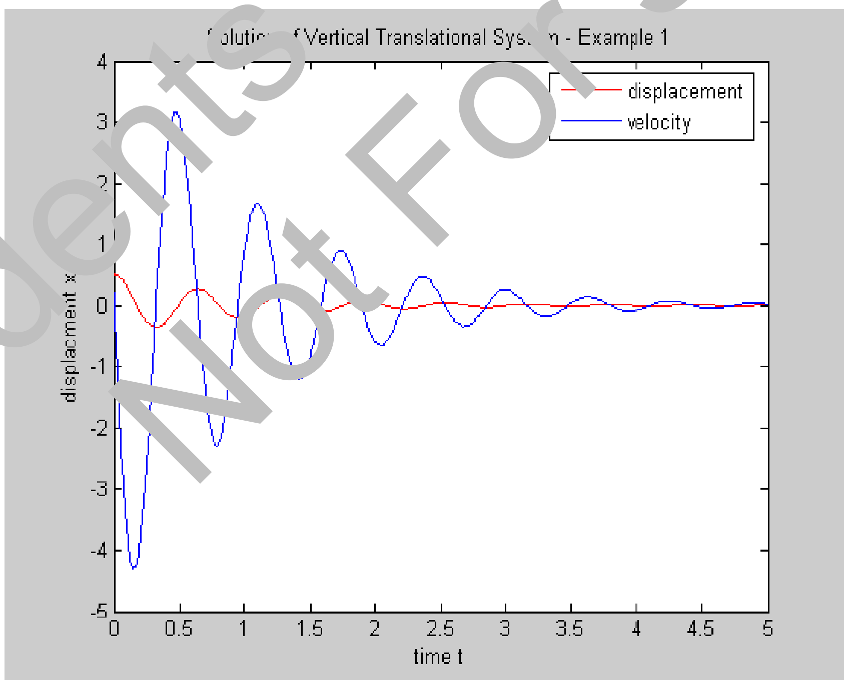
$$\dot{x}_2 = \ddot{z} = \frac{1}{M}(f_a(t) - Bx_2 - kx_1)$$

a) The system where the forcing term is zero

```
clear
[t, x] = ode45('TMechEx1', [0 5], [0.5; 0.2]);
plot(t,x(:,1), 'r', t, x(:,2), 'b')
title('Solution of Vertical Translational System - Example 1');
xlabel('time t');
ylabel('displacment x');
legend('displacement','velocity')
```

```
function dxdt = TMechEx1(t,x)
M=1;
K=100;
B=2;
dxdt = [ x(2); (1/M)*(-B*x(2)-K*x(1)) ];
```

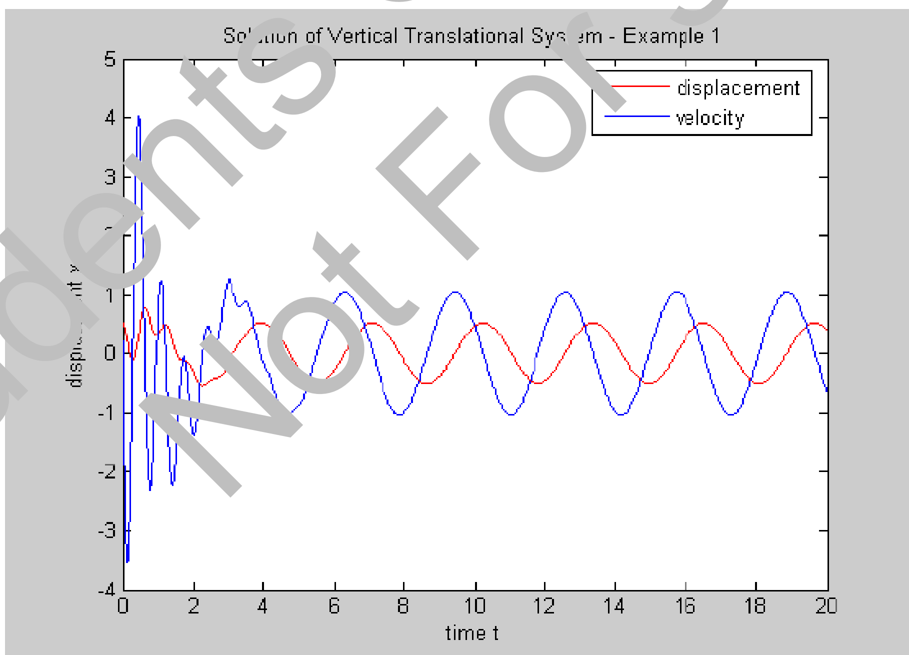
Free Response



b) The system where the forcing term is 50*sin(2t)

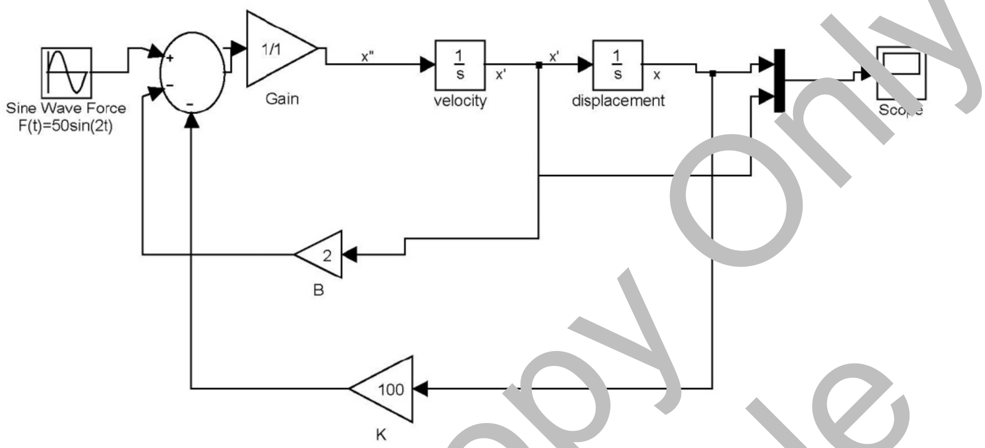
```
clear
[t, x] = ode45('TMechEx1',[0 20],[0.5; 0.2]);
plot(t, x(:,1), 'r', t, x(:,2), 'b')
title('Solution of Vertical Translational System - Example 1');
xlabel('time t');
ylabel('displacment x');
legend('displacement','velocity')
```

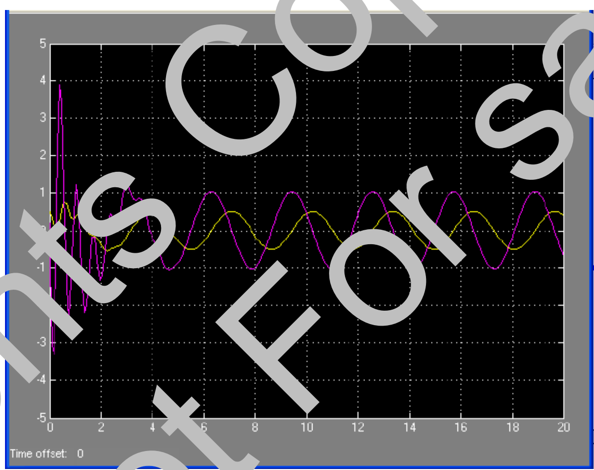
```
function dxdt = TMechEx1(t,x)
M=1;
K=100;
B=2;
dxdt = [ x(2); (1/M)*(50*sin(2*t)-B*x(2)-K*x(1)) ];
```

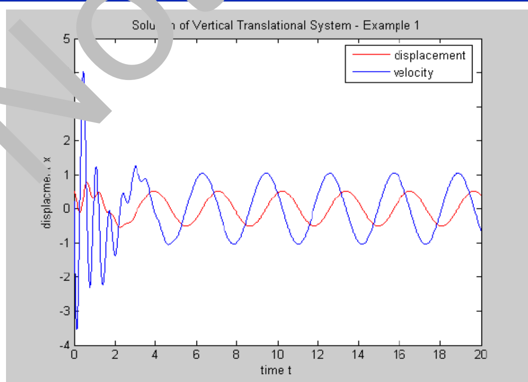


Example2:

Use SIMULINK to solve example 1 for the forced response part.

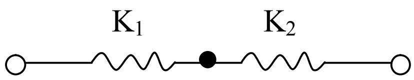




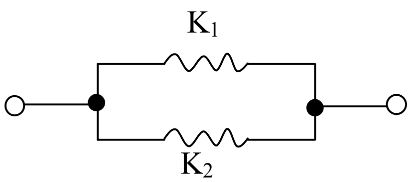


APPENCIX

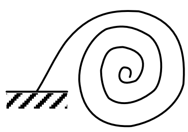
Table of soring stiffness



$$k = \frac{1}{1 / k_1 * 1 / k_2}$$



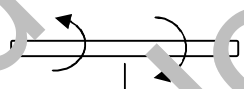
$$k = k_1 * k_2$$



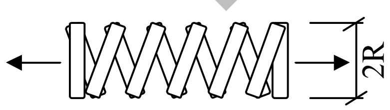
$k = \frac{EI}{L}$ I = moment of inertia of cross sectional area.

L = total length

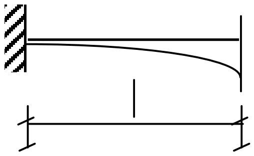
$k = \frac{EA}{L}$ A = Cross – sectional area.



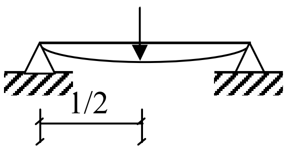
$k = \frac{GJ}{L}$ J = torsion constanl of cross section.



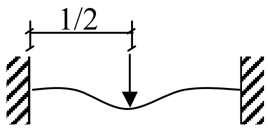
$k = \frac{Gd^4}{64 \, n \, R^3}$ n = number of turns.



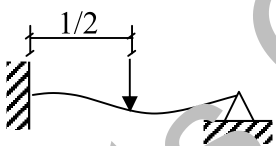
$k = \frac{3EI}{L^3}$ k at position of load.



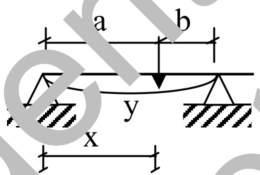
$k = \frac{48EI}{L^3}$



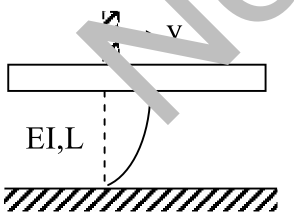
$k = \frac{192EI}{L^3}$



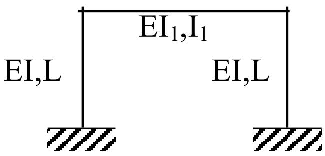
$k = \frac{768EI}{7L^3}$



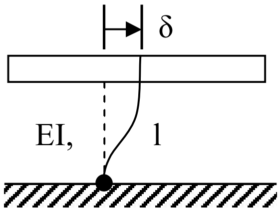
$k = \frac{3EIL}{a^2b^2}$ $y_x = \frac{pbx}{6EIL}(1^2 - x^2 - 2^2)$



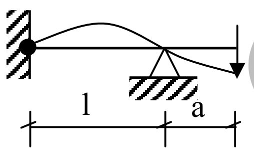
$k = \frac{3EI}{L^3}$



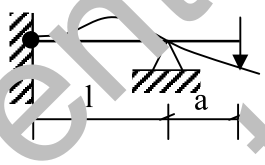
$$k = \frac{6EI}{1}\left(\frac{6+\frac{I}{I_1}.\frac{L_1}{1}}{\frac{3}{2}+\frac{I}{I_1}.\frac{L_1}{1}}\right)$$



$$k = \frac{12EI}{L^3}$$

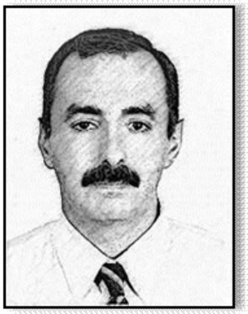


$$k = \frac{3EI}{(l+a)a^2}$$



$$k = \frac{24EI}{a^2(3l+8a)}$$

ABOUT THE AUTHORS



Dr. Waleed K. Ahmed obtained B.Sc. in Mechanical Engineering, College of Engineering, University of Baghdad, Iraq in 1992. Served over 6 years in the manufacturing and the quality control field. In 2000 he obtained M.Sc. in Applied Mechanics from the of the University of Technology, Baghdad, Iraq. Dr.Waleed appointed as a lecturer at the Materials Engineering Department, College of Engineering, University of Mustanseriyia, Baghdad, Iraq in 2001. In 2006 completed his Ph.D., where the research was done in collaboration with Nottingham University, UK. Besides, he worked as a consultant for many industrial companies Dr. Waleed moved to work in United Arab Emirates University in 2006. Moreover, he published more than 50 journals and conferences papers. Dr.Waleed his main interest in renewable energy, nanomaterials, failure analysis, FEA and fracture mechanics.

Professor Wail N. Al-Rifaie is the former President of University of Technology, Baghdad where holds a chair in the Building and Construction Engineering Department. He received his Ph.D in Structural Engineering from University of Wales, University College, Cardiff, U.K. in 1975. Professor Wail received the Telford Premium Prize from the Institution of Civil Engineers in 1976 on the strength of his doctoral work. He has been awarded numerous national honours including the Outstanding Professor Award (1996), the Science Merit Medal (2001) and the Science Decoration in the same year. He was a special professor at Nottingham. University, UK in years 2008-2011. Professor Wail has supervised over 70 post graduate thesis and published over 110 scientific papers, the majority of which concern ferrocement elements, including membrane roof structures, box beams, load bearing walls and columns, hydraulic containment structures and thin shells such as roof domes. Recently he is focusing on nano materials for construction.

MODELING OF MECHANICAL SYSTEMS

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