Prepared for submission to JHEP

Essential Astronomy & Astrophysics Through "Questions " and "Answers"

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Abstract:

In these notes, I discuss a number of selected of topics in Astronomy and Astrophysics that can be of interest to both science and non-science students. For a non-physics major student or layman, very short **answers** to these questions are presented, whereas for those with special interest in physics and astrophysics **more details** are provided. I also included many **appendices** with comprehensive discussion relevant to the questions discussed in these notes.

The figures in these notes are produced by the author, unless indicated.

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1 How did the ancients know that Earth was round ?

- **The idea of spherical Earth was first proposed by Pythagoras[a], however it was Mainly based on aesthetic ground.**
- **It was Aristotle[b] who argued, based on a number of observational facts that the shape of the Earth is spherical.**

[a] Greek philosopher who lived around 500 B.C,who made important discoveries in mathematics and astronomy. He best known for the Pythagorean theorem.

[b] Ancient Greek philosopher and scientist who lived between circa 384 B.C and 322 B.C, and is considered one of the greatest thinkers in history.

More Details

The observational facts are:

- Some **stars that are visible in the Northern hemisphere are not for an observer in the Southern hemisphere**. Similar argument holds the other way around. For instance, an observer located in the Southern hemisphere can not see the star "Polaris", but once in the Northern hemisphere it becomes visible. Thus, the view of constellations changes when going from northern hemisphere to the southern hemisphere, and vice versa.
- **The Earth's shadow on the moon surface during the different phases of the lunar eclipse always evolves from arched to circular**, regardless of the locations of the observer on the Earth[1]. Moreover, when the moon was in different positions during an eclipse, the sunlight struck the earth from different angles, and yet the shape of the Earth's shadow, as it moves across the moon, always looked part of a circle. This means that Earth has a shape that casts a circular shadow in every possible angle. This could only be caused by a spherical shape.
- When standing on the shore and watching a ship moving away, **the bottom of ship in sea starts to disappear first over the horizon**, and then, gradually, the mast.

[1] If the Earth had a shape of a disk, then the moon would have to be directly overhead to get a round shadow, which means the sun would be directly below the earth. However, if that was the case, the entire surface of the Earth would be dark during a lunar eclipse.

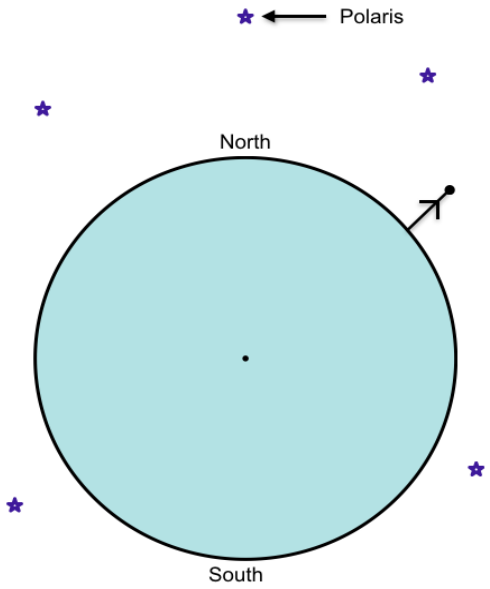


Figure 1: More stars are visible to an observer moving toward the north and looking to the sky toward the North. Similarly, more stars will be visible to her/him when moving toward the south and looking to the sky in the direction toward the south.

2 Who first estimated the distances and sizes of the Sun and the Moon?

Quick Answer

- **Around 250 B.C, Aristarchus of Samos[a] used observations of lunar eclipses and simple geometry to estimate the relative distances and sizes of the Earth, Moon, and Sun. He found that**

$$\frac{d_{\text{E-M}}}{D_E} \simeq 10; \quad \frac{d_{\text{E-S}}}{D_{\text{E}}} \simeq 191; \quad \frac{D_{\text{S}}}{D_{\text{M}}} \simeq 19; \quad \frac{D_{\text{E}}}{D_{\text{M}}} \simeq 2.85$$

  where,

  $d_{\text{E-M}}$ **= Earth-Moon distance,** $d_{\text{E-S}}$ **= Earth-Sun distance,**

  D_{E} **= Earth's diameter,** D_{M} **= Moon's diameter,** D_{S} **= Sun's diameter**

- **Although his results were off by one to two orders of magnitudes from the actual values, his method of calculation was correct.**
- **Equally important, his estimates showed that the universe is vast and the Sun is much larger than the Earth.**

[a] Samos is a Greek island in the Aegean Sea.

More Details

- Aristarchus's calculation of the distances and sizes of the Moon and Sun was based on four observations. These are as follows:

Observation #1 :

"When the moon is exactly half full as seen from the Earth, the angle between the lines of sight to the Sun and to the Moon is 87°."

Let us denote the Earth, Moon, and Sun by the letters E, M, and S denote, respectively, and by α the angle between the lines of sight to the Sun and to the Moon. When the Moon is exactly half full the angle $E\hat{M}S$ is exactly right angle, and so by measuring the angle α it would be possible to determine the distance to the Sun in terms of that of the Moon from the trigonometric relation

$$\frac{d_{\text{E-M}}}{d_{\text{E-S}}} = \cos\alpha \tag{2.1}$$

How did Aristarchus determined the angle α ? He used the following simple and cleaver idea. He calculated the angle ϵ shown in Fig. 2 below, which is equal to $(90^\circ - \alpha)$, in terms of the difference between the time the Moon takes to go from the first quarter to the third quarter, $\Delta t^{(1\to3)}$, and the time it takes from the third quarter to the first quarter, $\Delta t^{(3\to1)}$, given by the relation

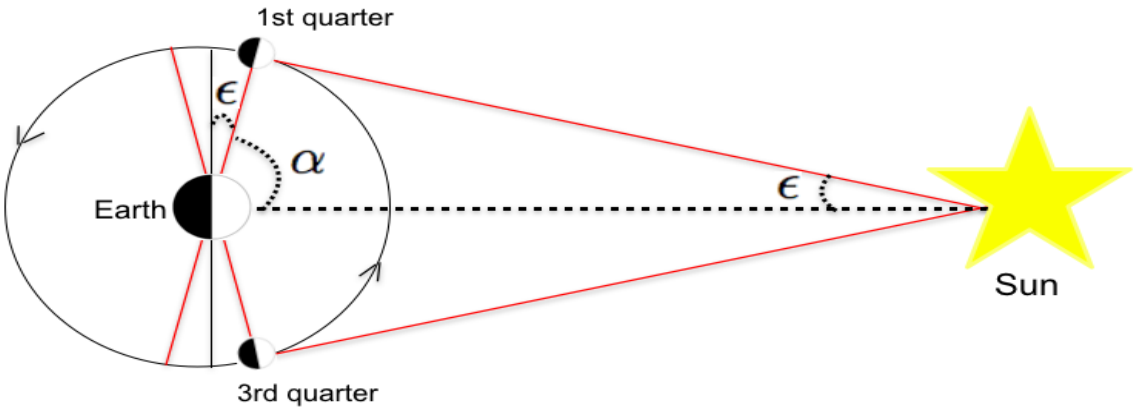


Figure 2: Determining the angle ϵ by measuring the time it takes the Moon to go from the first quarter to the third quarter and the time it takes from the third quarter to the first quarter. The diagram in this figure is not to scale.

$$\frac{\Delta t^{(1\to3)} - \Delta t^{(3\to1)}}{T_M} = \frac{4\epsilon}{2\pi} \tag{2.2}$$

Here $T_M = 29.5$ days, corresponding to the synodic orbital period of the Moon around the Earth. Aristarchus found that $(\Delta t_1 - \Delta t_2) = 1$ day, and deduced that

$$\epsilon = 90^\circ \times \left(\frac{1 \text{ day}}{29.5 \text{ day}}\right) \simeq 3^\circ \implies \alpha \simeq 87^\circ \tag{2.3}$$

Plugging this value into Eq. (2.1), gives

$$\left(\frac{d_{\text{E-S}}}{d_{\text{E-M}}}\right)^{\text{[Aristarchus]}} = 19.1 \tag{2.4}$$

This about a factor of 20 smaller than the actual value. The reason for this is ....

Observation #2 :

"During solar eclipse, the Sun have approximately the same angular size."

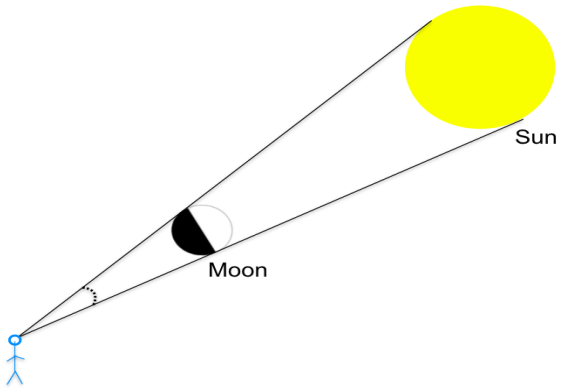


Figure 3: The Moon and the Sun have approximately the same angular size by an observer on the Earth. The diagram in this figure is not to scale.

Mathematically this statement implies that (see Fig. 3)

$$\frac{D_{\text{S}}}{D_{\text{M}}} = \frac{d_{\text{E-S}}}{d_{\text{E-M}}} \tag{2.5}$$

and using the value of $d_{\text{E-S}}/d_{\text{E-M}}$ calculated ration in Eq (2.4), we get

$$\left(\frac{D_{\text{S}}}{D_{\text{M}}}\right)^{\text{[Aristarchus]}} = 19.1 \tag{2.6}$$

This value is about a factor of **20 smaller** than the actual value. This discrepancy is due to the underestimated value of the ratio of the Earth-Sun distance to the Earth-Moon distance, discussed above.

Observation #3 :

"During lunar eclipse, the Moon travels a distance $2D_{\rm M}$ while passing the Earth's shadow."

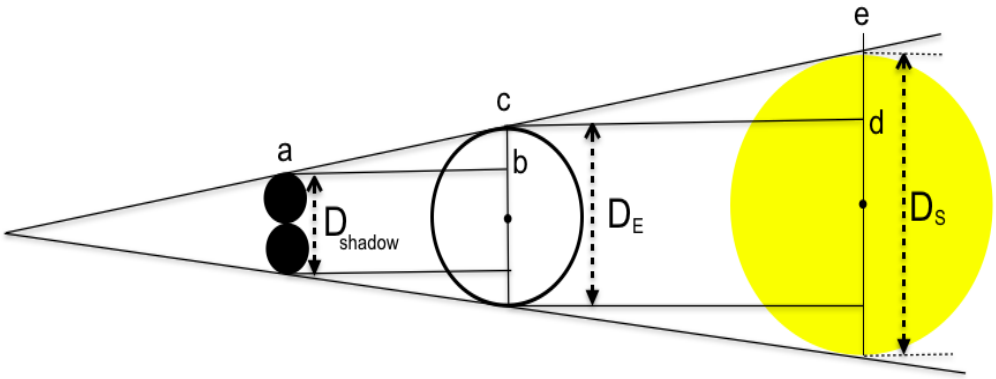


Figure 4: The Moon and the Sun have approximately the same angular size by an observer on the Earth. The diagram in this figure is not to scale.

From the similarity of the triangles $\Delta(abc)$ and $\Delta(cde)$ in Fig. 4, we have

$$\frac{d_{\text{E-M}}}{D_{\rm E}/2 - D_{\rm shadow}/2} = \frac{d_{\text{E-S}}}{D_{\rm S}/2 - D_{\rm E}/2} \iff \frac{d_{\text{E-M}}}{D_{\rm E} - D_{\rm shadow}} = \frac{d_{\text{E-S}}}{D_{\rm S} - D_{\rm E}} \tag{2.7}$$

Replacing $d_{\text{E-S}}/d_{\text{E-M}}$ by the ratio $D_{\rm S}/D_{\rm M}$ (see Eq. (2.5)), and $D_{\rm shadow} = 2D_{\rm M}$ (from observation #3), we can express the ratio $D_{\rm E}/D_{\rm M}$ as

$$\frac{D_{\rm E}}{D_{\rm M}} = \frac{D_{\rm S}/D_{\rm M}}{1 + D_{\rm S}/D_{\rm M}} \left(1 + \frac{D_{\rm shadow}}{D_{\rm M}}\right) = 3\frac{D_{\rm S}/D_{\rm M}}{1 + D_{\rm S}/D_{\rm M}} \tag{2.8}$$

Now we plug the value $D_{\rm S}/D_{\rm M} = 19.1$ that we obtained in Eq. (2.6)) into the above equation and find

$$\left(\frac{D_{\rm E}}{D_{\rm M}}\right)^{\text{[Aristarchus]}} = 2.85 \tag{2.9}$$

Aristarchus found $2.51 < D_{\rm E}/D_{\rm M} < 3.16$, which is consistent with the above result. The correct value of $D_{\rm E}/D_{\rm M}$ is 3.67, i.e 28% larger than the value given above. The reason for the difference between the calculated value and observation is size of the Earth's shadow, $D_{\rm shadow}$, which is almost three time the diameter of the Moon instead of two times.

Observation #4 :

"The Moon subtends one fifteenth part of a sign of the Zodiac."

Since there are 12 zodiac signs, it implies that, according to Aristarchus, the Moon has the angular diameter

$$\theta_M = \left(\frac{1}{15}\right)\left(\frac{360^\circ}{12}\right) = 2^\circ \tag{2.10}$$

With θ_M being a small angle, we can approximate $d_{\text{E-M}}$ by the length of the arc in a circle of radius D_{M} subtended by θ_M, and hence we write

$$D_{\text{M}} = d_{\text{E-M}}\ \theta_M = d_{\text{E-M}}\ \left(\frac{2^\circ}{180^\circ}\right) \times 3.14 \text{ rd} = 0.035\ d_{\text{E-M}} \tag{2.11}$$

With the use of this result and Eq. (2.9), we have

$$\left(\frac{d_{\text{E-M}}}{D_{\text{E}}}\right)^{\text{[Aristarchus]}} = \frac{d_{\text{E-M}}}{D_M} \times \frac{D_{\text{M}}}{D_{\text{E}}} = \left(\frac{1}{0.035}\right)\left(\frac{1}{2.85}\right) = 10 \tag{2.12}$$

which is about of factor 3 small than the actual value. Part of the discrepancy is due to the fact that the value of θ_M quoted by Aristarchus was largely overestimated by a factor of four[2].

- About century after Aristarchus, the Greek astronomer Hipparchus [**190 – 120** B.C] of Nicaea used the observation of solar eclipse to determine the Earth-Moon distance in terms of the radius of the Earth which was closer to the actual value than the value estimated by Aristarchus. He knew that at the city of "Hellespont", in modern day Turkey, the eclipse was total while at Alexandria (about 500 km apart) in Egypt, the eclipse was partial, covering about 4/5 of the Sun (i.e. partial eclipse). In Fig..., α denote the angle that corresponds to 1/5 of the angular size of the Moon that is uncovered as viewed from Alexandria. Hipparchus had measured the angular size of the Moon and the Sun and found that they subtend an angle of 0.5 degree. Thus,

$$\alpha = \left(\frac{1}{5}\right) \times 0.55^\circ = 0.11^\circ \tag{2.13}$$

Hipparchus also knew that the difference in latitudes between Hellespont and Alexandria is approximately 9 degrees. Consequently (see Fig....), we have

$$\begin{aligned} \frac{\text{HA}}{9^\circ} &= \frac{2\pi R_{\text{E}}}{360^\circ} \\ \frac{\text{HA}}{\alpha} &= \frac{2\pi d_{\text{E-M}}}{360^\circ} \end{aligned} \tag{2.14}$$

[2]The actual angular size of the Moon (and also the Sun, from observation #2) is 0.52 degree. If we use the correct value for the angle (i.e. $\theta_M = 0.52$), and $D_{\text{E}}/D_{\text{M}} = 3.67$ we obtain $(d_{\text{E-M}}/D_{\text{E}}) = 29.9$, which is in very good agreement with observation.

By solving for $d_{\text{E-M}}$ yields

$$\left(\frac{d_{\text{E-M}}}{R_{\text{E}}}\right)^{\text{[Hipparchus]}} = \left(\frac{9^\circ}{\alpha}\right) = 82$$

Compared to the actual value, this value is overestimated by about 30%, and the reason for this is because the Moon was assumed to be directly overhead at the time of the eclipse, but at some significant angle.

3 What is the average Earth-Sun distance in kilometers ?

Quick Answer

$d_{E-S} = 149.6$ **million km**

More Details

- **Measuring Earth-Sun distance using the transit of Venus**

  The transit of a planet is its passage from one end of the disk of the Sun to the other, and it can be observed only for planets whose orbits are inside Earth's orbit, i.e. for the planets Mercury and Venus.

  The idea of using the transits of Venus or Mercury to calculate the Earth-Sun distance was first proposed in **1663** by the mathematician **James Gregory**, but it was the English astronomer **Edmond Halley** who in **1716** suggested that of the two observing the transits of Venus will allow to determine the distance more accuracy[3]. The method requires measuring the times of the beginning and the end of the transit seen by two observers, A and B, from two places on Earth separated by large latitudes[4].

  Let us denote by $R_\odot$ the radius of the Sun, $\Delta_\odot \simeq 15$ arcmin half its angular size, d_{E-V} Venus-Earth distance, $\omega_V = 0.067$ arcsec/sec the angular velocity of venus, Δt_A and Δt_B be the duration of the transit as measured by the observers A and B, respectively, d_{AB} the distance between their locations, θ_S and θ_V are the parallax angles measured from the locations of the two observers for the Sun and Venus, respectively, $(\alpha_A - \alpha_B)$ is the angular separation between the two transits. Looking at Fig. ..., we have

$$d_{E-V} = \frac{d_{AB}}{\theta_V} \implies d_{E-S} = \frac{1}{\theta_V}\left(\frac{d_{AB}}{0.28}\right) \tag{3.1}$$

[3] The transits of Venus are incredibly rare because the orbits of the Earth and Venus are inclined with respect to each other, and occur in pairs about 8 years apart, and sometimes century apart. For example: (June 1761, June 1769), (December 1874, December 1882), ect.

[4] As we will discuss in **A 16**, the measurement of distance from two separated places is called **parallax** method and was used by Cassini and Richter to measure the distance to Mars, from which they inferred the Earth-Sun distance (see **A 17**).

Where in the second equality we made the approximation $\tan\theta_V \simeq \theta_V$ since d_{E-V} much larger than the size of the Earth. We also know that d_{VS} is approximately 0.72 times the distance Earth-Sun[5].From the triangles ACO and BCV we see that the angle θ_V can be written in terms of the angular separation as

$$\theta_V = |\alpha_A - \alpha_B| + \alpha_\odot \tag{3.2}$$

Since the angles α_A and α_B are very small[6], we can approximate the angular size of the projected paths of Venus on the Sun's disk from the start to the end of the transit as seen by the observers A and B o n the Earth by

$$\Delta\beta_A \simeq \omega_V\ \Delta t_A, \qquad \Delta\beta_B \simeq \omega_V\ \Delta t_B \tag{3.3}$$

Now applying Pythagoras theorem to the triangles in Fig. we write

$$R_\odot^2 = [d_{E-S}\ \alpha_A]^2 + [d_{E-S}\,(\Delta\beta_A)]^2 = [d_{E-S}\ \alpha_A]^2 + [d_{E-S}\,(\omega_V \Delta t_A)]^2 \tag{3.4}$$

$$R_\odot^2 = [d_{E-S}\ \alpha_B]^2 + [d_{E-S}\,(\Delta\beta_B)]^2 = [d_{E-S}\ \alpha_B]^2 + [d_{E-S}\,(\omega_V \Delta t_B)]^2 \tag{3.5}$$

from which it follows

$$(\alpha_A - \alpha_B) = \sqrt{\Delta_\odot^2 - \left(\frac{\Delta t_A \omega_V}{2}\right)^2} - \sqrt{\Delta_\odot^2 - \left(\frac{\Delta t_B \omega_V}{2}\right)^2} \tag{3.6}$$

The angle $\theta_\odot$ can be expressed as

$$\theta_\odot \simeq \frac{d_{AB}}{d_{E-S}} \simeq \left(\frac{d_{E-V}}{d_{E-S}}\right)\theta_V = 0.28\ \theta_V \tag{3.7}$$

After substituting the above expression of $\theta_\odot$ and the angular separation into Eq (3.2), the equation for Earth-Sun distance given in Eq (3.1) in reads

$$d_{E-S} = \left(\frac{0.72}{0.28}\right)\left[\frac{\sqrt{\Delta_\odot^2 - \left(\frac{\Delta t_A \omega_V}{2}\right)^2} - \sqrt{\Delta_\odot^2 - \left(\frac{\Delta t_B \omega_V}{2}\right)^2}}{(\omega_V \Delta t_B)^2 - (\omega_V \Delta t_A)^2}\right] d_{AB} \tag{3.8}$$

In this formula the only unknowns are the durations of the transits, which can be measured, and hence the distance to the Sun can be calculated. Note that in the above expression we have ignored the effect of the rotation of the Earth and assumed that the orbits of Venus and Earth were circular. Taking into account these corrections will improve the accuracy of the calculated value of

[5]In 1919, Kepler established a relation between the square a planet's period and its average distance to the Sun, also known as Kepler's third law of planetary motion. Consequently, the distances of the known planets at that time (Uranus and Neptune were not discovered yet) to the Sun in units of the Earth-Sun distance were known.

[6]This is because the ratio of the size the Earth to the distance to Venus is extremely small.

the astronomical unit.

In **1771**, the French astronomer **Jerome Lalande** used the combined the data of the transit that occurred in the years 1761 and 1769 and calculated the distance to the Sun, and hence the astronomical unit, AU, to be

$$[d_{ES}]^{(\text{Transit})} = 153 \text{ million kilometers} \pm 1 \text{ million km} \tag{3.9}$$

which is just 2% higher than the actual value.

- **More precise measurement of Earth-Sun distance**

  An accurate determination of the Earth-Sun distance has been obtained using radar echoes where radio waves are transmitted toward an astronomical body, such as a planet[7]. By measuring the time it takes the radio pulses to come

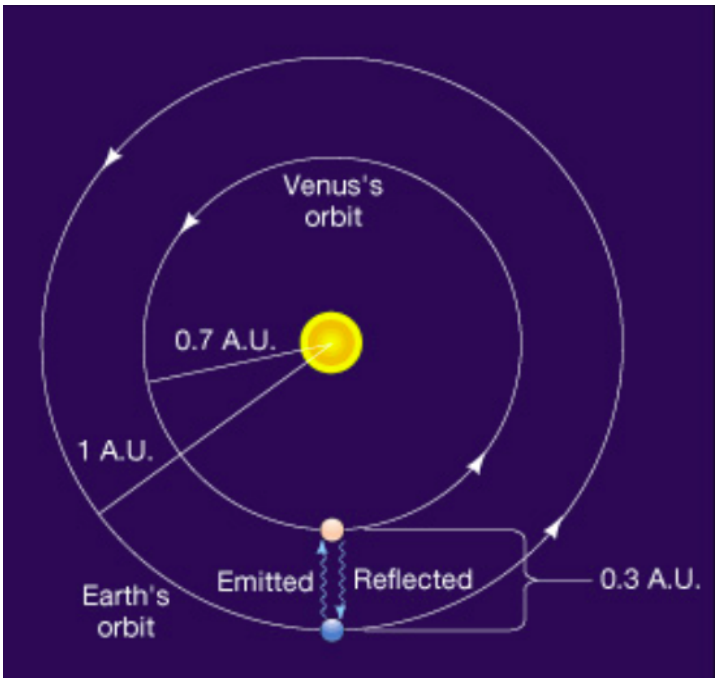


Figure 5: Using radar echoes to determine the Earth-Venus distance at the closest approach, and hence the determining the Earth-Sun distance.

back we can determine the distance to that planet. The first attempt to apply this technique was in 1958 by the MIT Lincoln Laboratory where they directed radar waves toward **Venus**, however the results were ambiguous. Three years later, NASA's Jet Propulsion Laboratory (JPL) directed a beam of radio waves towards Venus and, for the first time in history, they received the radar echoes

[7] One cannot use radar ranging to measure the distance to the Sun directly because radio signals are absorbed at the solar surface and not reflected to Earth.

after few minutes[8]. So by measuring the time it take the reflected radar pulses off Venus when it is closest to Earth[9] and multiplying it by the speed of light[10] we obtain twice its distance to Earth at closest approach. For instance, it was found that when Venus is closest to Earth, it took about 280 seconds to get the radar echoes. Multiplying this round-trip travel time of the radar signal by the speed of light (300,000 km/s), we obtain twice the distance from Earth to Venus. Thus

$$(1 - 0.72) \times d_{\text{E-S}} = \frac{1}{2}\left(3 \times 10^5 \text{ km}/s \times 280 \text{ s}\right) = 42 \times 10^6 \text{ km} \tag{3.10}$$

from which it follows

$$d_{\text{E-S}} \simeq 150 \times 10^6 \text{ km} \tag{3.11}$$

4 What is the average Earth -Moon distance ?

Quick Answer

$$d_{E-M} = 384,400 \textbf{ km}$$

More Details

- **Accurate Measurement of Earth-Moon distance**

  The Apollo missions 11 (1969), 14 (1971), and 15 (1972) deployed a retro-reflector arrays on the Moon to measure the distance between surfaces of Earth and the Moon from the duration that light takes from a very high-powered laser pulses directed to the mirrors on the Moon to bounce off. It take light reflected off the mirror anywhere between 2.34 to 2.71 seconds, depending how is the Moon at the time of the measurement. Thus, the distance is in the range

$$d_{E-M} \in [351,000 - 406,000 \text{ km}] \tag{4.1}$$

  This experiment, called the **Lunar Laser Ranging** (LLR), can measure the distance between Earth-and Moon with an impressive accuracy of a millimeter.

[8] After six minutes and half.
[9] The same method can also be applied when the Venus is at the maximum elongation.
[10] The speed of light is $300,000$ km/s.

- **The gradual recession of the Moon**

  The Moon is gradually recessing from Earth of 3 **cm/year**.....

  due to tidal effect..... which will be discussed later

5 How did the ancients estimate the size of the Earth?

Quick Answer

Around 240 B.C, Eratosthenes[a] of Cyrene[b] determined the circumference of the Earth by measuring the difference in the angles between the incident light and the vertical at two different locations, and using simple geometry.

[a] He was a prolific Greek scholar; he was a poet, an astronomer, geographer, philosopher, and mathematician. He was also the chief librarian of the great library of Alexandria.

[b] It's a city in modern day Libya.

More Details

- **Eratosthenes' estimate of the size of the Earth [240 BC]**

  One day, while reading a papyrus book[11] in the great library of Alexandria, **Eratosthenes** [276 – 194 BC], came across the statement that at noon during the summer solstice[12], the light shone straight down into the bottom of a very deep well[13] in the Egyptian city of Syene (modern-day Aswan)[14], which means that the Sun is directly overhead. At the same time in Alexandria (where he lived), almost due north of Syene, he observed that the Sun light casts a shadow. By measuring the length of the shadow casted by a tower in Alexandria, he determined the angle between the vertical tower and the Sun's ray[15] to be equal to **7.2** degrees. He also knew that the distance between Alexandria and Syene is 5000 Stadia, where a stadion[16] repesents the unit of lengths used at that time. Unfortunately, there is not an exact conversion of "1 stadion" to our unit metric system, however, many historians put it in a range between 0.16 km and 0.18 km. So, Eratosthenes reasoned that if 7.2° decomposes the

[11] Papyrus is a thick paper-like material that was used in ancient times as a writing surface.

[12] This is when the Sun reaches its highest position in the sky, which occurs between **June 20** and **June 22** in the Northern Hemisphere, and between **December 20** and **December 22** in the Southern Hemisphere.

[13] The water was illuminated but not the walls of the well.

[14] It is close to the tropic of cancer (23°.5 N).

[15] Knowing the height of the tower and the length of the shadow one can deduce the angle between the vertical tower and the Sun's ray.

[16] In Latin it is "stadium".

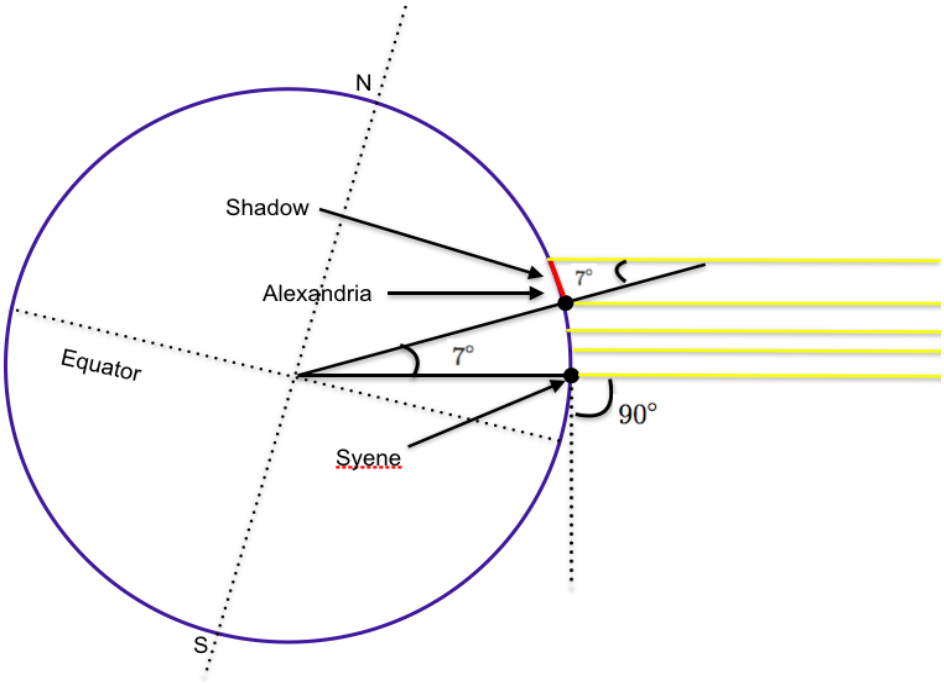


Figure 6: Measuring the size of the Earth.

circumference of the Earth, $\mathcal{C}$, to an arc of length 5000 stadia, then

$$\mathcal{C} = \frac{360^\circ}{7.2^\circ} \times 5000 \text{ Stadia} = 250,000 \text{ Stadia} \in [\mathbf{40,000 - 45,000} \text{ km}] \quad (5.1)$$

Thus, the radius of the Earth is

$$\mathbf{R}_\oplus = \frac{[40,000 - 45,000 \text{ km}]}{2\pi} \in [\mathbf{6370 - 7325} \text{ km}]$$

This is within about 8% accuracy of the true value, which is quite remarkable[17] considering the very simple method that he used for this measurement[18].

[17] This result is indeed remarkable given that it was based on some erroneous assumptions. For instance, it was assumed that Syene is due south of Alexandria; whereas it is actually 3° east of it. Also, at the summer solstice the Sun light is not precisely overhead at Syene, but about 0.4° from the vertical. Another inaccurate quantity is the angle that a shadow casts on the day of summer solstice at Alexandria; the true value is 7.76° instead of 7.2° that Eratosthenes used. It turns out that these errors almost cancel each out, and the estimate of the circumference would lie within the the range given in (5.1) when using the unit of kilometer.

[18] One might wonder what if Eratosthenes assumed that Earth is flat. In this case, for the tower to cast a shadow the Sun has to be be very close to the Earth Surface. With an angle of 7°, the distance from the Earth to the Sun would be

$$d_{ES} = \frac{5000 \text{ stadia}}{\tan 7^\circ} \in [\mathbf{6370 - 7325} \text{ km}]$$

which is of course unacceptable since at that time it was already known (**Aristarchus; 270 B.C**) that the d_{ES} was about 20 times the distance from Earth to the Moon (which we know now that it is smaller by a factor of about 20 than the actual value).

- **Al Biruni's[19] method for measuring Earth's radius [11th A.D]**

6 What is the simplest clear experimental evidence for Earth rotation ?

Quick Answer

In 1851, the French physicist Leon Foucault, hang up a very long pendulum from a ceiling at the Pantheon in Paris and let it swing, and showed that its path "appeared " to be slowly shifting, which was a clear evidence that the Earth rotates around its axis.

More Details

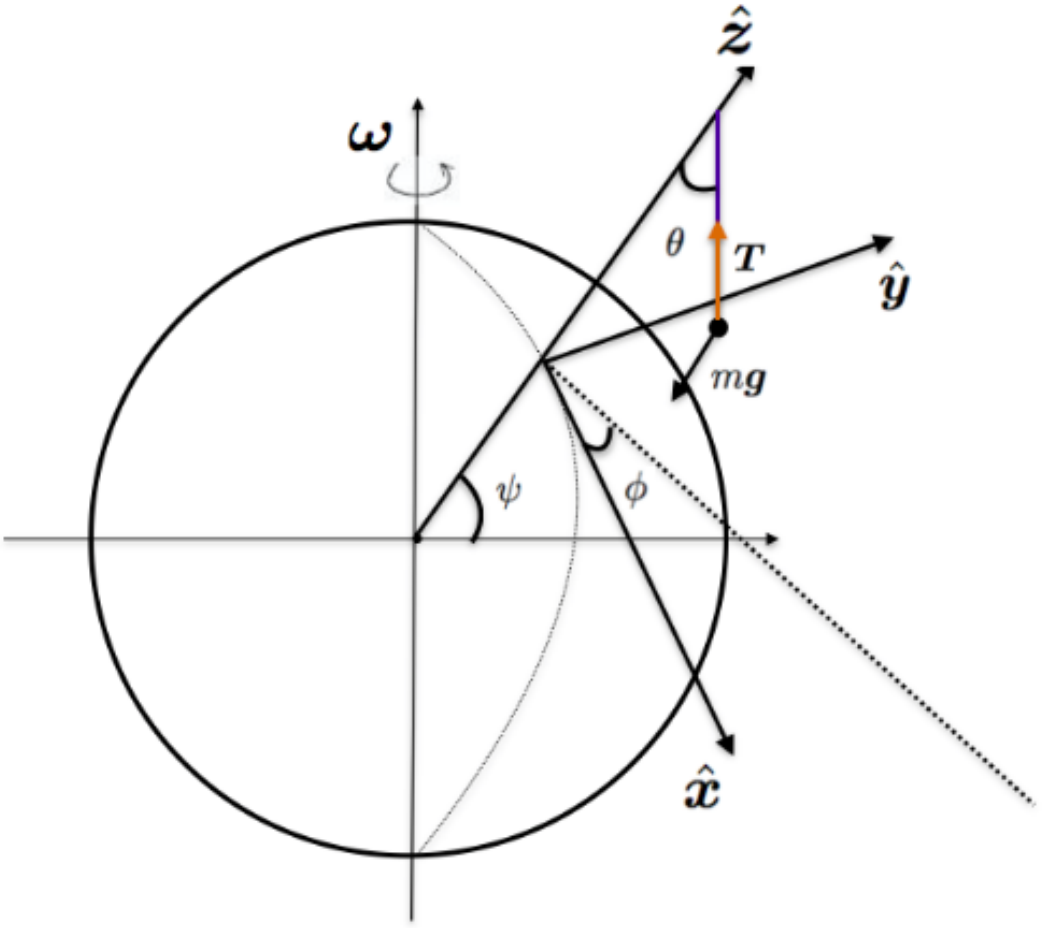


Figure 7: A schematic of the Foucault pendulum.

[19] Abu Raihan Al-Biruni (973 – 1048) was a famous astronomer, physicist and geographer.

- To understand how does Foucault's pendulum demonstrates the rotation of the Earth around its axis, we need to discuss the so called **Coriolis force**, which was named in honor of the French mathematician **Gustave Coriolis**[20] who proposed the existence of pseudo-force (i.e. an apparent force) that a body experiences when it is moving in a rotating frame. To be more specific, let us consider a body that is located at a point (r, θ, ϕ) in spherical coordinates. Its velocity as seen in a reference frame, $\mathcal{R}$, attached at the Earth's surface (see Fig. 7) is

$$\boldsymbol{v}|_{\mathcal{R}} = v_r \hat{\boldsymbol{r}} + v_\theta \hat{\boldsymbol{\theta}} + v_\phi \hat{\boldsymbol{\phi}} \tag{6.1}$$

Here $\hat{\boldsymbol{r}}, \hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\phi}}$ form an orthonormal basis in the spherical coordinate system. Then, the **Coriolis force** is given by

$$\boldsymbol{F}_{\text{coriolis}} = -2m\boldsymbol{\omega} \times \boldsymbol{v}|_{\mathcal{R}} \tag{6.2}$$

where $\boldsymbol{\omega} = \omega \hat{\boldsymbol{z}}$ is the angular velocity vector of the Earth. Using the vector identities

$$\hat{\boldsymbol{z}} \times \hat{\boldsymbol{r}} = \sin\theta \hat{\boldsymbol{\phi}}, \quad \hat{\boldsymbol{z}} \times \hat{\boldsymbol{\theta}} = \cos\theta \hat{\boldsymbol{\phi}}, \quad \hat{\boldsymbol{z}} \times \hat{\boldsymbol{\phi}} = -\sin\theta \hat{\boldsymbol{r}} - \cos\theta \hat{\boldsymbol{\theta}} \tag{6.3}$$

Then, Eq (6.4) reads

$$\boldsymbol{F}_{\text{coriolis}} = 2m\omega \left[v_\phi \sin\theta \, \hat{\boldsymbol{r}} + v_\phi \cos\theta \, \hat{\boldsymbol{\theta}} - (v_r \sin\theta + v_\theta \cos\theta) \, \hat{\boldsymbol{\phi}} \right] \tag{6.4}$$

For example, if a body is moving north, i.e. $v_r = v_\phi = 0$ and $v_\theta < 0$, the Coriolis force acts eastward.

- **Foucault** suspended an iron ball of mass $m = 28$ kg with a very long wire[21] of length $l = 67$ m from the dome of the Pantheon in Paris. Due to the rotation of the Earth, the plan of oscillations will rotate with respect to the surface beneath it. From the view point of an observer in the inertial frame, there are just two forces acting on the bob, the tension $\boldsymbol{T}$ in the string and the force of gravity $m\boldsymbol{g}$, whereas in the rotating frame of the earth, there are also the centrifugal and Coriolis forces. However, the centrifugal force is a second order in in ω, and so its effect will be neglected. Thus, the equation of motion of the mass m in the earth's frame $(\mathcal{R})$ is given by

$$\begin{aligned} m\ddot{\boldsymbol{r}} &= m\boldsymbol{g} + \boldsymbol{T} - 2m \, \boldsymbol{\omega} \times \dot{\boldsymbol{r}} \\ &= -mg\hat{\boldsymbol{z}} + T \left(\hat{\boldsymbol{z}} \, \cos\theta - \hat{\boldsymbol{x}} \, \sin\theta\cos\phi - \hat{\boldsymbol{y}} \, \sin\theta\sin\phi \right) - 2m \, \boldsymbol{\omega} \times \dot{\boldsymbol{r}} \end{aligned} \tag{6.5}$$

[20] Coriolis lived during the 18th and 19th centuries, and published his work on the force caused by the rotation of the Earth in 1835.

[21] Recall that the oscillation period of a pendulum in an inertial frame is $\propto \sqrt{l/g}$, and hence with very long wire it takes the pendulum longer time to swing back and forth. Making the pendulum very heavy surpasses the effect of air resistance.

For small oscillations, we can consider $\theta \simeq 0$, and in this case the component of the tension of the string along the z-axis cancels the weight. Then, in terms of components, Eq (6.5) reads[22]

$$\ddot{x} - 2\omega_z \dot{y} + \omega_0^2 x = 0, \quad \ddot{y} - 2\omega_z \dot{x} + \omega_0^2 y = 0 \tag{6.6}$$

where $\omega_z = \omega \sin\psi$, and $\omega_0^2 = g/L$ is the natural frequency of simple pendulum. Choosing the initial conditions $x(0) = x_0$, $y(0) = 0$, $\dot{x}(0) = 0$, and $\dot{y}(0) = 0$, the solution to the above system of differential equations is given by

$$\begin{aligned} x(t) &= \frac{x_0}{\omega_0}\left(\omega_z \sin\omega_z t \sin\omega_0 t + \omega_0 \cos\omega_z t \cos\omega_0 t\right) \simeq x_0 \cos\omega_z t \cos\omega_0 t \\ y(t) &= \frac{x_0}{\omega_0}\left(\omega_z \cos\omega_z t \sin\omega_0 t - \omega_0 \sin\omega_z t \cos\omega_0 t\right) \simeq -x_0 \sin\omega_z t \cos\omega_0 t \end{aligned}$$

The above solution shows that due to the Coriolis force, from the point of view of an observer on the ground it looks as the pendulum's plane of oscillations precess clockwise at constant angular velocity $\Omega = -\omega_z = -\omega \sin\psi$ with respect to the Earth. However, from a view point of an observer in a reference frame not attached to the Earth the plane of oscillation stays fixed and it is the Earth which rotates beneath it.

- This effect is very small when observed in short time interval, but as the time goes by, the orientation of the oscillation plane changes more and more. Thus, at latitude, ψ, at which the pendulum is placed, the time, T, it takes the oscillation plane of the pendulum to make a full rotation is

$$T = \frac{2\pi}{\omega|\sin\psi|} = \frac{T_\oplus}{|\sin\psi|} \tag{6.7}$$

where $T_\oplus$ is a sidereal period of the Earth, i.e. with respect to fixed stars. For instance, at the Panthéon in Paris, where the latitude is $\psi \simeq 49°$, the period of the oscillation is

$$T|_{\text{Paris}} \simeq 31 \text{ hrs } 48' \tag{6.8}$$

- It is worth mentioning that around **1650**, almost two centuries before Foucault, the astronomer **Giovanni Battista**, suggested that if the Earth is rotating around its axis, then **by dropping a ball from a a tower of about** 75 **meters tall** it would land about **one centimeter to the side**. However, at the time it was not easy to make a measurement with such precision to be able to prove the Earth's rotation around its axis.

[22] Here $x = l\sin\theta\cos\phi$ and $y = l\sin\theta\sin\phi$.

7 How fast does Earth revolve around the Sun?

Quick Answer

$$\upsilon^{(\oplus)}\big|_{\textbf{Equator}} = \mathbf{1675\ km/hr} \simeq 0.47\ \mathbf{km}/s$$

More Details

- The time it takes the Earth to complete one full rotation around its axis is **23 hours** and **56 minutes**. Note this is the sidereal day, and it is **different from** the **24 hours solar day**. The former corresponds to the time that a point on the surface of the Earth return to the same point with respect to to some background stars, whereas the solar day is the time it takes the Sun to return to the same point in the sky. However, since the Earth is orbiting the Sun, it takes 4 minutes for the Sun to return to the same point. Thus, knowing that the Earth's circumference at the equator is 40075 km (see **A** 2), the speed with which a point on its surface at this latitude goes in a full circle around rotational axis of the Earth is given by

$$\upsilon^{(\oplus)} = \frac{40075\ \text{km}}{(23+0.93)\,\text{hours}} \simeq 1675\ \text{km/hr}$$

- Because the Earth is spinning, we (and every object on it) are subject to the centrifugal force that tries to push us off into space. However this centrifugal force is just 0.3% the force of gravity, and hence, its effect is negligible.

- We do not feel the difference in speed either way because everything around us is moving at the same relative speed, even the atmosphere.

- If for some reason the Earth suddenly stopped rotating, we and every thing on the Earth will fly off with speed of 1675 km/hr, but not large enough for us be launched into space[23]. However, the atmosphere would continue moving at the original speed of the Earth's rotation, producing large wind currents.

[23] There are other dramatic effects that make life on Earth unbearable, for example, half the Earth will be permanently facing heat of the Sun, while the other half be exposed to the cold of the space.

8 What is the relationship between sidereal and synodic periods ?

Quick Answer

- **Synodic period of a planet**
- **Synodic period of a planet**
- **Formula** [Derived by **Copernicus**]

$$\begin{cases} \frac{1}{\mathbf{S}} = \frac{1}{\mathbf{E}} - \frac{1}{\mathbf{P}} \\ \frac{1}{\mathbf{S}} = \frac{1}{\mathbf{P}} - \frac{1}{\mathbf{E}} \end{cases}$$

where **E** is the sidereal period of the Earth, **P** sidereal period of the planet, and **S** its synodic period.

More Details

Derivation of the relation between synodic and sidereal periods

First, we note that Earth and any other planet move in their orbits at the rates

$$\omega_{\text{Earth}} = 360^\circ/\mathbf{E}, \qquad \omega_{\text{planet}} = 360^\circ/\mathbf{P} \tag{8.1}$$

Let us chose $t = 0$ to be the instant when a superior planet is in opposition. Then, at the next opposition, the Earth has completed one orbit and then traverse an additional angle $\Delta\theta_{\text{Earth}} = \mathbf{S} \times \omega_{\text{planet}}$ during the time interval $\delta t = (\mathbf{S} - \mathbf{E})$, i.e.

$$\omega_{\text{Earth}}\,(\mathbf{S} - \mathbf{E}) = \omega_{\text{planet}} \tag{8.2}$$

or, equivalently,

$$\frac{1}{\mathbf{S}} = \frac{1}{\mathbf{E}} - \frac{1}{\mathbf{P}} \qquad \textbf{[superior planets]} \tag{8.3}$$

For inferior planet, we just make the interchange $E \leftrightarrow P$ in the above formula, and obntain

$$\frac{1}{\mathbf{S}} = \frac{1}{\mathbf{P}} - \frac{1}{\mathbf{E}} \qquad \textbf{[inferior planets]} \tag{8.4}$$

Planet	P [Yrs]	S [Yrs; calculated]	S [Yrs; observed]
Mercury	0.24	0.3175	0.3175
Venus	0.62	1.604	1.599
Earth	1.00 (365.25 days)	—	—
Mars	1.88	2.111	2.135
Jupiter	11.86	1.092	1.092
Saturn	29.46	1.035	1.035
Uranus	84.01	1.012	1.012
Neptune	164.80	1.006	1.006

9 What is the Copernican model of the solar system ?

Quick Answer

- **In 1543, Nicolaus Copernicus[a], proposed that the Sun was stationary in the center of the universe and the Earth and all the known planets revolved around it in circular orbits.**
- **The Earth rotates around its axis, producing the motion of the celestial objects in the sky.**
- **This is known as the Heliocentric model of the solar system.**
- **At the time of Copernicus the only the known planets were: Mercury, Venus, Mars, Jupiter, and Saturn.**

[a] He was a Polish mathematician, astronomer, and a priest.

More Details

- **Some Historical Background**
  - **Aristarchus of Samos** [circa **300** B.C] was the first to propose the **heliocentric model** for the solar system, where the Sun is the center of the solar system with the Earth orbiting it. In addition, he suggested that the Earth rotates on its axis, causing day and night and accounting for the daily motion of the stars. His reasoning was that it was unnatural for the Sun to orbit the Earth which he estimated its size much smaller than the Sun.
  - His idea was rejected in favor of Aristotle's **geocentric model** in which the **stationary Earth is the center of the universe**[24] and the Sun,

[24] According to Aristotelian worldview, if the Earth spins around its axis from west to east, we should feel a wind blowing constantly from the east.

Moon, and all planets revolve around it[25]. One of the physical argument that many Greek astronomers had against Aristarchus model is that if the Earth was orbiting the Sun why no stellar parallax[26] has been observed[27].

– However, the **geocentric model** could not account for the **retrograde motion**, which is the occasional apparent westward motion of a planet against the stars. Around **150** BC, **Hipparchus of Rhodes** (**160 – 130 BC**) developed a model, building on ideas of the great geometer **Appolonius of Perga** who lived a century earlier[28], to account for the retrograde motion. He proposed that each planet moved with **constant speed** on a small circle, called an **epicycle**, whose center moved **uniformly** on a larger circle, called a **deferent** (see Fig..).

– In the **2**nd century A.D, **Claudius Ptolemy** (**100 – 170 AD**), noted that Hipparchus model did not fit the data well as the observed speed of the planets, Moon, and the Sun, change during different stages[29]. For that, he refined the epicycle-different mechanism by introducing the concept of **equant** (see figure ..), a point on the diameter of the deferent but at a position opposite to that of the Earth from the center of the deferent[30]. So according to Ptolemy, **epicycle** still move about the center of the **deferent**, but the uniform motion about the center of the deferent is now replaced by the uniform angular motion about **equant**. For the next fourteen centuries, this model remained the accepted description of the planetary motion in the solar system.

– Although the the **Ptolemaic geocentric system** was successful in predicting explaining the planetary motion, the epicycles, eccentrics, and equants that each planet requires to fit the observational data made the

[25] **Eudoxus** (circa. **400** BC), one of Plato's students, was the first to propose a universe where all objects in the sky sit on moving spheres, with the Earth stationary at the centre. This is known as "**the homocentric**" model of the universe. He needed a set of **27** interlocking spheres to explain the observed motion of the celestial bodies: the exterior sphere carries the fixed stars; each planet requires four spheres (five planets were known at that time); the Sun and the Moon require three spheres each.

[26] For the definition of stellar parallax see question 22.

[27] Of course, this argument turns out to be incorrect after the invention of the telescope, where the parallax of many astronomical objects were measured.

[28] He was Greek geometer and astronomer who lived during circa **247 222**BC, known for his **Conics**, a treatise in eight books where he introduced and named the conic sections, namely the ellipse, parabola, and hyperbola.

[29] Of course, Hipparchus was aware of this problem, in particular for the Sun and the Moon. To solve this issue he proposed put the Earth not at the center of the deferent, but off its center by a small amount (i.e. eccentric), which he estimated to be about 1/25 the radius of the Sun's deferent.

[30] So, as in the Hipparchus model, the Earth is not at the center of the deferent.

model incredibly complicated. This lead **Nicolaus Copernicus** in **1543** to propose a Sun-centered solar system. In the heliocentric model of Copernicus the apparent retrograde can be easily explained as due to the fact that the earth orbits the sun faster than the outer planets (i.e. Mars, Jupiter, and Saturn) so that the position of one of such planets appear to be moving backward when the Earth overtakes it[31]. Moreover, in the Copernican model the rotation of the Earth's rotation around its axis[32] accounts for the apparent daily rotation of the stars.

- In **1576**, **Tycho Brahe** (**1546 – 1601**), a Danish noble man and astronomer, set up a state-of-the art observatory to measures planetary and stellar motions with unprecedented accuracy[33]. To help him with this endeavor, he hired **Johannes Kepler**(**1571 – 1630**), a German mathematician,...

- After **Brahe** died[34], **Kepler** used the highly precise observations on the planet **Mars** by Brahe[35] and tryied to determine the shape of the orbit that best fit the data. He found that data would fit more accurately if **Mars** move in an **ellipse** rather than a circle[36].

- **Copernicus Derivation Planet's distance from the Sun**

  Before we show how Copernicus determined the distances from a planet to the Sun using Euclide's geometry, we need some definitions (see Fig. **??**).

  ⋆ **Inferior planets** are the planets whose orbits are closer to the Sun than Earth, i.e. the planets Mercury and Venus.
  ⋆ **superior planet** are those planets whose orbits are beyond that of the Earth.
  ⋆ **Elongation** is the angle between the sun and a planet as seen by an observer on earth , which for a superior planet it can vary from 0° to 180°, whereas for inferior planets the maximum elongations are **28°** for **Mercury** and **47°** for **Venus**.
  ⋆ **Opposition** is a configuration when a superior planet and the Sun are on opposite

[31] As seen from Earth, the two inner planets, Mercury and Venus, donâ€™t exhibit retrograde motion because they move faster than Earth.

[32] Why we don't feel a wind .....?

[33] Brahe built his observatory in an island near Copenhagen which was given to him by the king of Denmark, Frederick II who was a great patron of science and art.

[34] was long assumed to have died from poisoning. Recently, archaeologists pored over his skeleton to reveal the real cause of his early death: a fatal combination of obesity, diabetes, and alcoholism (see [5]).

[35] This includes 10 observations at oppositions, which Kepler used to determine the correct shape of Mars orbit.

[36] Initially, Kepler succeeded in finidng circular orbit for Mars that fitted all the observational data to within 2 arcminutes accuracy.

sides of the Earth, and in this case the planet would arise when the Sun sets.
⋆ **Quadrature** configuration occurs when the elongation is 90°, and hence an inferior planet can never be in quadrature.

– **Distance of an Inferior Planet to the Sun**

In Fig. we show an inferior planet at its greatest elongation, which we denote as $\theta_{\rm max}$. It can be shown using simple geometry that the angle between the line Earth-planet, EP, and the line Sun-planet, SP, form a 90° angle. Thus, by measuring $\theta_{\rm max}$, one can determine the planet-Sun distance, $d_{PS}^{(\rm inf)}$, in term of the Earth-Sun distance, $d_{\rm ES} = 1$ AU, i.e.

$$\mathbf{d_{PS}^{(inf)}} = \sin\theta_{\rm max} d_{ES} = \sin\theta_{\rm max}\ [\rm AU] \tag{9.1}$$

Using the measured values of $\theta_{\rm max}$ for Venus and Mercury, we obtain

$$\mathbf{d_{Mercury-Sun}} = 0.47\ [\rm AU]; \qquad \mathbf{d_{Mercury-Sun}} = 0.73\ [\rm AU] \tag{9.2}$$

It is remarkable that the above estimate for Venus is in excellent agreement with modern measurement of the distance from the Sun. For Mercury the calculated value is larger by 20% than the current measurement, which is due to the fact that the orbit of mercury is more elliptical[37] than the one Venus, which means that the $\theta_{\rm max}$ is not strictly a constant. So, a better estimate of Mercury-Sun distance can be obtained by by measuring the different consecutive maximal elongations over one year, take the average, and use it in the above for formula to compute the distance to the Sun. Observations give an average angle $\overline{\theta}_{\rm max} \simeq 22°$, which yields

$$\overline{d}_{Mercury-Sun} \simeq 0.38\ [\rm AU] \tag{9.3}$$

which is in very good agreement with the modern value.

– **Distance of a Superior Planet to the Sun**

At quadrature, the distance from a superior planet to the Sun, $d_{PS}^{(\rm sup)}$, is the hypotenuse of the right triangle ABC, and hence

$$\mathbf{d_{PS}^{(sup)}} = \frac{d_{ES}}{\cos\theta} = \left(\frac{1}{\cos\theta}\right)\ [\rm AU] \tag{9.4}$$

where θ is the angle between the planet and the Earth as viewed from the Sun in this configuration. Thus, by measuring the number of days, ΔT, it takes for a superior planet to move from an opposition to the next quadrature, we can express θ as

$$\theta = \frac{360°\Delta T}{E} - \frac{360°\Delta T}{P} \tag{9.5}$$

[37] Mercury 's orbit has eccentricity $e = \simeq 0.2$, the largest of all the planets in the solar system.

Here E and P are the sidereal periods of the Earth and the superior planet in days. Using the formula for the sidereal and synodic that we derived in the previous question, the above equation reads

$$\theta = \frac{360^\circ \Delta T}{\mathbf{S}} \tag{9.6}$$

with **S** is the synodicl period of the planet. Hence, the planet-Sun distance can be determined in terms of the measured quantities ΔT and **S**, and we get

$$\mathbf{d}_{\mathbf{PS}}^{(\mathrm{sup})} = \left[\cos\left(\frac{\Delta T}{\mathbf{S}} \times 360^\circ \right) \right]^{-1} \ [\mathrm{AU}] \tag{9.7}$$

- **Kepler's Laws of Planetary Motion**

  (i) **Law of Orbits [1609]**:
  The planets orbit the sun in **ellipses**, with the **Sun** at one **focus**.

  (ii) **Law of Areas [1609]**:
  The line joining the Sun and a planet sweeps **equal areas** in **equal times**.

  (iii) **Law of Periods [1619]**:
  The **square of the orbital period of a planet** is **proportional to the cube of the semi-major axis** of the ellipse.

In **1687**, **Isaac Newton** used his universal theory of gravitation to deriver Kepler's laws and also determined the constant of proportionality between the square of the orbital period of a planet, $[\tau_p^{(\mathrm{rev})}]^2$, and the the cube of the semi-major axis of the ellipse, a_p^3, as

$$[\tau_p^{(\mathrm{rev})}]^2 = \frac{4\pi^2}{G\left(M_\odot + m_p\right)} a_p^3 \simeq \frac{4\pi^2}{G M_\odot} a_p^3 \tag{9.8}$$

where $M_\odot$ and m_p are the masses of the Sun and the planet, respectively. The above formula is known as **Kepler's third law**.

10 What is the modern view of the solar system?

Quick Answer

It consists of

- **Sun**
- **8 planets**
- **More than 100 planetary satellites**
- **5 dwarf planets**
- **Asteroids, meteoroids, and comets**

More Details

- **Basic properties of the Sun**
  - It is **109** times bigger than Earth.
  - It makes up **99.8**% of the mass of the entire solar system.
  - It is located at **26, 000 ly** from the center of the Milky Way galaxy.
  - The Sun, and the whole solar system, orbits the center of the Milky Way galaxy once each **260 million years**.

- **The Planets**

  After Newton's time, with the improvement of telescopes, **Two more planets** were discovered: **Uranus [1781]**[38] and **Neptune [1846]**[39]. So in total there are **8 planets** (see table 1 for details), and they are divided into two types:

  (a) **Terrestrial Planets**[40] = **Mercury**[41], **Venus**, **Earth**, **Mars**. They have solid surfaces and are mainly composed of silicate compounds, and their average mass densities range between 3.5 and 5 g/*cc*.
  (b) **Jovian Planets**[42] = **Jupiter**, **Saturn**, **Uranus**, **Neptune**. They have no solid surfaces and their outer layers are composed of light gases. The average densities of Jovian planets are in the range 0.7 to 1.6 g/*cc*.

- **More Moons**

  Except Mercury and Venus all other planets have Moon. There are at least 175 known Moons orbiting six of the eight planets[43]. The 6 largest Moons in our solar system are:
  Ganymede [Jupiter], **Titan** [Saturn], **Callisto** [Jupiter], **Io** [Jupiter], **Earth Moon**, and **Europa** [Jupiter].

- **Asteroid Belt**

  These are rocky interplanetary bodies that are remnants of the planets formation, which orbit the Sun at a distance $[2.2 - 3.2$ AU], **forming a belt of**

[38] It was discovered accidentally by William Herschel
[39] Its existence was predicted by Leverier , a French mathematician, and discovered by....
[40] The term "terrestrial" comes from the work "terra" which means land.
[41] Note that although Mercury is closer to the Sun than Venus, the later is hotter. The reason for this is that Mercury has no atmosphere and so its surface reflects most of the radiation and heat into space, whereas Venus has ticker atmosphere which consisting mainly of carbon dioxide (see table 5), a powerful greenhouse gas, which trap the heat into the atmosphere and make it hotter.
[42] The term "jovian" is derived from Jupiter, and is associated with Jupiter-like planets.
[43] For example, Jupiter has more than 60 Moons, three of them larger than the Earth's Moon

Name	$<d_{pS}>$ [AU]	e	$\delta_{\rm inc}$ [°]	$R/R_{\oplus}$	$T^{\rm min}_{\rm surface}/T^{\rm max}_{\rm surface}$ [C]	$P_{\rm surface}$ [atm]	$\tau_p^{(rev(}$ [Eyears]
Mercury	**0.39**	**0.206**	**7.0**	**0.38**	**- 170/+430**	**0**	**0.24**
Venus	**0.7**	0.007	**3.4**	**0.95**	**+ 465**	**93**	**0.62**
Earth	**1**	**0.017**	**0.0**	**1**	**- 89/+ 58**	**1**	**1**
Mars	**1.5**	**0.093**	**1.9**	**0.53**	**- 143/+ 35**	**0.006**	**1.88**
Jupiter	**5**	**0.048**	**1.3**	**11.2**	**- 108**	**No-SS**	**11.86**
Saturn	**9.5**	**0.054**	**2.5**	**9.45**	**- 139**	**No-SS**	**29**
Uranus	**19**	**0.047**	**0.8**	**4.01**	**- 197**	**No-SS**	**84**
Neptune	**30**	**0.009**	**1.8**	**3.88**	**- 201**	**No-SS**	**164.8**

Table 1: Some general properties of the planets : $< d_{pS} >$ the average distance to the Sun, e is the eccentricity of the elliptical orbit, $\delta_{\rm inc}$ is the planet's orbit to the plane of Earth's orbit, R/R_{oplus} the ratio of the radius of a planet to that of the Earth, $T_{\rm surface}$ surface temperature, $P_{\rm surface}$ the atmospheric pressure at the surface, and τ_{rev} the period of revolution around the Sun in units of Earth years. The abrevition "NO-SS" stands for "No Solid Surface". Note the jovian planets have no solid surface the measurements of the temperature are taken from a level in the atmosphere equal in pressure to sea level on Earth.

about 1 **AU** lying **between the orbits of the planets Mars and Jupiter**, and there are about two millions asteroids in belt, about a **million of them have more than a kilometer wide**[44]. The first and largest asteroid in this belt was in 1801 by the Italian astronomer **Giuseppe Piazzi**, which he named it **Ceres** after the Roman goddess of agriculture and fertility. Sometimes asteroids collide with each other and break up into small objects called **meteoroids**. Their size ranges. from dust grains to about few meters wide objects. When a meteoroid enters the atmosphere .....

- **Dwarf planets**

  There are 5 dwarf planets in the solar system: **Pluto**, **Ceres**, **Pluto**, **Make-make**, **Haumea**, and **Eris**.

- **Comets**

  They are made of rocks, dust and ice; sometimes it is called **dirty snowball**, which go around the Sun in elliptical highly-eccentric orbits. As a comet gets closer to the Sun, the Solar radiation vaporizes some of the comet's surface material causing them to form a tail of dust and ionized gas. Comets originate in **the Kuiper Belt** at **30 – 50** AU and the Oort Cloud at about 50, 000 AU ,

[44] Based on geological evidence, around 66 million years ago Earth was hit by an asteroid of 10 km in diameter at a speed of 10 km/s that caused the extinction of all non-avian dinosaurs and about 70% of all plant and animal species.

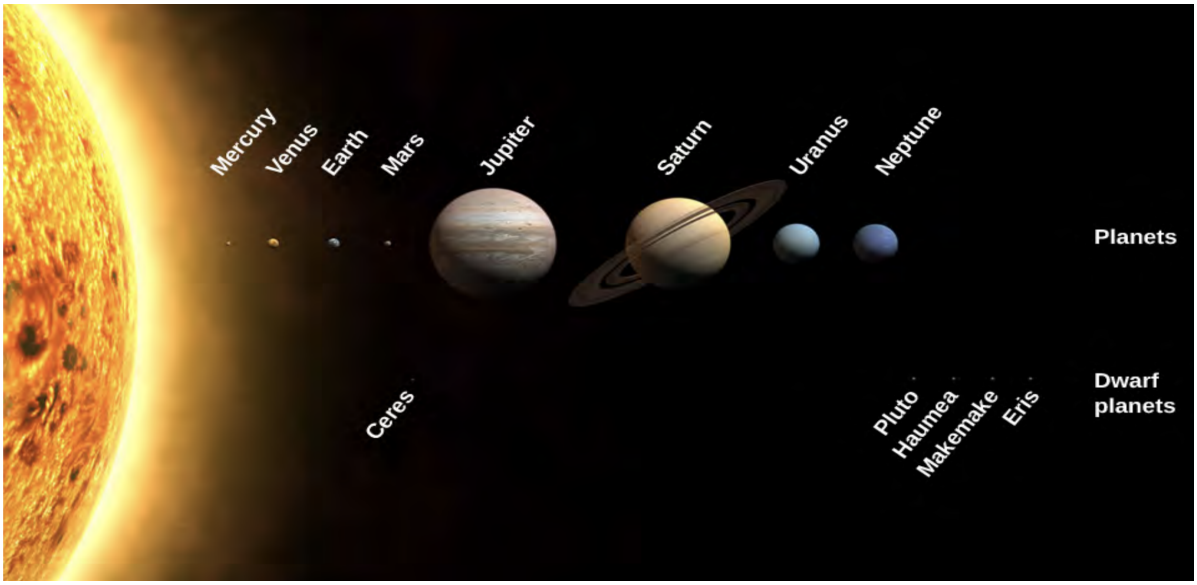


Figure 8: The Sun, the eight planets, and the five dwarf planets in the solar system [credit: NASA].

Name	$<d_{pS}>$ [AU]	R/R_{Mercury}	$T^{\text{average}}_{\text{surface}}$ [C]	Year of discovered	# of Moons
Ceres	**2.8**	**R**	**- 106**	**1801**	**0**
Pluto	**0.7**	**R**	**- 228**	**1930**	**5**
Haumea	**1**	**1**	**- 240**	**2004**	**2**
Makemake	**1.5**	**R**	**- 243**	**2005**	**1**
Eris	**5**	**R**	[**- 248** to **- 232**]	**2005**	**1**

Table 2: Some general properties of the 5 dwarf planet in our solar system and the years they were discovered.

orbiting the Sun in elliptical highly-eccentric orbits. Comets are usually named after their discoverers. For example, the famous **Halley's comet** can be visible from Earth every **75** years. The last time it was seen was in 1986, thus it is expected to return in 2061.

11 What is Bode's Law?

Quick Answer

- In **1772**, the German astronomer **Bode** proposed an **empirical rule** that gives approximate **distances of planets from the Sun**.

More Details

Start with the following sequence of numbers

$$0, 3, 6, 12, 24, 48, 96, 192, 384$$

Now, add 4 to each number above:

$$4, 7, 10, 16, 28, 52, 100, 196, 388$$

Then, divide each number by 10:

$$0.4, 0.7, 1.0, 1.6, 2.8, 5.2, 10.0, 19.6, 38.8$$

If we set the distance Earth-Sun = 1 (AU), then we see that the numbers in the above sequence approximately represent the distance of the planets from the Sun, except the the value 2.8. However, in 1801, an astronomical object at about 2.8 AU from the Sun was discovered. It can be expressed in a mathematical formula as

$$\mathbf{d_n} = \mathbf{0.4} + \mathbf{0.3} \times \mathbf{2^n}, \quad n \in 0, 1, 2, 3, 4, 5, 6 \tag{11.1}$$

Here the number 0.4 in the above relation represent the distance Mercury-Sun, n the number of planet of the planet with $n = 0$ for Venus, $n = 1$ for the Earth, and so on, and $\mathbf{d_n}$ is the distance of the nth planet from the Sun.

	n							
		0	1	2	3	4	5	6
	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Bode's law	0.4	0.7	1.0	1.5	2.8	5.2	9.6	19.2
Actual distance	0.4	0.7	1.0	1.6	2.8	5.2	10	19.6

12 What is the universal law of gravitation ?

$$\boldsymbol{F}_g = G\ \frac{m_1 m_2}{d^2}$$

where

m_1, m_2 : **masses of the two objects**

G : **Gravitational Constant**

d: the distance between the two masses

More Details

- In **1687**, **Isaac Newton** published the universal law of gravitation and his three laws of motion in "The Mathematical Principles of Natural Philosophy", commonly known as the **Principia**.

- In **1749**, the French geophysicist **Pierre Bouguer** tried to measure the density of the Earth using the deflection of a plumb line due to the attraction of a mountain. That would allowed him to determine the strength of the universal gravitational force. Unfortunately, he did not succeed as the deviations of pendulum were extremely tiny to be measured. The same method was applied by N. Maskelyne and C. Hutton in 1755 and determined that the average density of the Earth to be between $4.5 - 5$ g/cc, which is of the same order of the current measured value.

- It was **Henri Cavendish** who, in **1798**, performed an extremely delicate experiment that allowed a precise measurement of the gravitational constant [45]. **The setup** is as follows: A dumbbell consisting of two identical masses m attached to its ends and suspended horizontally from a very thin torsion wire. Two other much heavier balls of identical mass M are placed at the positions shown in the figure(see Fig. 9).[46] Theses masses produce attractive forces on the two masses m and cause the dumbbell to twist. The dumbbell will oscillate back and forth before finally settling down at some tiny angle θ_* away from the initial position due to the restoring torque provided by the wire. With θ_* being very small, the torque on the dumbbell that arises from the twist takes the form of Hooke's law

$$\tau = -\kappa\theta_* \tag{12.1}$$

Here κ is a constant that depends on the thickness and the material from which the wire is made of. If we denote by d_* the separation at equilibrium between the centers of the masses in each pair, then gravitational force between each pair of is

$$F_g = G\,\frac{Mm}{d_*^2} \tag{12.2}$$

Hence, the torque on the dumbbell due to the two gravitational forces is $(GMm/d_*^2)\,L$, where L is distance between the small balls of the dumbbell. Applying the equilibrium condition, i.e. the total torque equals to zero, we get

$$G = \frac{\kappa\ \theta_* d_*^2}{MmL} \tag{12.3}$$

[45] Actually, Cavendish intended this experiment to be a measurement of the Earth's density relative to water through the precise knowledge of the gravitational interaction.

[46] The experiment was originally devised by the geologist **John Mitchell** around 1780, but he died before he could begin taking measurements. However, Cavendish substantially improved Mitchell's apparatus.

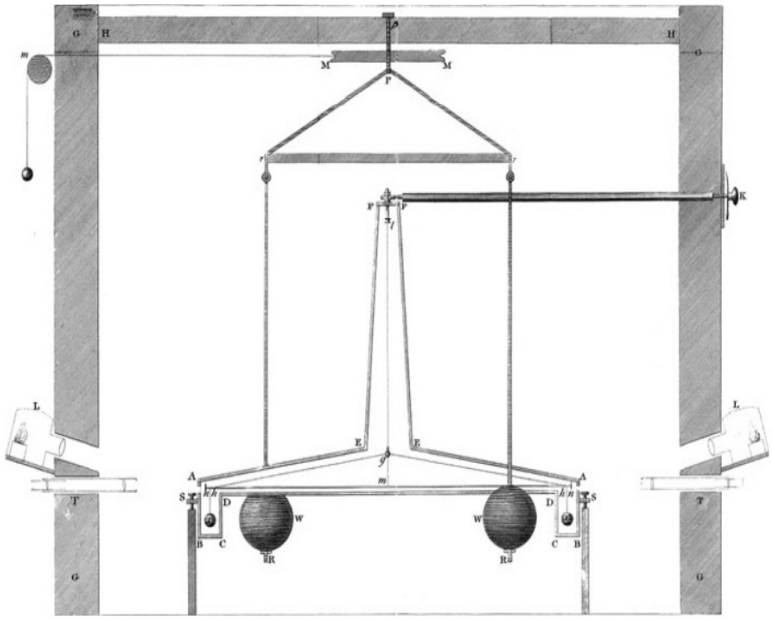


Figure 9: Cavendish' experimental apparatus taken from [6].

Note that for θ_* large enough for it to be measured, the torsion proportionality constant κ needs to be as small as possible, and so it is desirable to have l as large as possible[47]. For that, Cavendish made use of high precision vernier scale and a small telescope to aid the detection of the rotation angle, and measures the displacement ΔS_* of the small ball[48]. So by measuring the The constant κ can be obtained in terms the moment of inertia of the dumbbell, I, and its oscillation period, T_0, as follows. By twisting the dumbbell (in the absence of the large masses) by small angle and let it oscillate around its equilibrium position, the equation of motion reads

$$I\ddot{\theta} + \kappa\ \theta = 0 \tag{12.4}$$

This is just the equation of harmonic oscillator with frequency $\omega = \sqrt{\kappa/I}$, and so by measuring the period of the oscillations one can determine the constant

[47] For a given material, **the torsion constant** is proportional to the fourth power of its diameter and **inversely proportional to its length**.

[48] It is a very challenging task to precisely determine the deflection angle even with the telescope that Cavendish used. In modern experiments, to overcome this difficulty a small mirror is attached to the center of the rod to reflect a light beam on a wall where the angular motion can be more easily observed. If the wall is at a distance D from the mirror, then when the mirror rotates with an angle θ_*, the spot on the wall from the reflected light beam moves a distance $x = D\tan 2\theta_* \simeq 2D\theta_*$. A nice animation of this type of setup can be viewed at https://www.youtube.com/watch?v=EE9TMwXnx-s.

κ from the relation

$$\kappa = I\left(\frac{2\pi}{T_0}\right)^2 \tag{12.5}$$

Plugging this expression of κ into (12.3) yields[49]

$$G = \frac{4\pi^2 I d_*^2 \theta_*}{MmLT_0^2} \tag{12.6}$$

or, equivalently, in terms of the displacement ΔS_*, we obtain

$$G = \frac{8\pi^2 I d_*^2 \Delta S_*}{MmL^2 T_0^2} \tag{12.7}$$

In this experiment Cavendish used $m = 0.73$ kg, $M = 158$ kg, $d_0 := (d_* + \Delta S_*) = 22.5$ cm, $L = 186$ cm, and measured the oscillation period $T_0 = 14.6$ min and the displacement of the small ball $\Delta S_* = 4.1$ mm[50]. With these values one finds[51]

$$G^{(\text{Cavendish})} = 6.74 \times 10^{-11}\ N.m^2/kg^2 \tag{12.8}$$

which is very close to present-day measured value.

13 What is the minimum period of rotation a spherical astronomical object can have ?

Quick Answer

A spherical object with mass density ρ can not have a rotation period less than

$$T_{\text{min}} = \sqrt{\frac{11\ g/\text{cm}^3}{\rho}}\ \textbf{hours}$$

[49] If we approximate the moment of inertia by $I = m\,(L/2)^2 + m\,(L/2)^2 = ml^2/2$, then the expression of G_N reads

$$G = \frac{2\pi^2 I \theta_* d_*^2}{MT_0^2}$$

which is independent on the value of the mass m.

[50] He determined the displacement with to an accuracy of better than 0.25 mm.

[51] We should point out that Cavendish did not explicitly derive the expression of G, and in fact it is not even mentioned in his paper. Instead he computed the density of the Earth which is related to the gravitational constant (see ***A7***). The first reference to the universal gravitational constant was in 1873 by Alfred Cornu and Baptistin Baile (which was denoted by the letter f), about 75 years after Cavendish's work, and the modern notation with the letter G was introduced in 1894 Charles . V. Boys (see ref. [7]).

More Details

Let R be the radius of a spherical astronomical object with mass density ρ that we assume to be uniform though out its volume. When the object is spining it is subject to a centrifugal force, acting outward. For a point mass δm at the equator of the object, this force is radial of magnitude $F_c = \delta m a_c$, where a_c is the centrifugal acceleration, given by

$$a_c = \frac{v^2}{R} \tag{13.1}$$

In addition, gravity exerts a radially inward force $F_g = \delta m g$, with g being the gravitational acceleration at the surface of the object. It reads

$$g = \frac{GM}{R^2} = \frac{4\pi}{3} G\rho R \tag{13.2}$$

Here M is the mass of the object, and G is Newton's gavitational constant (see section 12). In obtaining the second equality we made use of the assumption that ρ is constant.

In order for the spinning object not to disintegrate, the gravitational force must be larger or equal to the centrifugal force, i.e.

$$v \leq R\sqrt{\frac{4\pi}{3} G\rho} \tag{13.3}$$

If T is the period of rotation, then we can use the fact that $vT = 2\pi R$ to express the above inequality as can we can

$$T \geq \sqrt{\frac{3\pi}{G\rho}} \quad \Longrightarrow \quad T_{\text{min}} = \sqrt{\frac{3\pi}{G\rho}} \tag{13.4}$$

From this we that the minimum period depends only on the mass density of the object and not its mass or radius. We also note that the denser the object is the larger its rotation frequency is.

In table z, we show the minimum period required for a spining terrestrial planet, a star similar to the Sun, and a neutron star. We also show the typical observed values of the their rotation periods.

14 What is the mass of the Earth ?

Quick Answer

$$M_\oplus \simeq 6 \times 10^{24} \text{ kg}$$

More Details

- Consider the Earth to be a perfect sphere of radius $R_\oplus$, and mass $M_\oplus$. Let a test particle of mass, m, free falls toward the ground from some distance much smaller than the Earth's radius. Then, according to the universal law of gravitation (see A6) and Newton?s second law of motion we have

$$\frac{GM_\oplus m}{(R_\oplus + r)^2} = mg(h) \tag{14.1}$$

where h is the distance from the ground to the mass m at some instant t during the free fall, g(h) is its acceleration due to gravity at the height h, and G is the universal constant of gravity. After canceling the common factor m from both sides of the equation, and neglecting the contribution of h as compared to $R_\oplus$, we obtain

$$M_\oplus = \frac{R_\oplus^2 g_\oplus}{G} \tag{14.2}$$

Here $g_\oplus = g(h = 0)$ is the acceleration of gravity at the surface of the Earth[52]. Thus, using $R_\oplus \simeq 6370$ km (see ***A2***), $G = 6.67 \times 10^{-11}$ $N.m^2.\text{kg}^{-2}$ (see ***A5***), and $g_\oplus \simeq 9.8$ m/s^2, we obtain

$$M_\oplus = \frac{6.37 \times 10^6 \text{ m}^2 \times 9.8 \text{ m}/s^2}{6.67 \times 10^{-11} \ \ N.m^2.\text{kg}^{-2}} \simeq 6 \times 10^{24} \text{ kg}$$

- Since we now know the mass of the Earth and its radius, we can calculate its average density from the formula

$$\overline{\rho}_\oplus = \frac{\mathbf{M}_\oplus}{(4\pi/3)\, R_\oplus^3} \simeq \mathbf{5.5} \text{ g/cm}^\mathbf{3} \simeq \mathbf{5.5} \times \rho_{H_2O} \tag{14.3}$$

Since most of the rocks have density of about 3 g/cm^3, we conclude that the Earth?s interior must be more dense than typical crustal rocks in order for the whole thing to average to about 5.5 g/cm^3.

[52] The value of $g_\oplus$ can be easily measured by either recording the positions, $x(t)$, that a test particle moves at different instances, t, during the free fall, or, by measuring the period of oscillations of a simple pendulum of a given length since its period is a function of the acceleration of gravity at the Earth's surface.

- **Another method**: If we know the distance from the Earth to the Moon, we can determine Earth's mass using Kepler's third law and the measured value of the gravitational constant. In the same way, one can infer the mass of a stars that form a orbiting binary star-system with another one (see ***A12*** for details).

15 What is gravity at the surface of an astronomical object ?

Quick Answer

$$\mathbf{g^{(surface)} = \frac{GM}{R^2}}$$

More Details

- The above expression for the gravitational acceleration can be obtained using Newton's universal law of gravitation and Newton's second law of motion, which for a test mass m located at (or near) the surface of an astronomical object of mass M, reads

$$-\frac{GMm}{R^2}\,\hat{\boldsymbol{r}} = m\boldsymbol{a} \tag{15.1}$$

where $\hat{\boldsymbol{r}}$ is a unit vector directed from the center of the celestial body to the test mass. Hence, the acceleration of gravity at the Earth's surface is given by

$$\boldsymbol{a} = -\frac{GM}{R^2}\,\hat{\boldsymbol{r}} \equiv -g^{(\text{surface})}\,\hat{\boldsymbol{r}} \tag{15.2}$$

with $g^{(\text{surface})}$ is the magnitude of the gravitational field at the Eearth's surface.

- In table 3, we give the values of the acceleration of gravity for the Moon and the planets in our solar system.

Object	g [m/s]	v_e (km/s)
Sun	**273.4**	**617.5**
Mercury	3.7	4.3
Venus	8.9	10.3
Earth	**9.8**	**11.2**
Mars	**3.7**	**5**
Jupiter	24.8	59.6
Saturn	9	35.6
Uranus	8.7	21.3
Neptune	11	23.8
Moon	**1.6**	**2.4**

Table 3: The values of the acceleration of gravity and the escape velocity for the Sun, the Moon, and the Planets.

16 What is the escape velocity from the Earth?

Quick Answer

$$v_{\rm esc}^{(\oplus)} = \mathbf{11.2}\ \mathrm{km/s}$$

More Details

- The **escape velocity** is the minimum speed v_e that an object needs to have to break free from the surface of a large body such as moon, planet or a star. Suppose that an object of mass m leaves the surface of stationary object of mass $M >> m$, with initial speed v_0. If, for simplicity, we consider the astronomical object to be spherical of radius R, and assume it has no atmosphere, then the energy conservation $E(r = R) = E(r), \forall r > R$, implies that

$$\frac{1}{2}mv_0^2 - G\frac{Mm}{R} = \frac{1}{2}mv_\infty^2 \tag{16.1}$$

where v_∞ is the speed of the particle at $r = \infty$, i.e. at the point where the mass m has completely escaped the effect of the gravity due to the mass M. Since the right hand side of the above equation can not be negative, the escape velocity correspond to the value of v_0 for which $v_\infty = 0$, i.e.

$$v_{\rm esc} = \sqrt{2\frac{G_N M}{R}} \tag{16.2}$$

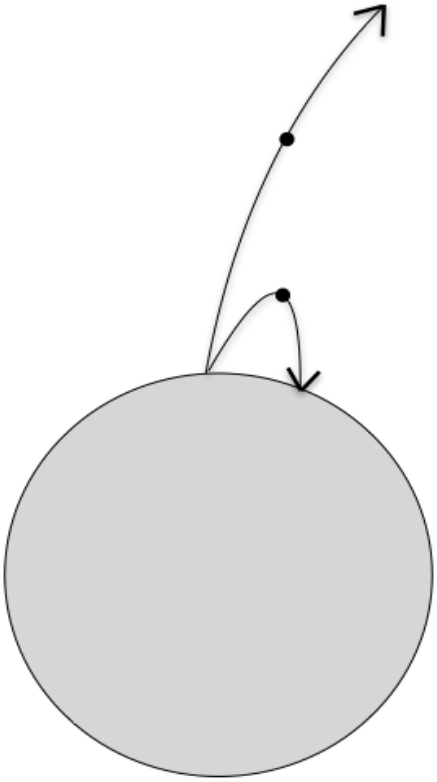


Figure 10: Escape velocity.

which is independent of the mass of the escaping object[53]. Applying the above formula for the escape velocity to the Earth, and using $M_{\oplus} \simeq 6 \times 10^{24}$ kg (see ***A6***), we obtain

$$\upsilon_{\rm esc}^{(\oplus)} = 6.9 \text{ m}/s = 11.2 \text{ km}/s \tag{16.3}$$

- In table 3, we give the values of the escape velocity for different planets in the solar system and some other astronomical objects[54].
- In the absence of the atmosphere, to get a spacecraft from Earth's surface into Earth's orbit would require that the speed of the rocket to be 7.8 km/s[55].

17 What is the age of the Earth?

Quick Answer

[53] It is important to note that the above calculation neglects the effect of air resistance.

[54] **Titan** is the Saturn's largest moon, and **Europa** is one of Jupiter's moon.

[55] Because o the Earth's atmosphere, the rocket needs to have a speed of about 10 km/s to overcome the air resistance.

Radioactive Element	Symbol	Half-life
Radon-222	^{222} Rn	3.8 days
Carbon-14	^{14} C	**5, 730 years**
Uranium-235	^{235} U	**0.7 billion years**
Potassium-40	^{40} K	**1.25 billion years**
Uranium-238	^{238} U	**4.47 billion years**
Thorium-232	^{232} Th	14.1 billion years
Rubidium-87	^{87} Rb	47 billion years

Table 4: Common radioactive elements (found in rocks, water, and air) .

$$\tau_{\oplus} \simeq \mathbf{4.5} \text{ billion years}$$

- The oldest known rocks on Earth were dated at **4.28 billion years**. They were discovered in 2008, in northern Quebec, Canada.
- The oldest minerals found on Earth are **zircon** with an age of **4.40 billion years**. They were discovered in 2014 in the Jack Hills of western Australia.

More Details

- **Radiometric dating**:

  The dating of a material, such as a rock or a fossil, is based on the method of **radiometric dating**, which is the measurement of the radioactive decay[56] of an unstable isotope[57] (parent) into more stable element. In table 5 we list some of the radioactive elements that are widely used in radiometric dating.

- **The equation of a radioactive decay process**:

  Consider a system of radioactive nucleus, P, with $N_P(0)$ their number at the instant $t = 0$. We assume that each particle decays into a stable daughter nucleus, D, with equal probability per unit time, λ, also known as the decay constant for the decay process. Then, during the time interval $[t, t + dt]$, the decrease in the number of the P particles, N_P, and the growth in the number of the D particle, N_D, are given by

[56] Radioactivity was discovered in **1896** by the French physicist Henri Becquerel.

[57] Isotopes are atoms that their nuclei contain the same number of protons but have different number of neutrons. This means they have different mass but their chemistry is unchanged since the outer shell configuration will be the same.

$$\begin{cases} dN_P = -\lambda N_P dt \\ dN_D = \lambda N_D dt \end{cases} \tag{17.1}$$

Note that $d\left(N_P + N_D\right) = 0$, which is implies that the total number of particles (i.e. parents and daughters) is conserved, i.e.

$$N_P + N_D = N_P(0) + N_D(0) \tag{17.2}$$

Now the integration of (17.1) yields[58]

$$\begin{cases} N_P(t) = N_P(0)\, e^{-\lambda t} \\ N_D(t) = N_D(0) + N_P(t)\left(e^{-\lambda t} - 1\right) \end{cases} \tag{17.3}$$

This exponential distribution of the parent particles can be interpreted in term of a **decay probability**, $d\mathcal{P}(t)$, for a single particle to disintegrate between the instant t and $t + dt$, given by

$$d\mathcal{P}(t) = \lambda e^{-\lambda t} dt \tag{17.4}$$

Thus, the probability for particle to decay within a time t is[59]

$$\mathcal{P}(t) = \int_0^t d\mathcal{P}(t)\; dt = 1 - e^{-\lambda t} \tag{17.5}$$

Using this probability distribution, the mean lifetime of the radioactive particle P can be calculated as[60]

$$\tau = \int t\; d\mathcal{P}(t) = \boldsymbol{\lambda^{-1}} \tag{17.6}$$

Another related quantity is the half-life, $\tau_{\mathbf{1/2}}$, defined as[61]

$$\tau_{\mathbf{1/2}} \simeq 0.7\; \tau \tag{17.7}$$

[58] In obtaining the equation of $N_D(t)$, we made use of Eq (**??**).

[59] This is also known as **Poisson distribution**, and it is normalized to unity.

[60] Equivalently, it can be obtained as

$$\tau = \frac{1}{N_P(0)} \int_{t=0}^{\infty} t\; dN_P = \frac{1}{N_P(0)} \int_{t=0}^{\infty} t\; dN_P = \frac{1}{N_P(0)} \int_{t=0}^{\infty} t\; \lambda N_P\; dt = \lambda \int_{t=0}^{\infty} t\; e^{-\lambda t}\; dt = \lambda^{-1}$$

[61] This relation can be shown as follows:

$$N(\tau_{1/2}) = N_0 e^{-\lambda \tau_{1/2}} := \frac{N_0}{2} \quad \rightarrow \quad \tau_{\mathbf{1/2}} = \frac{\ln \mathbf{2}}{\lambda}$$

Now, suppose we know $N_P(0)$ or its ratio compared to some stable element in the sample[62], P', then from Eq (17.3) the the age is simply

$$t = \tau \ \ln\left(\frac{N_P(t=0)}{N_P(t)}\right) = \tau \ \ln\left[\frac{(N_P/N_{P'})\,(t=0)}{(N_P/N_{P'})\,(t)}\right] \tag{17.8}$$

We can generalize the above results to the case when the parent nucleus decay to many daughter nuclei, $D_1, D_2, ..., D_n$, with decay constants $\lambda_1, \lambda_2, ..., \lambda_n$, i.e.

$$P \to D_i + ..., \qquad i = 1, 2, ..., n$$

Then,

$$\begin{cases} dN_P = -\lambda N_P dt, \\ dN_{D_i} = \lambda_i N_P dt, \qquad i = 1, 2, ..., n \\ d\left(N_P + \sum_{i=1}^{n} N_{D_i}\right) = 0 \end{cases} \tag{17.9}$$

where $\lambda = \sum_{i=1}^{n} \lambda_i$ is the total decay constant for these processes. Thus, integrating the above differential equations gives

$$\begin{cases} N_P(t) = N_P(0)\ e^{-\lambda t} \\ N_{D_i}(t) = N_{D_i}(0) + \frac{\lambda_i}{\lambda} N_P(0)\left(1 - e^{-\lambda t}\right), \quad i = 1, 2, ..., n \\ N_P + \sum_{i=1}^{n} N_{D_i} = N_P(0) + \sum_{i=1}^{n} N_{D_i}(0) \end{cases} \tag{17.10}$$

- **Carbon-14 dating** [$\leq$ **70, 000** years[63]]:

  The interaction of high energy cosmic rays (mostly protons, and helium nuclei) with the atmosphere produce showers of many types of subatomic particles, including energetic neutrons[64]. These neutrons collide with the nitrogen atoms in the atmosphere producing a hydrogen and a carbon-14, i.e.[65]

$$n + {}^{14}N \to {}^{14}C + p$$

  These ${}^{14}C$ atoms combine with oxygen molecules, forming carbon dioxide, which spreads around at low altitudes, where it is used by the plants, and hence by all living things.

  At any particular time, the ratio of ${}^{14}C$ to ${}^{12}C$ is approximately $\mathbf{10^{-12}}$ in all living organisms. We will denote this ratio by $\mathbf{R_0}$. However, once an organism dies the carbon-14 starts to decay to nitrogen via the β-process:

$${}^{14}C \to {}^{14}N + e^{-} + \bar{\nu}$$

[62] For example, one of the stable particle of the decay product.

[63] Beyond an age of 50, 000 years the amount of ${}^{14}C$ left in them is so tiny that it can not be used for a reliable dating.

[64] This happens at high altitudes around $9 - 15$ km in the atmosphere.

[65] For thermal neutrons, $\sigma(n + {}^{14}N \to {}^{14}C + p) \simeq 1.8 \times 10^{-24}\ cm^2 \equiv 1.8$ barn.

where $\bar{\nu}$ is the anti-neutrino, a neutral spin 1/2 particle with almost vanishing mass. Thus, by measuring the ratio $R(t) = \frac{^{14}C(t)}{^{12}C}$, then compare it to the assumed initial value R_0 in a living organism, scientists can determine the time elapsed since it died. Using Eq (17.8) and the value of $\tau_{1/2}$ of ^{14}C from table 5, we obtain

$$[t]_{^{14}\mathrm{C}/^{12}\mathrm{C}} = 8022 \ \ln\left(\frac{R_0}{R(t)}\right) \textbf{ years}$$

- **Potassium-Argon dating**: [$\geq$ **10^4** years]:
  - Potassium is present in Earth's crust[66] with a concentration of about 1.5%, and its naturally occurring radioisotopes, **^{40}K** make up 0.12% of natural potassium, and hence it can be used in dating rocks. About 11% of ^{40}K decays to ^{40}Ar and 89% to ^{40}Ca via the following processes:

$$^{40}K \rightarrow ^{40}Ca + e^- + \bar{\nu}; \quad [\beta \text{ process; } 89\%; \quad \lambda_\beta = 4.96 \times 10^{-10} \text{ years}]$$
$$^{40}K + e^- \rightarrow ^{40}Ar + \nu; \quad [\text{electron capture; } 11\%; \quad \lambda_e = 0.58 \times 10^{-10} \text{ years}]$$

  - However, because ^{40}Ca can be present both as radiogenic and non-radiogenic[67], and so only the first decay process can not be used for dating rocks. Instead, geologists make use of the decay to argon by electron capture and measure the ^{40}Ar/^{40}K ratio in materials that trap argon. Furthermore, since argon is a noble gas it can escape easily from the extremely hot molten magma, and so it is reasonable to assume that there was no argon initially when the rocks formed. Thus, the argon in a rock today is that which was produced by ^{40}K since the rock has cooled down.
  - Now, applying Eq (17.10) to the decay processes above, we obtain

$$N_{Ar}(t) = \frac{\lambda_e}{\lambda} N_K(t) \left(e^{\lambda t} - 1\right) \tag{17.11}$$

    where $\lambda = \lambda_e + \lambda_\beta = 5.54 \times 10^{-10}$ years is the total decay constant of ^{40}K to argon and calcium. Solving for t, we obtain

$$\begin{aligned}[t]_{\mathrm{Ar/K}} &= \frac{1}{\lambda} \ln\left[\frac{N_{Ar}(t)}{N_K(t)}\left(\frac{\lambda}{\lambda_e}\right) + 1\right] \\ &= 1.8 \ln\left[9.55\left(\frac{N_{Ar}(t)}{N_K(t)}\right) + 1\right] \textbf{ billion years}\end{aligned}$$

[66] As well in oceans and all organic materials.

[67] It means can be produced by another radioactive decay process. So it would be impossible to distinguish what proportion of ^{40}Ca that was produced by potassium.

– **Uranium-lead dating**: [$\geq 10^7$ years]

The naturally occurring uranium consists primarily of two radioisotopes: ^{238}U and ^{235}U, which produce ^{206} Pb and ^{207}Pb, respectively, with the emission of α particles, via long decay chains. Another isotope of lead is ^{204}Pb which is non-radiogenic, and so using Eq (17.3) one can get two independent dates from the uranium-lead system:

$$\left(\frac{^{206}\text{Pb}}{^{204}\text{Pb}}\right)(t) - \left(\frac{^{206}\text{Pb}}{^{204}\text{Pb}}\right)(0) = \left(\frac{^{238}\text{U}}{^{204}\text{Pb}}\right)(t)\left[e^{\lambda_{238}t} - 1\right] \tag{17.12}$$

$$\left(\frac{^{207}\text{Pb}}{^{204}\text{Pb}}\right)(t) - \left(\frac{^{207}\text{Pb}}{^{204}\text{Pb}}\right)(0) = \left(\frac{^{235}\text{U}}{^{204}\text{Pb}}\right)(t)\left[e^{\lambda_{235}t} - 1\right] \tag{17.13}$$

Both these equations is has the form $y = mx + y_0$, where y is the ratio of the amount of lead produced from radioactive decay to ^{204}Pb, and they are known as the **isochron equations**.Taking the ratio of the above equations, we obtain

$$\frac{\left[\frac{^{207}\text{Pb}}{^{204}\text{Pb}}\right]_{\text{Now}} - \left[\frac{^{207}\text{Pb}}{^{204}\text{Pb}}\right]_{\mathbf{0}}}{\left[\frac{^{207}\text{Pb}}{^{204}\text{Pb}}\right]_{\text{Now}} - \left[\frac{^{207}\text{Pb}}{^{204}\text{Pb}}\right]_{\mathbf{0}}} = \left(\frac{^{235}U}{^{238}U}\right)\frac{e^{\lambda_{235}t} - 1}{e^{\lambda_{238}t} - 1} = \left(\frac{\mathbf{1}}{\mathbf{137.8}}\right)\frac{\mathbf{e^{\lambda_{235}t} - 1}}{\mathbf{e^{\lambda_{238}t} - 1}}$$

where we used the present day ratio $^{235}U/^{238}U$ is 137.8, as it has been measured in terrestrial, lunar, and meteorite samples. The above equation is known as **Pb-Pb isochrone equation**. For a rock or meteorite, this equation represents a straight line with $[^{207}\text{Pb}/^{204}\text{Pb}]_{\text{Now}}$ is, say on the y-axis, and $[^{206}\text{Pb}/^{204}\text{Pb}]_{\text{Now}}$ on the x-axis, and the left-hand side of the equation is the its slope. So if the initial ratio of Pb are known or inferred from minerals that essentially had no lead initially present in them[68], then one can determine their age.

18 What is the critical temperature of a planet's atmosphere above which a particular atmospheric component will be lost into space ?

Quick Answer

$$T_{\text{critical}} = \frac{1}{54}\frac{GMm}{k_B R}$$

More Details

[68] Such as zircon, apatite, or monazite.

- **The most probable speed of a molecule in a gas**

  Whether a planet has an atmosphere depends on the ratio of the molecular speed in the gas and the escape velocity from the planet. Thus, we will first calculate the mean speed of molecules in a gas. For that, we recall that in thermal equilibrium, the velocity distribution of particles of mass m is

$$f(v)d^3\boldsymbol{v} = \mathcal{N}\left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-mv^2/2k_BT} v^2 dv \sin\theta d\theta d\phi \tag{18.1}$$

  Integrating over the angles θ and ϕ, we obtain the speed distribution function

$$f(v)dv = \mathcal{N}\sqrt{\frac{2}{\pi}}\left(\frac{m}{k_B T}\right)^{3/2} e^{-mv^2/2k_BT} dv \tag{18.2}$$

  The most probable speed, v_0, is given by the speed at which the derivative of $f(v)$ vanishes. We find[69]

$$v_0 = \sqrt{\frac{2k_BT}{m}} \longrightarrow \boxed{f(v) = \frac{1}{\pi^{3/2}}\frac{1}{v_0^3}\ \exp\left(-\frac{v^2}{v_0^2}\right)} \tag{18.3}$$

  The speed v_0 given above can also be expressed in terms of the molar mass $\mathcal{M}$ of the gas molecules as

$$v_0 = \sqrt{\frac{2\mathcal{R}T}{\mathcal{M}}} \tag{18.4}$$

  where $\mathcal{R} = 8.314$ J/(gram-mol.K) is called the universal gas constant. Thus, at a given temperature, the more massive the molecules of a gas, the lower is their mean speed. This is why it is more easy for hydrogen and helium to escape the Earth rather than does carbon dioxide, when the temperature of the gas is large enough so that $v_0 > v_{esc}$. With the Earth's average temperature of 15° (i,e, approximately 280 K), the oxygen and nitrogen gases, have average speeds of about 380 m/s and 406 m/s, respectively. These are much smaller than the escape velocity from the Earth (see **A(09)**), which explains why these gases are still present in the atmosphere.

- **Naive estimate of the critical temperature**

  A particle can escape the surface of a planet if its kinetic energy exceeds the gravitational energy of the planet. For a gas molecules, this is equivalent to

[69] Note that the most probable speed is not the average speed. The later is given by

$$\overline{v} = \int_0^\infty v\ f(v)dv = \sqrt{\frac{8k_BT}{m}} = 2v_0$$

the condition

$$v_{\rm rms} \geq v_{esc} = \sqrt{\frac{2GM}{R}} \tag{18.5}$$

which in terms of temperature, yields

$$T \geq \frac{2GMm}{3k_B R} \tag{18.6}$$

Hence, for $T \leq 1000$ K, a very tiny fraction of the molecules in the gas have such high speed, and one might think this will have no effect on particles loss from the atmosphere.

In reality, even if $v_0 < v_{esc}$, there will always be particles in the high-speed tail of the distribution in (18.2) that have speed larger than the escape velocity which leak into space. Although the fraction of such particles is tiny, the remaining particles will redistribute them selves so that the high-speed tail get reproduced so that their speeds follow a Maxwell-Boltzmann distribution, and again the particles in the high-speed tail will escape into space. Therefore, a more relevant quantity to estimate is the time scale for the gas molecules to escape the planet's atmosphere, as will be discussed below.

- **Jeans thermal escape flux**

  A particles can escape the planet's atmosphere if the magnitude of the vertical component of its velocity, $v_z = v\cos\theta$, exceeds the escape velocity from the planet, and does not collide with other particles in its motion upward. This region where it is unlikely for a gas particle to collide with another one is known as **exosphere**. If we denote by n_{exo} the density of the gas particle in the exobase[70], then the escape rate is

$$\Phi_{esc} = n_{\rm exo} \int_{v_{esc}}^{\infty} v_z \; f(v) v^2 \sin\theta d\theta \; dv \; d\phi \tag{18.7}$$

  where

$$Y_e = \left(\frac{v_{esc}}{v_0}\right)^2 \tag{18.8}$$

  After integrating (18.7) yields

$$\Phi_{esc} = \frac{n_{\rm exo} v_0}{2\sqrt{\pi}} \left(1 + Y_e\right) e^{-Y_e} \tag{18.9}$$

[70] This is the altitude above which it is unlikely for the particles to collide with each other.

The above expression is called **Jeans thermal escape flux**. Note that the smaller is Y_e, the more significant is the escape rate.

As an example, consider the gas molecules N_2, which is the most common gas in the Earth's atmosphere. With the exobase at altitude of about 500 km at which $T \sim 1000$ K, we have

$$\upsilon_0[\mathbf{N_2}] = \mathbf{0.78}\ \mathrm{km/s} \quad \longrightarrow \quad Y_e[\mathbf{N_2}] \simeq \mathbf{196} \tag{18.10}$$

Thus, the escape rate of the nitrogen molecules is

$$\Phi_{esc}[N_2] \simeq 8 \times 10^{-56} \left(\frac{n_{\rm exo}}{2.5 \times 10^{25}\ m^{-3}} \right)\ m^{-2} s^{-1} \tag{18.11}$$

We see that even if we the value $n_{\rm exo} = 2.5 \times 10^{25}\ m^{-3}$, which is the number density of the molecular nitrogen gas at the sea level, the escape rate is vanishingly small. To estimate the number of N_2 particles escaping from the atmosphere per unit time, we multiply Φ_{esc} by the surface area of the exobase, $4\pi\,(R_\oplus + H)^2$, where $R_\oplus$ is the Earth's radius[71]. We get

$$\frac{d\mathcal{N}[N_2]}{dt} \sim -10^{-41} \left(\frac{n_{\rm exo}}{2.5 \times 10^{25}\ m^{-3}} \right) \frac{\text{Molecule}}{\text{billion year}} \tag{18.12}$$

which shows that the Earth will hold its nitrogen (and oxygen) gas much longer than the age of the universe.

- **The time scale for particles to escape entirely**

  Let h be thickness of the region in the atmosphere where the probability for particles to collide with each other is extremely small. Then, a rough estimate of the time it takes for such particles to escape entirely from the atmosphere into space is

$$\tau_{esc} \sim \frac{h\overline{n}}{\Phi_{esc}} \tag{18.13}$$

  where $\overline{n}$ is the average number density of the gas above the exobase where particles collision is very rare. Substituting the expression of τ_{esc} in the above equation gives

$$\tau_{\rm esc} \sim 3 \left(\frac{\overline{n}}{n_{\rm exo}} \right) \frac{\sqrt{Y_e}}{1+Y_e} e^{Y_e} \left(\frac{h}{500\ \mathrm{km}} \right) \left(\frac{10\ \mathrm{km}/s}{\upsilon_{esc}} \right)\ \text{minute} \tag{18.14}$$

[71] In reality, at altitudes above 200 km the number density of nitrogen drops significantly compared at sea level.

Planet	Components of the Atmosphere	Comments
Mercury	42% O_2, 29% Na, 22% H	Extremely thin atmosphere[72](Surface pressure $\simeq$ **0**)
Venus	96% CO_2 , 3.5% N_2	Runaway greenhouse effect from CO_2
Earth	78% N_2 , 21% O_2	Thin atmosphere
Mars	**95**% CO_2 , 2.7% N_2, 1.6% Ar	Very thin atmosphere (Surface pressure $\simeq \mathbf{7 \times 10^{-3}}$ atm)
Jupiter	**90**% H_2 , $\sim$ 10% He	Height from its base $>$ 5000km
Saturn	**96**% H_2 , $\sim$ 3% He	Magnetic field smaller than Jupiter
Uranus	**83**% H_2 , 15% He_2, 2.5% CH_4	Blue-Green color of atmosphere from CH_4
Neptune	**80**% H_2 , 19% He_2, $\sim$ 1% CH_4	Bright blue atmosphere from CH_4

Table 5: The gases in the atmospheres of the planets in our solar system.

Let us estimate for how long an Earth-like planet can hold an atmosphere of hydrogen gas. We will assume that initially that the atmosphere contained a substantial amount of hydrogen, and the temperature is about 1000 K at the base of the exosphere[73]. Using Eq (18.4), we obtain[74]

$$v_0^{(H)} \simeq 4 \text{ km}/s \tag{18.15}$$

Now, if we assume that hydrogen is abundant at altitudes between 200 km and 1000 km, i.e. $h \simeq 800$ km, we obtain

$$\tau_{\rm esc}^{(H)}|_{\oplus} \sim \text{ 12 days} \tag{18.16}$$

However, from observations, we know that hydrogen is one of the abundant gases in the exosphere, which seems to be in contradiction with our estimate above. This is because in obtaining $\tau_{\rm esc}^{(H)}|_{\oplus}$ we have assumed that there were no source for hydrogen production in the atmosphere, which is not correct, whereas at latitudes above 500 km, hydrogen is constantly produced from the collision of high-energy photons from the Sun with water vapor and methane molecules.

19 What type of electromagnetic spectrum reach the Earth's surface ?

Quick Answer

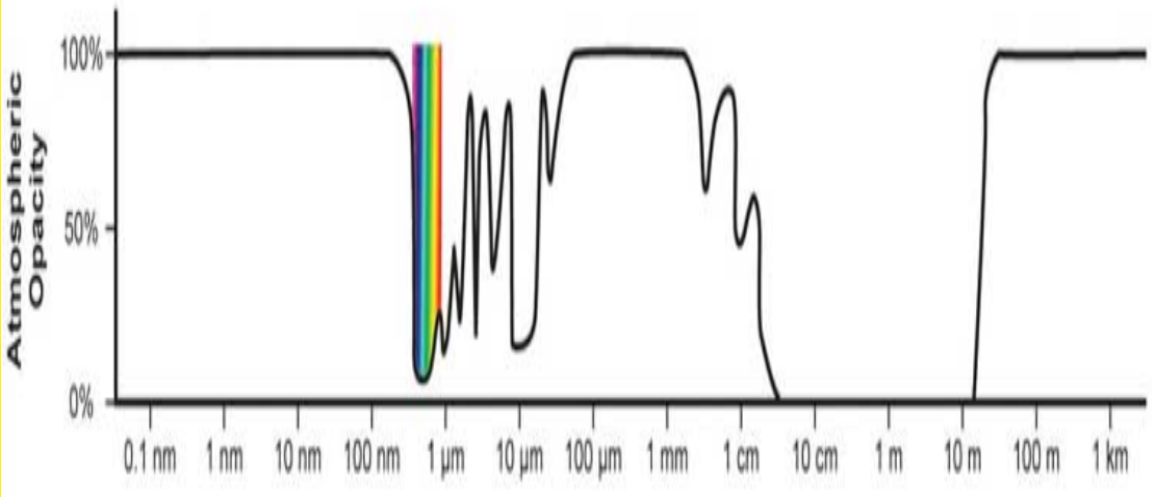
[73] Jeans wrongly assumed that the Earth's upper atmosphere was very cold and concluded that no hydrogen leaked into space.

[74] The molecular mass of hydrogen is $\mathcal{M} = 1$ g/mole.

- **The range of the electromagnetic (EM) spectrum:**

Type	Wavelength	Frequency range	Energy
Gamma ray	< 10 pm	[> 30 EHz]	> 10 keV
X ray	10 pm – 10 nm	[30 EHz – 30 PHz]	10^2 eV – 10 keV
Ultraviolet	10 – 400 nm	[30 PHz – 750 THz]	3 – 10^2 eV
Visible	400 – 700 nm	[750 – 430 THz]	2 – 3 eV
Infrared	700 nm – 1 mm	[430 THz – 300 GHz]	0.1 – 2 eV
Microwave	1 mm – 1 cm	[300 GHz – 300 MHz]	100 μeV – 0.1 eV
Radio	1 cm – 100 km	[300 MHz – 3 KHz]	< 100 μeV

- **Transparency of the Earth atmosphere to EM radiation**



The Gamma-rays, X-rays and UV with $\lambda \leq 300$ nm get absorbed by the ozone molecules in upper atmosphere, whereas most of the infrared radiation are absorbed by the H_2O and CO2 molecules.

More Details

- **Example of typical astronomical sources of the EM spectrum**

Type	Typical Source	Temperature
Gamma ray	matter falling into black hole	$> 10^8$ K
X ray	gas in cluster of galaxies	$10^6 - 10^8$ K
Ultraviolet	very hot stars	$10^4 - 10^6$ K
Visible	stars; hot planets	$10^3 - 10^4$ K
Infrared	very cool stars; planets; cool cloud of dust	$10 - 10^3$ K
Microwave/Radio	cosmic microwave background ; electron moving in magnetic field	< 10 K

- **Some major space-based telescopes across the EM spectrum**

Name	Space Agenc	Cost[Billion]	Launched-Termiated	Wavelength range
COBE	NASA	0.16 ($)	**1989** - 1993	Microwave
HIPPARCOS	ESA	0.60 (€)	**1989** - 1993	Optical
HST	NASA	**10** ($)	**1990** -	UV, Optical, NIR
Chandra X-ray	NASA	1.65 ($)	**1999** -	X-ray
XMM-Newton	ESA	0.77 (€)	**1999** -	X-ray
WMAP	NASA	0.15 ($)	**2001** - 2010	Microwave
INTEGRAL	ESA	0.30 (€)	**2002** -	Gamma-ray
Spitzer Space Telescope	NASA	0.72 ($)	**2003** -	IR and Sub-mm
FGST (formerly: GLAST)	NASA	0.69 ($)	**2008** -	Gamma-ray
Planck	ESA	0.70 (€)	**2009** - 2013	Microwave
Herschel Space Observatory	ESA	1.10 (€)	**2009** - 2013	IR and Sub-mm
GAIA	ESA	0.65 (€)	**2013** -	Optical

- **Some of the major ground-based telescopes across the EM spectrum**

Name	Dishes/Mirrors [in meter]	Location	Start-End	Wavelength range
VLA	**27** dishes ([**25**]; Y shape)	New Mexico (**USA**)	**1981** -	Radio
Keck	**2** Telescopes (I & II)/[**10**]	Hawaii (**USA**)	**1993** &**1996** -	Optical/NIR
VLT	**4** Telescopes [**8.2**]	Atacama desert (**Chile**)	**1998** -	Optical/NIR/MIR
HESS	**5** Telescopes (1 [**28**] + 4 [**12**])	**Namibia**	**2003** -	Gamma-ray(indirect)
MAGIC	**2** Telescopes [**17**]	Canary Islands (**Spain**)	**2004** & **2009** -	Gamma-ray(indirect)
SALT	**1** Primary Mirror [**9.2**]	(**South Africa**)	**2005** -	Optical/NIR
VERITAS	**4** Telescopes [**12**]	Arizona (**USA**)	**2007** -	Gamma rays(indirect)
GTC	**1** Primary Mirror [**10.4**]	Canary Islands (**Spain**)	**2007** -	Optical/NIR
ALMA	**66** dishes (54[**12**] + 12 [**7**])	Atacama desert (**Chile**)	**2011** -	NIR-Radio
FAST	**1** dish [**500**]	Guizhou (**China**)	**2016** -	Radio

- **Acronyms**

  * KHz → Kilohertz = 10^3 Hz.
  * MHz → Megahertz = 10^6 Hz.
  * GHz → Gigahertz = 10^9 Hz.
  * THz → Terahertz = 10^{12} Hz.
  * PHz → Petahertz = 10^{15} Hz.
  * EHz → Exahertz = 10^{18} Hz.
  * NIR → Near Infrared.

* MIR → Medium Infrared.
* NASA → National Aeronautics and Space Administration.
* ESA → European Space Agency.
* JAXA → Japan Aerospace Exploration Agency.
* COBE → Cosmic Background Explorer.
* HIPPARCOS → HIgh Precision PARallax COllecting Satellite.
* HST → Hubble Space Telescope.
* XMM → X-ray Multi-Mirror Mission Observatory - Newton.
* INTEGRAL → International Gamma Ray Astrophysics Laboratory.
* WMAP → Wilkinson Microwave Anisotropy Probe.
* FGST → Fermi Gamma-ray Space Telescope.
* GLAST → Gamma-ray Large Area Space Telescope.
* GAIA → Global Astrometric Interferometry for Astrophysics.
* VLA → Very Large Array.
* VLT → Very Large Telescope.
* VERITAS → Very Energetic Radiation Imaging Telescope Array System.
* SALT → South Africa Large Telescope.
* GTC → Gran Telescopio Canarias.
* ALMA → Atacama Large Millimeter Array.
* FAST → Five-hundred-meter Aperture Spherical radio Telescope.
* MAGIC → Major Atmospheric Gamma Imaging Cherenkov telescope.
* HESS → High Energy Stereoscopic System.

20 What are some of the astrophysical sources of radio waves ?

Quick Answer

Planets, Stars , Pulsar , Gas clouds, Comets, Galaxies, Cosmic background, and Supernova remnants.

More Details

Depending on the source we distinguish two types of radio emissions: thermal and non-thermal. Below I discuss with some details each one of them.

- **Thermal emsission**
- **Non-thermal emission**

21 What is the Ozone layer ?

Quick Answer

- **Ozone is a molecule composed of three atoms of oxygen (O_3)**[a]
- **10% in the troposphere, and 90% in the stratosphere**

[a]It was discovered in **1840** in Munich by the German chemist C. .F. Schonbein.

More Details

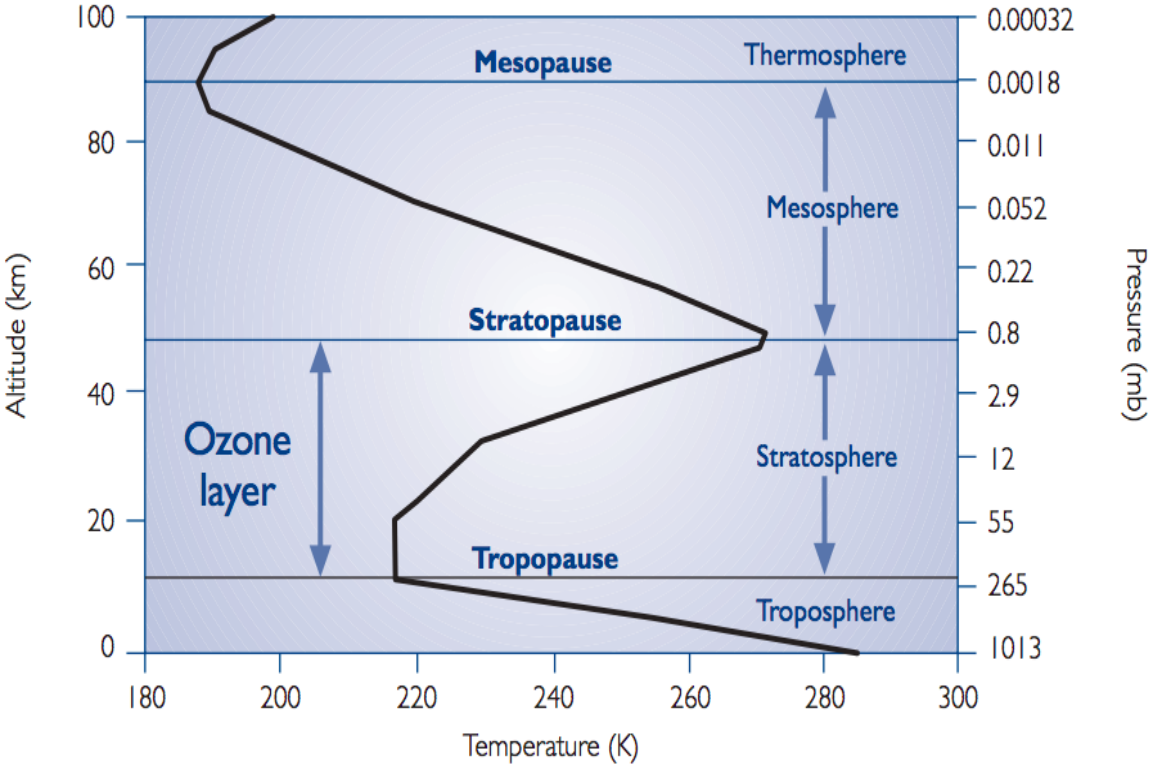


Figure 11: The layers of the Earth's atmosphere. Source: Reference[17].

- **Ozone formation and destruction**

  The formation of ozone molecules in the stratosphere (see Fig.. **??**) is produced in a two step process: In the first step, UV photons from the Sun[75] strike the O_2 molecules[76] in the stratosphere, dissociating some of them into individual oxygen atoms; b) (Almost all) the produced oxygen atoms, O, combine with oxygen molecules to produce O_3 molecules. The ozone generation also get

[75]UV radiation constitute about of 5% of the Sun's radiation.
[76]which make up about 21% of the atmosphere.

destructed by the UV photons as well via its collision with oxygen atoms, which keeps the concentration of the ozone in balance. This cycle of the formation and destruction of the ozone molecules is known as **Chapman mechanism**[77], represented by the following chemical reactions:

$$\text{Reaction 1}: \quad O_2 + \gamma_{UV}[\lambda_{UV} < 240 \text{ nm}] \rightarrow 2\ O \tag{21.1}$$

$$\text{Reaction 2}: \quad O + O_2 + M \rightarrow \mathbf{O_3} + M \tag{21.2}$$

$$\text{Reaction 3}: \quad \mathbf{O_3} + \gamma_{UV}[\lambda_{UV} < 320 \text{ nm}] \rightarrow O_2 + O \tag{21.3}$$

$$\text{Reaction 4}: \quad O + \mathbf{O_3} \rightarrow 2O_2 \tag{21.4}$$

Here M denotes a non-reactive particle that can take up some of the energy released in the reaction[78], γ_{UV} denotes a UV photon, and λ_{UV} is its wavelength. So the net effect of the above process is an equilibrium between atomic oxygen, ozone, and diatomic oxygen, which can represented by the reaction

$$O + \mathbf{O_3} \rightarrow \ \mathbf{2}\ \mathbf{O_2} \tag{21.5}$$

As a result of this balance between the amount of ozone produced and destroyed, its total concentration remains relatively constant at a given latitude and altitude in the stratosphere. The greatest concentrations are at altitude between 15 km and 25 km, forming ozone layer[79] and most of it is formed over the equator where the amount of radiation from the Sun is higher.

- **Impact of ozone depletion**

  Scientists commonly subdivide the ultraviolet light into three wavelength ranges[80], classified as UV-A , UV-B, and UV-C. The UV-C rays are the most dangerous present the highest is completely absorbed by the ozone layer, whereas UV-A and UV-B penetrate the atmosphere and reaches the Earth's surface, with about 95% is of the type UV-A, as can be seen in Fig. 12. Although it is believed that UV-A rays they do not cause significant skin damage over short time exposure, they contribute to skin aging and wrinkles, and they increase skin cancer risk when exposed to large amount. The UV-B can cause sunburn and damage skin's cell DNA. Thus, a decrease in the ozone layer would have serious effect on life on Earth, in particular due the increase of the intensity of UV-B radiation at the Earth's surface.

[77] In honor of Sydney Chapman who proposed it in 1930.

[78] M is usually N_2 or O_2.

[79] The ozone layer was discovered in 1913 by the French Physicists C. Fabry and H. Buisson.

[80] Although these divisions has its grounding in Physics but they turn out to be very useful in biology.

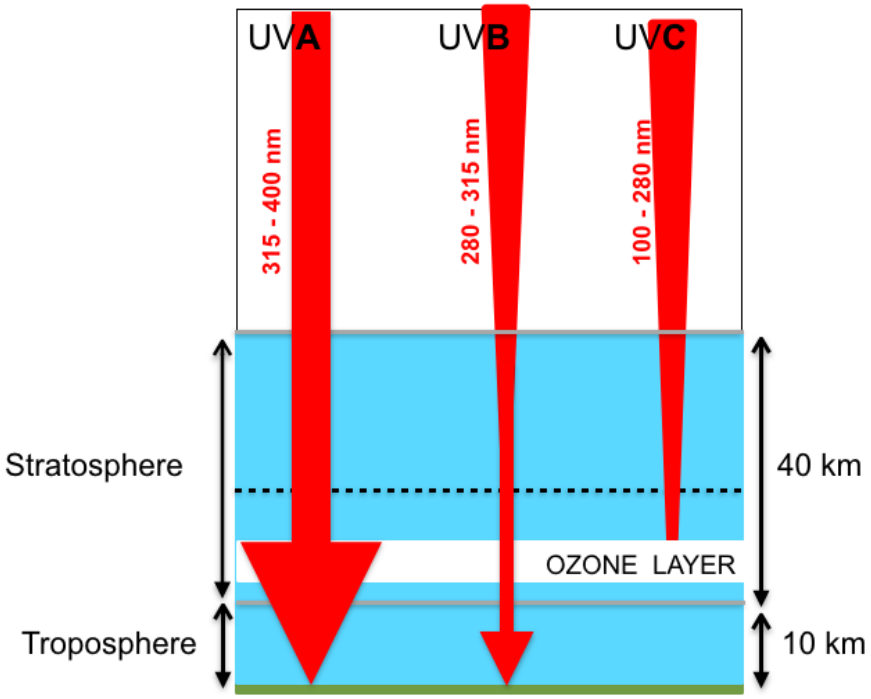


Figure 12: The three types of UV radiations and ozone layer.

- **Causes of ozone depletion**

  One of the major causes of ozone depletion in the stratosphere is chlorofluorocarbons, CCl_2F_2, usually abbreviated as CFCs, which is used in many applications, including **refrigeration** and **air conditioning**. The reason for the CFC being a source of ozone-depletion is the presence of **chlorine**[81] atoms that catalyzes the breakdown of ozone to diatomic oxygen via the reaction

$$\begin{aligned} CCl_2F_2 + \gamma_{UV} &\rightarrow C\ ClF_2 + +Cl \\ CCl_3F + \gamma_{UV} &\rightarrow C\ Cl_2F + +Cl \\ O_3 + Cl &\rightarrow ClO + O_2 \\ ClO + O &\rightarrow Cl + O_2 \end{aligned}$$

  A CFC molecule does not interact with anything in the lower atmosphere and eventually reaches the upper atmosphere where the Sun's radiation is intense enough to break it apart producing chlorine. Moreover, since the CFC has a lifetime of about 20 to 100 years, from the chain reaction above, at very low temperature, as it is the case over the Antarctic, a single chlorine atom can

[81] Actually, the **radicals** nitrogen oxide (**N**O), hydrogen oxide (**H**O), and bromine oxide (**Br**O), are also effective ozone-killers.

destroy hundred thousands of ozone molecules before it form one of other chlorine compounds and exit the stratosphere. In fact, in 1985, British scientists discovered that the ozone layer over the Antarctic was thinning. This ozone decline over the south pole was called **ozone hole**[82].

Fortunately, in 1987 many industrial nations came together and agreed to phase out the CFCs and signed the Montreal protocol, and over the next decades most of the world's nations have signed up.

22 What is a parallax angle? a parsec? a light year ?

Quick Answer

- **In astronomy, the parallax is half the angular displacement of nearby star as seen from the Earth at one time of year, and six months later.**
- **A parsec, denoted by pc, is the distance to an object that has a parallax angle of 1 arsecond.**
- **A light year is the distance that light travels in one year.**
- **The relation between parsec and light year is given by**

$$\mathbf{1pc = 3.26\ ly}$$

More Details

- **Parsec (pc), astronomical unit (AU), and light years (ly)**

  If D denotes the distance to an astronomical object, and p the the parallax angle, then we have

$$\begin{aligned} \mathbf{D} &= \frac{\text{average distance between the Earth and the Sun}}{p} \\ &= \frac{1\text{ AU}}{p[\text{in radian}]} = 1\text{ AU}\left(\frac{\text{arsec}}{p}\right)\left(\frac{1}{\text{arsec}}\right) \qquad (22.1) \\ &= \frac{60\times 60\times 180}{\pi}\times 1\text{ AU}\left(\frac{\text{arcsec}}{p}\right) \qquad (22.2) \end{aligned}$$

  Thus,

[82] The first ozone hole over the Arctic was observed in 2011 after an unusually cold Arctic winter.

$$\boxed{\mathbf{D} = 206,265 \text{ AU} \left(\frac{\text{arcsec}}{p}\right)} \tag{22.3}$$

Now, since by definition 1 pc is the distance that corresponds to a parallax angle $p = 1$ arcsec, we deduce

$$\boxed{1\text{pc} = 206,265 \text{ AU}} \tag{22.4}$$

To express distances in light years, we use the fact that

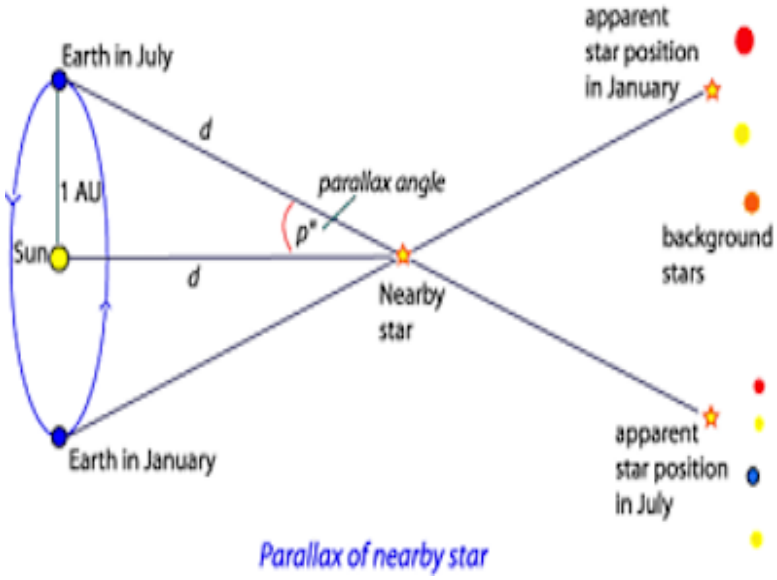


Figure 13: Parallax.

$$\begin{aligned} 1 \text{ ly} &= \text{speed of light} \times \text{one year time interval} \\ &= 2.99 \times 10^8 m/s \times 3.15 \times 10^7 \ s \\ &\simeq 9.42 \times 10^{15} \ m \end{aligned}$$

Hence, we obtain

$$\mathbf{D} = \frac{206,265 \text{ AU}}{9.42 \times 10^{15} \ m} \left(\frac{\text{arsec}}{p}\right) \text{ly} = 3.26 \text{ ly} \left(\frac{\text{arsec}}{p}\right)$$

Setting $p = 1$arcsec in the above expression results in a distance of 1 pc, from which it follows that

$$\boxed{1\text{pc} = 3.265 \text{ ly} \simeq \mathbf{3 \times 10^{16} \ m}} \tag{22.5}$$

- **The first Measurement of parallax of a star**

  In 1838, the German astronomer **Friedrich Bessel**, successfully measured the parallax of a star, and that was for the star **61 Cygni** in the constellation of Cygnus. He obtained $p^{(\mathbf{61Cygni})} = 0.286$ arcsec, which corresponds to a distance of **11.4** light years from Earth.

- **Measuring the parallax of Mars**

  In **1972**, **Cassini** and with his colleague Jean **Richer**, made a simultaneous measurement of the position of Mars at opposition[83] from Paris (France) and Cayenne (French Guiana), respectively, and determined that the parallax angle of Mars[84] with respect to Paris-Cayenne baseline to be

$$p[\text{Mars}] \simeq 25 \text{ arcsec} = \frac{25 \times \pi}{180 \times 3600} = 1.2 \times 10^{-4} \text{ radian}$$

  Given that the distance between Paris and Cayenne is 6437 km, he deduced that the distance between Earth and Mars is

$$\begin{aligned} D_{\text{Earth}\to\text{Mars}} &= \frac{\text{distance between Paris and Cyenne}}{p[\text{Mars}]} \\ &= 6437 \times \frac{180 \times 3600}{24 \times 2\pi} \simeq 55 \text{ million km} \end{aligned} \tag{22.6}$$

  The measurement of the Earth-Mars distance can be used to calibrate the astronomical unit *AU* in kilometers, and hence determine the distances from the planets to the Sun. Cassini Knew that at opposition Mars is 3/8 AU from Earth, and hence he deuced that

$$D_{\text{Earth}\to\text{Sun}} \simeq 140 \text{ million km} \tag{22.7}$$

  which is only 7% smaller than the accepted modern value.

- **Measuring distances to other stars**

  - **Earth Based Telescopes**[85]: By using **adaptive optics** to remove the distorting effect due to the atmosphere[86], it is possible to measure parallax angles as small as **0.01** arcsec, corresponding to a distance of **100** pc.

  - **Space Based Telescopes**: Being in space, these telescopes avoid many of the observational problems associated with ground-based measurements, and can measure distances to objects up to about **few** $\times$ **100 pc** away. For instance, the **Hipparcos satellite**[87] was able to measure the parallax of more than 100,000 stars with a precision of **0.001** arcsecond,

[83] It means when it was on the opposite side of the Earth from the Sun, and hence closest to Earth.

[84] Venus can be in inferior conjunction with Sun, i.e. on a line between the Earth and the Sun, and as a result it can be closer to the Earth than Mars. So one would expect its parallax to be larger than that for Mars, but unfortunately it is usually invisible in such position.

[85] With ordinary telescope, one can observe stars at distances about **30** pc $\simeq$ **100** ly away from us.

[86] This effect is known among astronomers as the "seeing" effect.

[87] It was launched by the European Space Agency (ESA) in 1989 and operated until 1993.

Name	Parallax [arcsec]	Distance [lyr]	Distance [pc]
Proxima Centauri	0.770	**4.24**	**1.30**
Alpha Centauri A & B (binary)	0.750	**4.37**	**1.33**
Bernard's star	0.546	**5.96**	**1.83**
Wolf 359	0.419	**7.86**	**2.39**
Lalande 21185 (binary)	0.395	**8.31**	**2.53**
Sirius A & B (binary)	0.382	**8.66**	**2.62**
Luyten $726-8$	0.374	**8.73**	**2.68**
Ross 154	0.345	**9.68**	**2.90**
Ross 248	0.316	**10.32**	**3.16**
Epsilon Eridani	0.305	**10.70**	**3.28**

Table 6: The distance of the 10 closest stars to our solar system

corresponding to a distance of a kpc, which is about 1/30 the size of our galaxy (see **Q9**).This is in addition to about million stars that Hipparcos measured their distances, but with lower accuracy. The successor of Hipparcos, the **Gaia satellite** (see **Q12**), measured parallaxes with an accuracy of **4** μarcsecond for about a billion stars down to $20th$ magnitude ((see **Q19**)), giving distances up to **25 kpc** with 10% accuracy[88].

In table 6 we show the parallax angles and the corresponding distances to the ten closest stars to Earth.

23 What is the mass of the Moon ?

Quick Answer

7.4×10^{22} **kg** $\simeq$ **1%** $\mathbf{M_\oplus}$

More Details

- **Newton's estimate**

  The first attempt to estimate the mass of the Moon was by Isaac Newton by using the measurements of the sea tides. He showed that the height the tides produced by an astronomical body is proportional to its mass, and determined that the ratio of the mass of the moon to the mass of the earth is approximately

[88]The final Gaia catalogue is expected to be released around 2022.

1/40, which is 50% smaller than the actual value. The details of this finding are a bit complicated but we will discuss with some details the tidal forces and their effects on the Earth in the next section.

- **An estimate**
  Let us consider the Moon to have a spherical shape with radius R_M, and density ρ_M, then its mass is given by

$$M_{\text{Moon}} = \frac{4\pi}{3}\rho_{\text{M}} R_M^3 \tag{23.1}$$

  Since it is widely accepted that the Moon formed out the debris left over from the impact of Mars-sized astronomical object with the Earth approximately 4.5 billion years ago[89] the Moon seems to be formed out Earth, it is reasonable to assume that ρ_M is similar to the density of a typical Earth's rocks, i.e. $\rho_M \sim 3$ g/cm^3 and use the fact that R_M is about 27% the radius of the Earth[90], we obtain

$$M_{\text{Moon}} \sim 7 \times 10^{22} \text{ kg} \tag{23.2}$$

- **Better estimate**

24 What is stellar magnitude?

Quick Answer

[89] This is known as the **Giant-impact hypothesis. The Mars-sized that collided with the proto-Earth was named Theia, which Greek mythology is the mother of Selene, the goddess of the Moon.**

[90] The estimate of the size of the Moon was known since the ancient Greek (more than 2000 years ago) from the timing the lunar eclipse. See also **A 2**.

We define $\mathcal{L}$ be the **intensity** of the electromagnetic radiation emitted by an astronomical object[a], i.e. the amount of energy the object radiates per unit time. For an observer, the energy detected per unit time and unit area, denoted by $\mathcal{F}$, is the **flux** of the observed radiation.

- **Apparent magnitude of star X [m]:**

$$\mathbf{m^{(X)}} = -\mathbf{2.5}\log_{\mathbf{10}}\left[\frac{\mathcal{F}^{(\mathbf{X})}}{\mathcal{F}^{(\text{Vega})}}\right]$$

- **Absolute magnitude of star X [M]:**

$$\mathbf{m^{(X)} - M^{(X)}} = -\mathbf{2.5}\log_{\mathbf{10}}\left(\frac{\mathbf{d}}{\mathbf{10}\text{ pc}}\right)^{\mathbf{2}}$$

Here **d** is the distance from the star X to the observer.

[a] It is also known as the **luminosity** the absolute brightness.

More Details

- **Apparent magnitude and absolute magnitude**
  Around 150 BC, the Greek astronomer **Hipparchus** classified stars by their brightness, with the brightest were termed **first magnitude**, next brightest were **second magnitude** through to the **sixth magnitude** which corresponds to the faintest stars a naked eye can see[91]. Hipparchus magnitude scale-system reflected the fact that when the intensity of a light source increases linearly, human eye's response is not linear, instead the eye sensitivity is logarithmic.

- With the invention of the telescope, astronomers were able to see much fainter stars which lead them to extend the magnitude scale system and decided to formulate it in a more precise way. For that, in 1856, the English astronomer **Sir Norman R. Pogson** proposed to define a 5 magnitude difference between two stars corresponds to a ratio of 100 to 1 in brightness. In general, if two stars A and B have **apparent magnitudes** $\mathbf{m}^{(A)}$. and $\mathbf{m}^{(B)}$, respectively, and $\mathcal{F}^{(A)}$ and $\mathcal{F}^{(B)}$ are corresponding fluxes, then

$$\frac{\mathcal{F}^{(A)}}{\mathcal{F}^{(B)}} = 10^{0.4\left(\mathbf{m}^{(B)}-\mathbf{m}^{(A)}\right)} \iff \mathbf{m}^{(A)} - \mathbf{m}^{(B)} = -2.5\log_{10}\left(\frac{\mathcal{F}^{(A)}}{\mathcal{F}^{(B)}}\right) \quad (24.1)$$

[91] This ranking is counter intuitive as one expect the magnitude to increase with the increase of brightness.

The magnitude system was extended to negative values to accommodate very bright objects[92]. By convention, astronomers use the star **Vega**[93] as a reference and take its **apparent magnitude** equal to zero[94]. Thus, the apparent magnitude of a star X can be expressed as

$$\mathbf{m}^{(X)} = -2.5 \log_{10} \left[\frac{\mathcal{F}^{(X)}}{\mathcal{F}^{(\text{Vega})}} \right] \tag{24.2}$$

This process of measuring the apparent brightness of an object. is called **photometry**[95]Note that the apparent magnitude of an object depends on its distance to the observer, and therefore, in order to be able to compare their luminosity, astronomers introduced the **absolute magnitude** scale, denoted by M, which is defined as the brightness that a star appears to have if it were placed $10 \text{ pc} = 3 \times 10^{17} \text{ m}$ (see **A 17**) away from the observer, i.e

$$\mathbf{m} - M = -2.5 \log_{10} \left(\frac{\mathcal{F}^{(\text{observed})}}{\mathcal{F}^{(\text{at d = 10 pc})}} \right) \tag{24.3}$$

So the absolute magnitude indicates the star's intrinsic brightness, i.e. it is independent of how far the star is from the observer. Now, we recall that for a source of known luminosity $\mathcal{L}$, which emits radiation isotropically at a distance d from an observer, the flux $\mathcal{F}$ received reads

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi d^2} \tag{24.4}$$

which we substitute into (24.3) and get

$$\mathbf{m} - M = 5 \log_{10} \left(\frac{d}{\text{pc}} \right) - 5 \tag{24.5}$$

The quantity $(\mathbf{m} - M)$ is called the **distance modulus** and represents a measure of distance to the object.

- **Filters**

  * Because stars have different temperatures, and hence they emit radiation strongly in different wavelength ranges, instruments that measure the

[92]The magnitude system is useful only for wavelengths in the visible, UV, and infrared spectrum. For other wavelengths astronomers use the flux units: **Jansky** ($1Jy = 10^{-26}\ W/m^2/Hz$ for radio waves, and $erg/cm^2/s$ for X-ray, and gamma-ray.

[93]The star vega is at a distance of about 25 lys from Earth, and it is part of the constellation Lyra in the northern hemi-sphere.

[94]Because in the original magnitude classification the star Vega was one of the brightest star.

[95]The maximum apparent magnitude of a naked human eye is **+6** in the best observing conditions, and **+3** in an urban environment. In contrast, The Hubble Space Telescope (HST) can reach apparent magnitude of **+30** in the visible band

fluxes arriving from a star have filters that can allow only some narrow range (or band) of wavelengths to pass through. The amount of spectrum that a filter allows to pass is called the **bandpass** and the central wavelength of the band pass is known as the **effective wavelength**. We denote by m_K (or sometimes by just a capital letter K) and M_K the apparent and absolute magnitudes in the spectral K-band, respectively.

* The most widely used filter systems are the [**U**, **B**, **V**, **R**, **I**] in the optical[96] and the [**Y**, **J**, **H**, **K**] in the near-infrared. For instance, the **V** filter allows only light in the wavelength range [**505 – 595** nm] to pass through the filter[97].

* In defining the magnitude at some wavelength λ one needs to choose the zero-point of magnitude. For a long time astronomers commonly used the **Vega- magnitude system**, in which $\mathbf{m}^{(\mathbf{Vega})} = \mathbf{0}$ **in all passband**[98]. So, the apparent monochromatic magnitude of an astronomical object reads

$$\mathbf{m}_{\text{Vega};\lambda} = -2.5 \log_{10} \left[\frac{\mathcal{F}_\lambda}{\mathcal{F}_\lambda^{(\text{Vega})}} \right] \tag{24.6}$$

where $\mathcal{F}_\lambda$ is the flux per unit wavelength. Thus, the zero point of this system depends on the flux of the star Vega and is different in different bands. For example, in the **V passband**, we have

$$\mathcal{F}_\lambda^{(\text{Vega; } \mathbf{V\text{-band}})} = 3.63 \times 10^{-9} \text{ erg}.s^{-1}.\text{cm}^{-2}.\text{Å}^{-1} \tag{24.7}$$

In terms of frequency, we have

$$\mathcal{F}_\nu \, |d\nu| = \mathcal{F}_\lambda \, |d\lambda| \quad \Longrightarrow \quad \mathcal{F}_\nu = \frac{\lambda^2}{c} \mathcal{F}_\lambda$$

where in the second equality we made use of the relation $\lambda = c/\nu$, where c is the speed of light. Thus, for the star Vega, the zero pointy flux density in the V-band is

$$\mathcal{F}_\nu^{(\text{Vega; } \mathbf{V\text{-band}})} = 3.63 \times 10^{-20} \text{ erg}.s^{-1}.\text{cm}^{-2}.\text{Hz}^{-1}$$

In table 7. we give the effective wavelength (λ_{eff}), the bandpass ($\Delta\lambda$), and the averaged flux density ($\mathcal{F}_\nu$) in each band of the UBVRI photometric system.

[96] This filter system was introduced in 1953 by H. Johnson and W. Morgan (see ref. [21]).

[97] In day light human eye is most sensitive to radiation with wavelength of about 550 nm, and it decreases towards the red.

[98] The Vega-magnitude system was generalised to a class of stars called **A0V** stars.

Filter	λ_{eff} [nm]	$\Delta\lambda$ [nm]	$\mathcal{F}_\nu$ $[W/m^2/Hz^{-1}]$
U	300	**50**	$\mathbf{1.81 \times 10^{-23}}$
B	430	**72**	$\mathbf{4.26 \times 10^{-23}}$
V	550	**86**	$\mathbf{3.64 \times 10^{-23}}$
R	650	**133**	$\mathbf{3.08 \times 10^{-23}}$
I	820	**140**	$\mathbf{2.55 \times 10^{-23}}$

Table 7: The characteristics of the UBVRI filters in the **Vega**-magnitude system.

* Another magnitude system which is becoming increasingly popular is the **AB**-magnitude system defined as[99]

$$\mathbf{m}_{AB;\nu} = -2.5 \log_{10} \left[\frac{\mathcal{F}_\nu}{(3.6308 \times 10^{-23}\ \text{W/Hz/}m^2)} \right] \tag{24.8}$$

or, equivalently,

$$\mathbf{m}_{AB;\nu} = -2.5 \log_{10} \mathcal{F}_\nu - \mathbf{48.6} \tag{24.9}$$

where $\mathcal{F}_\nu$ is in erg.s^{-1}.cm^{-2}.Hz^{-1} unit. Note that in contrast to the Vega-magnitude system no stellar spectral distribution is used as a reference, and instead the zero-point flux is a constant flux at all frequencies, and was chosen in such away **AB**-magnitude and **Vega**-magnitude are the same at the wavelength $\lambda = 5500$ Å.

* An example: The apparent magnitude of the Sun in the **V band** reads

$$m_V^{(\odot)} = -2.5 \log_{10} \left[\frac{\mathcal{F}_V^{(\odot)}}{\mathcal{F}_V^{(\text{Vega})}} \right] \tag{24.10}$$

Using the value of $\mathcal{F}_V^{(\text{Vega})}$ in (24.7) and the measured solar flux in the V-band $\mathcal{F}_V^{(\odot)} = 185$ erg.s^{-1}.cm^{-2}.Å^{-1}, we get

$$m_V^{(\odot)} = \mathbf{-26.7} \tag{24.11}$$

Now substituting the above value of $m_V^{(\odot)}$ and the mean Earth-Sun distance ($d = 1$ AU $\simeq 4.85 \times 10^{-6}$ pc) into Eq (24.5), we find

$$M_V^{(\odot)} = \mathbf{+4.83} \tag{24.12}$$

Sometime it is useful to compare the magnitudes of stellar objects with that of the Sun, which can be expressed as[100]

$$M_K - M_K^{(\odot)} = -2.5 \log_{10} \left[\frac{\mathcal{L}_K}{\mathcal{L}_{K,\odot}} \right] \tag{24.13}$$

[99] Note that the AB-magnitude system is defined in terms of the flux per unit frequency.
[100] The absolute magnitudes of the Sun in different filters can be found in [**?**].

Celestial Object	distance [pc]	m_V	M_V	$\mathcal{L}/\mathcal{L}_\odot$
Sun	4.85×10^{-6}	**−26.7**	**+4.83**	1
Moon	1.25×10^{-8}	**−12.6**	**It is not a star**	**It is not a star**
Venus	3.51×10^{-6}	**−4.6**	**It is not a star**	**It is not a star**
Proxima Centauri	1.29	**+11.0**	**+15.5**	$\mathbf{5.4 \times 10^{-5}}$
Alpha Centauri A	1.32	**−0.01**	**+4.4**	**1.52**
Alpha Centauri B	1.32	**+1.33**	**+5.7**	**0.5**
Sirius A	2.64	**−1.6**	**+1.45**	**23.6**
Polaris	133	**+2**	**−3.6**	$\mathbf{10^x}$
Betelgeuse	197	**+0.41**	**−5.2**	$\mathbf{10^4}$
Rigel	265	**+0.14**	**−6.8**	$\mathbf{4.5 \times 10^4}$

Table 8: The brightness of some celestial objects and their distance from Earth.

In table 8 we list some some celestial objects with their corresponding apparent magnitude, absolute magnitude, and their luminosity relative to the Sun's luminosity.

∗ If the flux of radiation is measured across all range of the electromagnetic spectrum, then the magnitude is called the **bolometric magnitude**, and denoted by m_{bol} and M_{bol} for the apparent and absolute magnitude, respectively.

∗ **Color indices**
The difference in magnitude between two filters, which is equivalent to measuring the flux ratio passing through the two filters, is called a **color index**. The color index of a star is an indication of its surface temperature[101]:
→ **hot stars** have $(B - R) < 0$, they are bright in the blue, and faint in the red,
→ **cool** stars have $(B - R) > 0$, i.e. they are faint in the blue and bright in the red.

By approximating stars as black bodies, Ballesteros (see ref. [23]) derived a formula of a star's surface temperature in terms of the color index $(B - V)$, given by

$$T^{(\text{surface})} = 4600 \text{ K} \left[\frac{1}{0.92(B-V) + 1.7} + \frac{1}{0.92(B-V) + 0.62} \right] \quad (24.14)$$

[101] Another way to estimate the surface temperature of a star is to use Wien's displacement law (see **A** 23).

For instance, applying the above formula for the **Vega** star, which by definition has color index zero, yields $T_{\textbf{Vega}}^{(\text{surface})} \sim 10,000$ K.

- **The Effect of Atmospheric Absorption**
  The definitions of the magnitudes discussed previously assumed that the fluxes are measured in the absence of the Earth's atmosphere or at the top of it. So for observations performed on the Earth, the incoming electromagnetic radiation suffer. absorption and scattering, which result in atmospheric extinction of the fluxes at the Earth's surface. Furthermore, the interstellar space is not perfectly empty as it contains gas and dust grains, and hence there will be attenuation of the observed electromagnetic flux radiated by the stellar objects. These effects can be corrected by redefining the distance modulus as[102]

$$m_\lambda = M_\lambda + 5\log_{10}\left(\frac{d}{10\text{ pc}}\right) + A_\lambda \tag{24.15}$$

  where the **extinction term** $A_\lambda = 1.086\tau(\lambda)$, with $\tau(\lambda)$ is called the **optical depth** and is given by

$$\tau_\lambda = \int_0^r n(r')\sigma(r',\lambda)\, dr' \tag{24.16}$$

  Here, $n(r)$ is the number density of the particles in the medium at distance r from the observer and $\sigma(r,\lambda)$ is the scattering cross section[103] of a photon of wavelength λ on a particle of the medium.

[102] Eq (24.15) is the consequence of the defining equation of the extinction term, i.e.

$$m_\lambda = m_{\lambda;0} + .A_\lambda$$

with $m_{\lambda;0}$ is the apparent magnitude in the absence of any medium of particles between the stellar object and the observer.

[103] The cross section is proportional to the probability for the scattering to occur.

25 What is the average solar power density at the Earth's atmosphere?

Quick Answer

1368 W/m²

More Details

The value of the solar power density reaching the surface of the Earth's atmosphere is known as **the solar constant**, and it is usually denoted by G_S. If there was no atmosphere, the total power of solar radiation reaching the Earth?s surface would be the solar constant times the cross sectional area of the Earth[104], which yields 1.7×10^{17} Watt. Thus, in one year, the top of the Earth?s atmosphere receives on average approximately 5.46 Yotta Joules $= 1.52 \times 10^{18}$ kWh[105]. Hence, the **mean irradiance at the surface of the Earth** is

$$\begin{aligned} \overline{G_S^{(0)}} &= \frac{\text{Fraction of sunlight incident of the Earth}}{\text{Area of the Earth facing the Sun}} \\ &= \frac{1.7 \times 10^{17} \text{ W}}{\left[2 \times 3.14 \times (6,400 \times 10^3 \text{ m})^2\right]} \simeq 680 \text{ W}/m^2 \end{aligned} \tag{25.1}$$

Because Earth has an atmosphere, around **30**% of the solar energy gets **reflected** by the clouds into space and **20**% is **absorbed** by **water vapor, dust and O3 molecules in the air**. Thus, the solar energy incident on the Earth[106] per day is[107]

$$H_S^{\text{with atm}}|_{\text{atm}} = 50\% \tag{25.2}$$

This means that even if just **0.1**% of the solar energy that reaches the ground is used it **can power the entire world**.

26 How much electromagnetic power is radiated by the Sun?

Quick Answer

- **The amount electromagnetic energy radiated per second** by an object X is called the **luminosity** of the object, and it is usually denoted by $\mathcal{L}_X$
- **For the Sun**, $\mathcal{L}_\odot = 3.9 \times 10^{26}$ W

[104] That is $\pi R_\oplus^2$, with $R_\oplus \sim 6,400$ km is the radius of Earth.
[105] Units conversions: Yotta $= 10^{24}$; kWh $= 3.6 \times 10^6$ Joule.
[106] Also called the insolation.
[107] Assuming that the day is an average 12 hours at any location on the Earth.

More Details

Since we know the Earth-Sun distance, $d_{\text{E-S}} \simeq 1.5 \times 10^{11}$ m (see A 3) , and the solar constant, $\overline{G_S^{(0)}} = 1368$ W/m^2 (see A 21), the Sun's luminosity is given by

$$\mathcal{L}_\odot = \overline{G_S^{(0)}} \times 4\pi d_{\text{E-S}}^2 \simeq 3.9 \times 10^{26} \text{ W} \tag{26.1}$$

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27 How much is the surface temperature of the Sun?

Quick Answer

$$T_\odot^{(\text{surface})} \simeq 5800 \text{ K}$$

More Details

- The observed electromagnetic radiation emitted by the Sun has approximately a blackbody spectrum, and hence we can apply Wien's displacement law[108]:

$$\lambda_{\max} T = 0.29 \text{ cm}.K \tag{27.1}$$

  where $\lambda_{\max}$ is the wavelength in the spectrum with the maximum intensity. For instance, the sun emits maximum radiation at the visible yellow wavelength, which corresponds to $\lambda \sim 500$ nm, and hence we find

$$T \simeq 5800 \text{ K} \tag{27.2}$$

- This method of estimating the surface temperature can be also applied to stars as long as the spectrum of emitted the radiation is a Planck function. However, in reality the electromagnetic spectrum from stars contains absorption lines, and so it may not be possible to observe a peak in the spectrum. Moreover, the interstellar dust may distort the spectrum light, an effect known as the interstellar reddening, which can produce erroneous temperature[109]. Instead, astronomers use absorption lines to determine spectral classification.

[108] One can measure the light spectrum emitted by the star, fit it with Planck?s distribution and deduce the temperature.

[109] Applying Wien's law to a reddened spectrum will yield a temperature that is too cool.

28 Is there water on Mars?

Quick Answer

- **Yes, but in icy form on its surface at the north polar ice cape and small quantities as vapor in its atmosphere.**
- With **the current atmospheric pressure at the surface of Mars** (6 – 14 millibar)[a], **most the ice sublimes**[b](see below for details).
- There are some recent evidence[c] for liquid water on Mars deep underground.

[a] 1 bar = 0.987 atm.
[b] It means ice turns directly into water vapor without first transitioning into a liquid.
[c] 2019

More Details

- Water can exist in three states: solid, liquid, and gas. At certain temperature and pressure, water can exists in the three phases simultaneously. This is achieved at temperature of 273.16 K and pressure 0.006 atm. This point is called the **the triple point** of water and is shown in Fig. 15 for the pressure-temperature phase diagram[110].
- From Fig. 15 we see that the atmospheric surface pressure on Mars is very close to the triple point of water, and this implies that is it is hard for liquid water to exist. Thus, in the present-day Martian surface liquid water is not stable and can be only be in a solid form (i.e. ice) or in gas form (i.e. vapor).
- However, at lower elevation. such as area created by impact of a large asteroid the pressure can be higher than the the pressure of the triple point of water and in this case if the temperature is about right (according to the phase diagram), then transition from ice to liquid water may be possible if the term. For instance, the pressure at the **Hellas Basin** is 12 mbar and hence in principle liquid water could form.

- In **2018**, Mars Express orbiter [2003- present], reported the detection of liquid water, 1.5 km below the southern polar caps and expanding 20 km horizontally.

[110] Note that the curve separating the solid and liquid phases is tilted to the left (in reality it is slightly tilted). This implies that ice is less dense than liquid water; this is why icebergs float on the oceans surface, and also this is why lakes and rivers freeze from top to bottom, allowing the life-form to survive even when the surface has frozen.

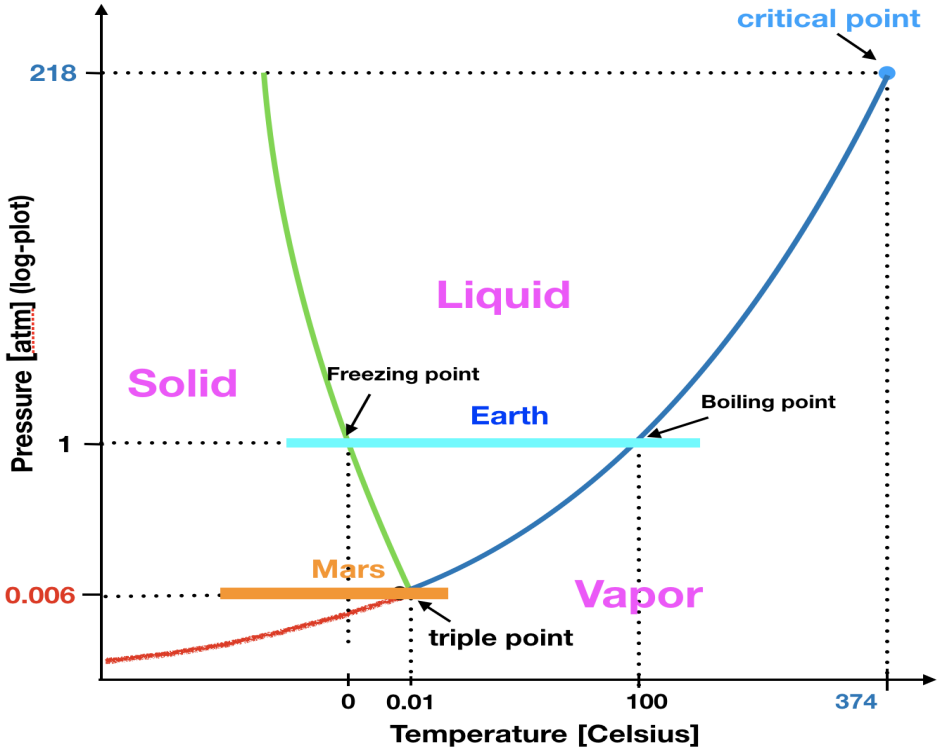

Figure 14: Schematic phase diagram for water. The atmospheric surface pressure on Earth and on Mars are shown in blue and orange colors, respectively.

Name	Launching Date	Agency	Type	Operational Until
Mars Odyssey	**2001**	**NASA**	**Orbiter**	**2025**
Mars Express	**2003**	**ESA**	**Orbiter**	**2026**
Opportunity	**2003**	**NASA**	**Rover**	**2018**
Mars Reconnaissance	**2005**	**NASA**	**Orbiter**	**2030s**
Curiosity	**2011**	**NASA**	**Rover**	+ **2019**
ExoMars Trace Gas Orbiter	**2016**	**ESA/Roscosmos**	**Orbiter**	+ **2020s**

Table 9: Some of successful mission to Mars.

The discovery has been made using **Marsis**[111], a radar instrument placed on board of the Mars Express satellite. Marsis bounces off radio waves off a selected area of the planet and then analyzes the received echoes.

- **Successful missions to Mars in the last 20 years**[See table 9]

[111] It stands for **A**dvanced **R**adar for **S**ubsurface and **I**onosphere **S**ounding instrument which was developed and built by Italian scientists from the the European Space Agency (ESA).

29 How do we know if a star or a galaxy is moving toward or away from us?

Using the Doppler shift .....

More Details

- **Doppler effect for sound waves**[112]

  Let us denote T^{emit} the period of wave at the source, λ_{emit} the wavelength of the wave at the source, λ_{rec} the wavelength of the wave detected by the observer, c_s the speed of the sound wave, v_{src} the speed of the source, v_{rec} the speed of the observer, d the distance between the observer and the source at the instant when the first crest is emitted by the source (i.e. at $t = 0$), t_{rec} is the instant at which the first crest is received by the observer, t_{rec} is the instant at which the second crest is received by the observer.

  - **Source moving and the observer standing still**

  - **Observer moving and the source standing still**

- **Doppler effect for EM waves**

30 What is the spectral classification of stars ?

Quick Answer

- **The strengths of the hydrogen lines depends on the surface temperature of the star.**
- **Hence, the apparent color of a star gives an approximate estimate of its surface temperature.**
- **The order of spectral types of stars from hottest to coolest is labeled**[a]

  OBAFGKM

[a] To remember this order one can use the following mnemonic: **O**h **B**e **A** **F**ine **G**irl **K**iss **M**e

More Details

[112] In 1842, the Austrian Mathematical Physicist **Christian Doppler** wrote an article on the behavior of sound waves and showed that the relative motion of the source and the observer, the detected and the emitted frequencies of the wave are different.

31 What is the Eddington limit ?

$\sim$ XXXX

More Details

-

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32 What is the minimum temperature for hydrogen fusion inside a star ?

$\sim$ 10 million degrees

More Details

- **The estimate of temperature from classical Physics**
  The nuclear fusion of hydrogen is the process in which protons combine to form helium nucleus. This can occur via the strong nuclear force, which has has a range of about the size of a medium nucleus, i.e.

$$d_{\text{strong}} \sim \mathbf{10^{-13}}\ \text{cm} \tag{32.1}$$

  Since electric charge of nuclei is of the same sign (positive), there will be an electrostatic potential energy barrier

$$E_{\text{Coulomb}} = \frac{Z_1 Z_2 e^2}{d_{\text{strong}}} \simeq 2 \times 10^{-13}\ \text{J} = 1.2\ \ (Z_1 Z_2)\ \ \text{MeV} \tag{32.2}$$

  To overcome this Coulomb barrier, the protons must have large kinetic energy, which inside a star it corresponds to a very high temperature. So if we consider a gas of protons in the core of a star. at temperature T, then according to kinetic theory of gases the average kinetic energy per particle is proportional to the absolute temperature, and it is given by

$$< K_p > = \frac{3}{2} k_B T \simeq \text{keV} \left(\frac{T}{10^7\ \text{K}} \right) \tag{32.3}$$

  However, as we will show in A **26**, the temperature in the core of a typical star, such as the Sun, is of order 10^7 K (for details see **A** XX), and hence the protons can not fuse in such a star nuclear. In fact, even for $\mathbf{T \sim 10^9}$ K, the particles (protons) will not have energy to penetrate of the Coulomb barrier, and hence the nuclear fusion can not occur; at least classically.

- **Estimate of the temperature using quantum mechanics**
  The solution to the above problem is quantum physics, where microscopic particle can have wave-like properties; known as wave- particle duality, which was suggested in 1921 by Louis de Broglie. It states that matter behave both like particles as well as waves. For a particles with momentum p we associate a wave with the wavelength

$$\lambda_{db} = \frac{h}{p} \tag{32.4}$$

  Thus, for the nuclear fusion between two nuclei to occur they only need to be within distances that are comparable to their de Broglie wavelengths. Now we require the kinetic energy of the two interacting particles be equal to the Coulomb potential at a separation distance of one de Broglie wavelength,

$$\frac{Z_1 Z_2 e^2}{\lambda_{\rm db}} = \frac{1}{2}\mu v^2 = \frac{p^2}{2\mu} = \frac{h^2}{2\mu\lambda_{\rm db}^2} \tag{32.5}$$

  from which we infer the distance the particle need to be of one another

$$\lambda_{\rm db} = \frac{h^2}{2\mu Z_1 Z_2 e^2} \tag{32.6}$$

  Substituting the $\lambda_{\rm db}$ into the expression of the Coulomb potential yields

$$E_{\rm Coulomb} = \frac{2Z_1^2 Z_2 e^4 \mu}{h^2} \tag{32.7}$$

  Now, to determine the minimum temperature needed for a nuclei to penetrate the Coulomb barrier we set the above relation of $E_{\rm Coulomb}$ equal to the thermal average kinetic energy per particle given in (32.3). We get

$$T_{\rm min} = \frac{4Z_1^2 Z_2 e^4 \mu}{3h^2 k_B} = 0.96 \times 10^7 \text{ K} \left(\frac{Z_1^2 Z_2^2}{1}\right)\left(\frac{\mu}{m_p/2}\right) \tag{32.8}$$

  which is of order the central temperature of a typical star like our Sun. Hence, we expect proton-proton fusion to take place in the core of the Sun[113].

  The above estimate for the fusion temperature seems to suggest that at $T \simeq 10$ million Kelvin any pair of protons will fuse. However, as mentioned above, the process of fusion is aided by the quantum mechanical effect mentioned above, and hence the correct way to estimate the interaction rate of such process which is proportional to the probability for a nuclei to go through the Coulomb barrier. According to quantum mechanics, the charged particle has a non-zero transmission probability to go through the electrostatic barrier even if the thermal

[113] As we will see in **A XX** the fusion of hydrogen through a series of steps.

kinetic energy is order of magnitudes smaller than what is required to overcome it. This phenomenon is known as the quantum tunneling. The derivation of the tunneling probability and the interaction rate will be discussed in Appendix X.

33 What is the smallest mass that a star can have?

- **$M_* \geq 0.08\ M_\odot$ (i.e. at least 80 times Jupiter mass).**
- **Below this minimum value there is not enough thermal energy to start the nuclear fusion of hydrogen in the star's core[a].**
- **The "failed stars" (i.e. with $M < 0.08\ M_\odot$) are called "brown dwarfs".**

[a] Some astrophysicists consider the 13 $M_{\rm Jupiter}$ is the appropriate cut off between a star and planet as a brown dwarf with mass larger than this cut off can burn deuterium (a.k.a heavy hydrogen; with nucleus having one proton and one neutron) producing some small amount of energy.

More Details

- **Electron degeneracy**.
- **Estimate of minimum mass of a star**

34 What is the largest mass a star can have?

- **$M_* \leq 120\ M_\odot$.**
- **Above this maximum value there will be excessive energy production that prevents any further mass accretion. In this case hydrostatic equilibrium can not be maintained.**
- .
- .

More Details

35 What is a white dwarf?

- **It is the stellar core remnant of star with mass $M_* < 8\ M_\odot$.**
- **It is mostly helium. It can also contain carbon and oxygen.**

More Details

36 What is a black hole (BH)?

A region of space from which nothing, including light, can escape, which is why we perceive them as black.

More Details

- **Historical background [18th – 20th century]**
  - It was the English philosopher, geologist and Clergyman John Michel who in **1784** proposed the concept of **dark star**, an object compressed into sufficiently small size that it would have an escape velocity that exceeds the speed of light.
  - The size that an object of mass M would have for it to be a BH is known as the **Schwarzschild radius** can be determined using Newtonian mechanics by setting the object's escape velocity $v_{\rm esc} = 2GM/r$ equal to the speed of light, c, and we obtain

$$r_s = \frac{2GM}{c^2} \simeq 3\ \text{km} \left(\frac{M}{M_\odot}\right) \tag{36.1}$$

    For instance, to make a black hole out of the the Sun it has be compressed to a radius of **3** km, and an Earth mass black hole has to have a radius of approximately 1 cm. However neither the Earth nor the Sun have the necessary gravitational force to overcome the electron and the neutron degeneracy pressures, respectively.
  - According to the theory of general relativity (GR), introduced by Albert Einstein in **1915**, a BH is a result of the curvature of spacetime caused by a huge mass.

– In **1915**, Karl Schwarzschild provided the first exact solution to the Einstein field equations of GR, for the case of a spherical non-rotating mass. He showed that the points at the Schwarzschild radius surface, i.e. $r = r_s$, defines the the event horizon beyond which light can not escape.

– The term "black hole" was coined by the American physicist John Wheeler in a lecture he gave in **1967**.

– The first BH to be discovered was in **1971**, named Cygnus-X1, located in the Cygnus constellation at about **6000** lyrs from us. It turns out that Cygnus X-1 is a source of X-ray[114] and form a binary system that includes blue supergiant variable star[115] which it orbits at about 0.2 AU.

– In **1974**, two astronomers, Bruce Balick and Robert L. Brown, published a paper describing a bright radio source in the constellation Sagitarius at the very center of the Milky Way. It turns out this radio source is a **supermassive black hole** in the center our galaxy, and was named **Sgr A*** black hole (see Fig. **??**). Observation of other galaxies showed they too have supermassive black home at their center.

- **Schwarzschild solution**
- **The Tolman-Oppenheimer-Volkoff equation**
- **Near the black hole**

37 How are black hole (BH) discovered?

By studying the way matter moves in its vicinity. For example, X-ray emission .....

More Details

[114] This was actually first spotted in 1964 during a rocket flight.

[115] It was designated HDE 226868.

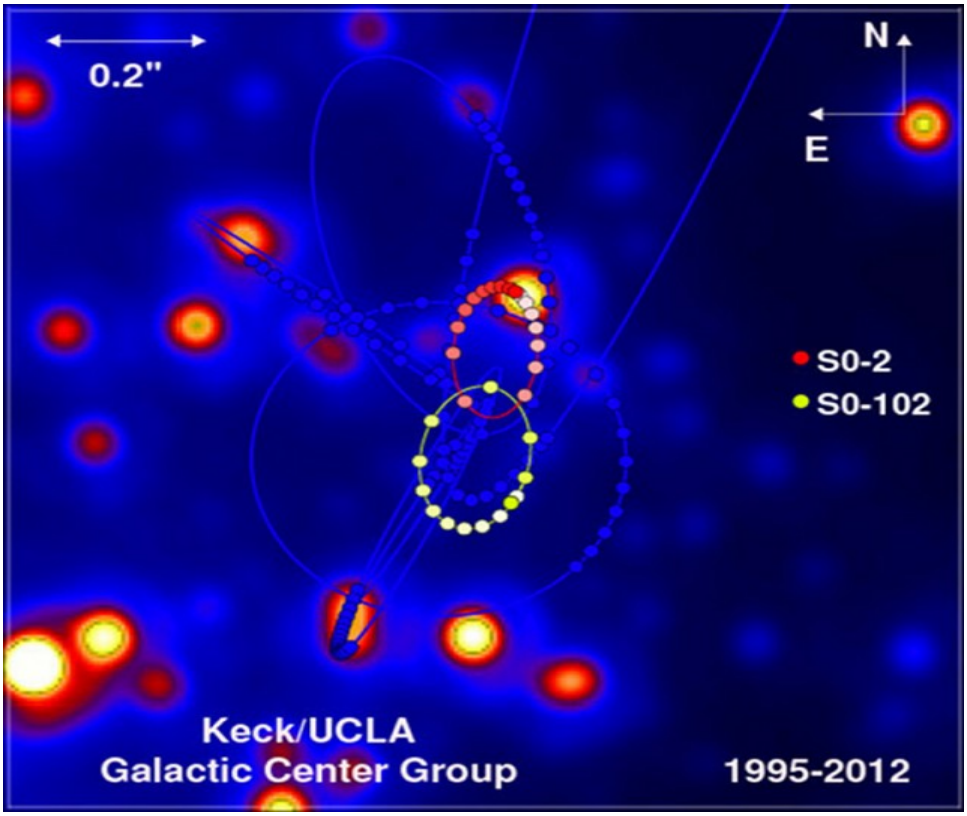

Figure 15: The inferred orbits of Stars (the red and yellow ellipses show the orbits of SO-2 and SO-102 stars, respectively) orbiting the supermassive black hole in the center of our galaxy [Credit: Andrea Ghez and colleagues using the Keck Telescopes].

38 What is the Event Horizon Telescope (EHT)?

- **The EHT is a large telescope array, consisting of a global network of radio telescopes, forming an Earth-sized virtual telescope.**
- **The first image of the event horizon of a black hole.**

More Details

The EHT collaboration involves more than 200 researchers from Africa, Asia, Europe, and North and South America, from the following radio observatories:

Telescope	Country	Wavelengths	Comments
ALMA	Chile	$0.3 - 3.6$ mm	Interferometer of **66** radio telescopes (54 antennas with 12 m diameter)
APEX	Chile	$0.2 - 1.5$ mm	A single dish of 12 m diameter
SMT	USA	sub-mm	A single dish of 10 m diameter
JCMT	USA	$0.4 - 1.4$ mm	A single dish of 15 m diameter
IRAM 30m	Spain	$1 - 3.7$ mm	A single dish of 30 m diameter
IRAM NOEMA	France	sub - few mm	Interferometer of 8 radio telescopes (each antenna has 15 m diameter)

The acronyms:
* ALMA → Atacama Large Millimeter Array.
* APEX → Atacama Path Finder Experiment.
* SMT → Submillimeter Telescope.
* JCMT → James Clerk Maxwell Telescope.
* IRAM → Institute for Radio Astronomy in the Millimeter Range.
* NOEMA → *NOrthernExtendedMillimeterArray*.

39 What is the resolving power of a telescope?

It is the ability of a telescope to show the fine details of an object and differentiate the images of two objects close to each other.

More Details

40 What are the major components of a radio Telescope?

A radio telescope consists of three main components:

1. **A radio reflector dish**
2. **A radio Antenna (receiver)**
3. **A recorder**

More Details

41 What is the epoch of reionisation?

- **It is the period when the universe changed from being mostly neutral to mostly ionized.**
- **It is approximately 600 million years after the big bang.**
- **It is caused by the first stars which are believed to have been very massive and highly luminous at wavelength below 912 A^0**

More Details

42 How many galaxies are there in our visible universe?

About two trillion galaxies

More Details

- Before the 20 th century it was thought that the universe is the Milky way, until 1925 when Hubble .....

43 What is the closest known galaxy to the Milky Way?

Andromeda galaxy

More Details

-
-
-

44 What is the farthest known galaxy in the universe?

- **The galaxy GN-z 11, discovered by the Hubble space telescope in 2018.**
- **It is at about 32 billion light years from us.**

More Details

The light emitted by the GN-z 11 galaxy and detected by the Hubble space telescope left this galaxy 13.4 billion years ago. Now one might wonder how come .....

45 What is a neutron star?

XXXXXXXXXXXXXXXXXXXXXX

More Details

46 How old is the universe?

XXXXXXXXXXXXXXXXXXXXXX

More Details

47 How big is the universe?

XXXXXXXXXXXXXXXXXXXXXX

More Details

48 What is the Cosmic Microwave Background?

XXXXXXXXXXXXXXXXXXXXXX

More Details

49 What is the epoch of dark ages?

xxxxxxxxxxxxxxxxxxxxxxxx

More Details

50 What is era of big bang nucleosynthesis?

xxxxxxxxxxxxxxxxxxxxxxxx

More Details

51 Appendices

51.1 Tunneling Through Coulomb Barrier

- **Rectangular Potential Barrier**
  We will first consider the behavior of quantum particle at a one-dimensional rectangular potential barrier of height V_0 (see Fig.)

$$V(x) = \begin{cases} 0. & x < 0 \\ V_0, & 0 < x < a \\ 0, & x > a \end{cases} \tag{51.1}$$

We assume that the particle has energy $E < V_0$, moving from the left toward the potential. Thus, the solutions to the time-independent Schrodinger equation in the regions $x < 0$ (region I), $0 < x < a$ (region II), and $x > a$ (region III), are given by[116]:

$$\psi_I(x) = Ae^{ikx} + Be^{-ikx}, \tag{51.2}$$
$$\psi_{II}(x) = Ce^{-\kappa x} + De^{\kappa x}, \tag{51.3}$$
$$\psi_{III}(x) = Ee^{-\kappa x} \tag{51.4}$$

Here $k = \sqrt{2mE}/\hbar$ is the momentum of a free particle, $\kappa = \sqrt{2m\left(V_0 - E\right)}/\hbar$, the A-term is the incident wave moving to the right and the B-term is the reflected wave moving to the left. Using the fact that wave function and its derivative must be continuous at $x = 0$ and $x = a$, i.e. $\psi_I(0) = \psi_{II}(0), \psi_I'(0) = \psi_{II}'(0), \psi_{II}(a) = \psi_{III}(a)$, and $\psi_{II}'(a) = \psi_{III}'(A)$ and $\psi_{II}(a) = \psi_{III}(a)$, we obtain

$$\begin{cases} A + B = C + D \\ ik\left(A - B\right) = -\kappa\left(C - D\right) \\ Ce^{-\kappa a} + De^{\kappa a} = Ee^{ika} \\ -\kappa\left(Ce^{-\kappa a} - De^{\kappa a}\right) = ikEe^{ika} \end{cases} \tag{51.5}$$

Note that there four equations for five unknowns, and hence we can only express four the five variables in terms one of them. Our aim is to determine the transmission probability of the particle through the potential barrier, which is

[116] In one dimension, the time-independent Schrodinger equation reads

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \left[E - V(x)\right]\psi = 0$$

defined as the ratio between the magnitudes of the transition current probability and the incident current probability[117], i.e.

$$T(E) = \left| \frac{\psi^*_{III}(x)\psi'_{III}(x) - \psi_{III}(x)\psi^{*'}_{III}(x)}{\psi^*_I(x)\psi'(x)_I - \psi_I(x)\psi^{*'}_I(x)} \right| = \frac{|E|^2}{|A|^2} \tag{51.6}$$

By solving the system of equations (51.5) for the coefficient E in terms of A, and after some tedious but straightforward calculation we obtain

$$\begin{aligned} \frac{E}{A} &= \frac{-i4k\kappa \exp(-ik)}{(\kappa - ik)^2 \exp(\kappa a) - (\kappa + ik)^2 \exp(-\kappa a)} \\ &= \frac{-i2k\kappa \exp(-ik)}{(\kappa^2 - k^2)\sinh \kappa a - 2i\kappa k \cosh \kappa a} \end{aligned} \tag{51.7}$$

which after substitution into (51.6), we get

$$\begin{aligned} T(E) &= \frac{4\kappa^2 k^2}{(\kappa^2 + k^2)^2 \sinh^2 \kappa a + 4\kappa^2 k^2} \\ &= \frac{16\kappa^2 k^2 \exp(-2\kappa a)}{(\kappa^2 + k^2)^2 (1 + \exp(-4\kappa a)) - \exp(-2\kappa a)(2\kappa^2 + 2k^4 - 12\kappa^2 k^2)} \end{aligned} \tag{51.8}$$

The above relation can be simplified for a "thick barrier", i.e. for which the the width of the potential barrier is larger than the characteristic length scale $1/\kappa$, so that we can neglect $\exp(-2\kappa a)$ and $\exp(-4\kappa a)$ in the denominator of the expression (51.8), and it becomes

$$T(E) \simeq \frac{16\kappa^2 k^2 \exp(-2\kappa a)}{(\kappa^2 + k^2)^2} \tag{51.9}$$

or, equivalently,

$$T^{(\text{thick})}(E) \simeq 16 \left(\frac{E}{V_0}\right)\left(1 - \frac{E}{V_0}\right) \exp\left(-2a\frac{\sqrt{2m(V_0 - E)}}{\hbar}\right) \tag{51.10}$$

On the other hand, if the barrier is very thin, i.e. $\kappa a \ll 1$, the expression of the transmission coefficient in (51.8) can be approximated by[118]

$$T^{(\text{thin})}(E) \simeq \exp\left(-2a\frac{\sqrt{2m(V_0 - E)}}{\hbar}\right) \tag{51.11}$$

[117] The transition current probability is defined as

$$\boldsymbol{j}(\boldsymbol{r}) = \frac{-i\hbar}{2m}\left[\psi^*(\boldsymbol{r})\boldsymbol{\nabla}\psi(\boldsymbol{r}) - \psi(\boldsymbol{r})\boldsymbol{\nabla}\psi^*(\boldsymbol{r})\right]$$

[118] We used the approximation $\exp -2(\kappa a) \simeq 1$ and $\exp -4(\kappa a) \simeq 1$ in the denominator of the expression (51.8).

We note that the transmission probability falls exponentially with the thickness and the height of the barrier[119].

- **Arbitrary Potential Barrier**
  The above analysis can be extended to a general potential barrier by dividing it into many small rectangular segments (see fig.), so that the total transmission probability, which we will denote by $\mathcal{P}_{\text{tunnelling}}$, can be obtained by taking the product of the transmission coefficients of the segments and in the thin barrier approximation (51.11), i.e.

$$\mathcal{P}_{\text{tunnelling}} = T_1 T_2 .....T_N \simeq \exp\left(-\sum_{i=1}^{N} 2p(x_i)\ \delta x_i\right) \tag{51.12}$$

  where x_i is the midpoint of the ith rectangular segment of with δx_i, and $p(x_i) = \sqrt{2m\,(V(x_i) - E)}/\hbar$ is the particle's momentum at this point. The entering and exit of the classically forbidden region, denoted by x_1 and x_2, respectively, are called the turning point, since at these locations the particle's energy is equal to the potential energy, and hence its kinetic energy vanishes. By taking $N \to \infty$, or equivalently $\delta x_i \to 0$, the summation in the exponential becomes an integration and we get

$$\mathcal{P}_{\text{tunnelling}} \simeq \exp\left(-2\int_{x_1}^{x_2} p(x)\ dx\right) = \exp\left(-\frac{2}{\hbar}\int_{x_{\text{in}}}^{x_{\text{exit}}} \sqrt{2m\,(V(x) - E)}\ dx\right) \tag{51.13}$$

- **Coulomb Potential Barrier**
  The electrostatic potential energy of a system of two nuclei with charges $Z_1|e|$ and $Z_2|e|$ is given by[120]

$$V(r) = \frac{\alpha\hbar c Z_1 Z_2}{r} \tag{51.14}$$

  where $\alpha \simeq 1/137$ is the fine structure constant. The two turning points are $r_1 = d_{\text{strong}} \sim$ fm, which is the distance scale at which the strong nuclear force operates[121] , and the point at which the particle's energy is equal to the electrostatic potential energy, i.e.

$$r_2 = \frac{\alpha\hbar c Z_1 Z_2}{E} \tag{51.15}$$

[119]The tunneling probability of electrons is larger inside semiconductors since the electron has an effective mass smaller than its rest mass (e.g. in a semiconductor like GaAs, the effective mass of the electron is approximately 6 % of its rest mass), making the exponential less suppressed.

[120]We used the relation $4\pi\epsilon_0\alpha\hbar c = e^2$.

[121]For the α decay process, r_1 is the nucleus radius, which can be estimated using the semi-empirical formula $R_N = 1.25A^{1/3}$ fm based on the the liquid drop model of nucleus.

Then, with the use of Eq (51.13), the tunneling probability reads

$$\mathcal{P}_{\text{tunnelling}} \simeq \exp\left[-\frac{2}{\hbar}\int_{d_{\text{strong}}}^{r_2}\sqrt{2m\left(\frac{\alpha Z_1 Z_2}{r}-E\right)}\,dr\right] \tag{51.16}$$

which can be re-written as

$$\mathcal{P}_{\text{tunnelling}} \simeq \exp-(2\gamma); \qquad \gamma = \frac{\sqrt{2\mu E}}{\hbar}\int_{d_{\text{strong}}}^{r_2}\sqrt{\frac{r_2}{r}-1}\,dr \tag{51.17}$$

Here $\mu = m_1 m_2/(m_1+m_2)$ is the reduced mass of the system of two nuclei. To evaluate the above integral, we make the change of variable $r = r_2\sin^2\theta$, with the integration lower and the upper limits of the integral $\theta_{\text{min}} = \sqrt{d_{\text{strong}}/r_2}$ and $\theta_{\text{max}} = \pi/2$, respectively, and obtain

$$\gamma = \frac{\sqrt{2\mu E}}{\hbar}\left[\frac{\pi}{2}r_2 - \sin^{-1}\left(\frac{d_{\text{strong}}}{r_2}\right) - \left(\sqrt{\frac{d_{\text{strong}}}{r_2}-\left(\frac{d_{\text{strong}}}{r_2}\right)^2}\right)\right] \tag{51.18}$$

Since r_2 is usually much small $\ll d_{\text{strong}}$, we can make the approximation $\sin^{-1}\left(\sqrt{d_{\text{strong}}/r_2}\right) \simeq \sqrt{d_{\text{strong}}/r_2}$ and neglect the term $(d_{\text{strong}}/r_2)^2$, which simplifies the above expression as

$$\gamma \simeq \frac{\sqrt{2\mu E}}{\hbar}\left[\frac{\pi}{2}r_2 - 2\sqrt{r_2 d_{\text{strong}}}\right] \tag{51.19}$$

or, equivalently[122],

$$\gamma = \frac{\pi\alpha Z_1 Z_2\sqrt{2\mu c}}{2\sqrt{E}} + \text{constant} \tag{51.20}$$

Thus, for energies well below the Coulomb potential energy[123], the probability of tunneling through the barrier has the form[124]

$$\mathcal{P}_{\text{tunnelling}} \propto \exp\left(-\sqrt{\frac{E_G}{E}}\right) \tag{51.21}$$

[122]The constant in (51.20) is given by

$$\text{constant} = -2\sqrt{\frac{2\alpha\mu c Z_1 Z_2 d_{\text{strong}}}{\hbar}} = -1.17\,\sqrt{\frac{d_{\text{strong}}}{\text{fm}}}$$

[123]This is the case for energies in the core of stars.

[124]The derivation of this expression is given in Appendix XX. In some nuclear physics and nuclear astrophysics literature the tunneling probability is expressed as

$$\mathcal{P}_{\text{tunnelling}} \propto \exp\left(-2\pi\eta(E)\right)$$

where $\eta(E) = Z_1 Z_2/\hbar v$, known as the Sommerfeld parameter.

with $v = \sqrt{2E/\mu}$ denotes the magnitude of the relative velocity of the two nuclei, and

$$E_G = (\pi\alpha Z_1 Z_2)^2\, 2\mu c^2 \simeq \left(Z_1^2 Z_2^2\right)\left(\frac{\mu}{m_p/2}\right) 493 \text{ keV} \tag{51.22}$$

is the so-called **Gamow energy**.

51.2 The Rate of Nuclear Fusion in the Core of Stars

Let n_a and n_b be the number densities of nuclei a and b, respectively, in a plasma, and σ the cross section of their nuclear fusion. The volumetric reaction rate for the fusion process, i.e. the number of fusion reactions occurring per unit time per unit volume, is given by

$$\mathcal{R}_{ab} = \left(\frac{1}{1+\delta_{ab}}\right)\sigma v n_a n_b \tag{51.23}$$

The cross section is conventionally written as

$$\sigma(E) = S(E)\sigma^{(\text{geo})}\mathcal{P}_{\text{tunnelling}} \tag{51.24}$$

where $\sigma^{(\text{geo})}$ is the geometric cross section given by the area of a disk of radius equal to the de Broglie wavelength, $\lambda_{\text{dB}} = h/p = h/\mu v$, i.e.

$$\sigma^{(\text{geo})} = \pi\lambda_{\text{dB}}^2 = \pi\frac{\hbar^2}{\mu^2 v^2} \propto \frac{1}{E} \tag{51.25}$$

The factor in $S(E)$ in Eq (51.24), known as the astrophysical S-factor, contains all the effects due to the finite size and structure, which makes it very difficult to calculate it. Moreover, experimentally it is hard to measure the S-factor in laboratory at low energies that correspond to temperatures inside typical stars. Instead, it is determined by extrapolating laboratory-measured values at high energy down to low energies. For non-resonant reactions this factor is weakly varying function of energy[125] Since the particles are in thermal equilibrium, the expression of (51.26) has to be folded with the Maxwell-Boltzmann distribution,

$$\begin{aligned} f(v)dv &= \left(\frac{\mu}{2\pi k_B T}\right)^{3/2} \exp\left[-\mu v^2/2k_B T\right]\, 4\pi v^2\, dv \\ &= \frac{2}{\sqrt{\pi}}\frac{\sqrt{E}}{(k_B T)^{3/2}}\, \exp\left[-E/k_B T\right]\, dE \end{aligned}$$

[125] An estimate of the S-factor for proton-proton reaction is (see Ref.)

$$S^{(pp)}[E] = 4 \times 10^{-25} \text{ MeV.barn}$$

where barn $= 10^{-24}$ cm denotes the unit of cross section.

and we find the thermally averaged rate

$$< \mathcal{R}_{12} >= \left(\frac{\mu}{8\pi}\right) \int dE \ \exp\left(-\sqrt{\frac{E_G}{E}}\right) \exp\left(-\frac{E}{k_B T}\right) \tag{51.26}$$

Note that the first exponential increases for higher thermal velocities, whereas the second exponential (coming from the Boltzmann distribution of velocities) decreases. Thus, the fusion reaction rate is important only in some energy range, and it reaches a maximum at a particular energy, E_0, known as Gamow peak. The value of E_0 can be determined by taking the derivative of the product of the exponential terms with respect to energy and set it to zero, i.e.

$$\frac{d}{dE}\left[\exp\left(-\sqrt{\frac{E_G}{E}}\right) \exp\left(-\frac{E}{k_B T}\right)\right]_{E_0} = 0 \tag{51.27}$$

We obtain

$$E_0 = \left[\frac{\sqrt{E_G} k_B T}{2}\right]^{2/3} = 1.22 \left[Z_1^2 Z_2^2 \mathbf{T_6}^2\right]^{1/3} \mathcal{A}[\text{keV}] \tag{51.28}$$

where $\mathcal{A} = A_1 A_2/(A_1 + A_2)$, with A_i the atomic masses of the nuclei 1 and 2, respectively, which is the reduced atomic mass of the system of two nuclei, and T_6 is the temperature in million of degrees. Note that E_0 is larger than the average particle's thermal energy but much smaller than the Gamow energy of the Coulomb barrier. Hence, for particles with energies close to E_0, we expect thermonuclear reactions to be the most effective. Moreover, we can approximate the exponential term in (51.26) with a Gaussian distribution centered around E_0, i.e.

$$\exp\left[-\sqrt{\frac{E_G}{E}} - \left(\frac{E}{k_B T}\right)\right] \simeq C \exp\left[-\left(\frac{E - E_0}{\Delta/2}\right)^2\right] \tag{51.29}$$

Setting $E = E_0$ and using the relation between E_G and E_0 in (51.28), we get

$$C = \exp\left(-\frac{3E_0}{k_B T}\right) \tag{51.30}$$

For The width of the Gaussian distribution, Δ, it can be determined by taking the second derivative of the both sides of Eq (51.29), evaluated at $E = E_0$, which yields

$$\Delta = \frac{4}{\sqrt{3}} \sqrt{E_0 k_B T} \tag{51.31}$$

.

51.3 Sunyaev-Zel'dovich (SZ) Effect

- It's the inverse Compton effect for CMB photons against electrons in the **hot** gas of clusters.
- Optical depth:

$$\tau = n\sigma l \tag{51.32}$$

  For clusters, $l =$ few Mpc $\sim 10^{25}$ cm, $n < 10^{-3}$ cm, and $\sigma = 6.65 \times 10^{-25}$ cm^2, which yields

$$\tau \simeq 0.01 \tag{51.33}$$

  Thus, there is 1% likelihood that a CMB photon crossing the cluster is scattered by an electron.
- Since $E_e \gg E_{CMB}$, an electron in the cluster transfer part of its energy to a CMB photon that is crossing it. To first order, this gain in energy, or, equivalently, in frequency, is given by

$$\frac{\Delta\nu}{\nu} = \frac{k_B T_e}{m_e c^2} \simeq \frac{5 \text{ keV}}{500 \text{ keV}} = 10^{-2} \tag{51.34}$$

- The resulting CMB anisotropy anisotropy is

$$\frac{\Delta T}{T} =\simeq \tau \frac{\Delta\nu}{\nu} \simeq 10^{-4} \tag{51.35}$$

51.4 Specific Intensity and Radiation Pressure

51.5 Radiation Transport

51.6 Dark Matter in Cluster of Galaxies from Hot X-ray Observations

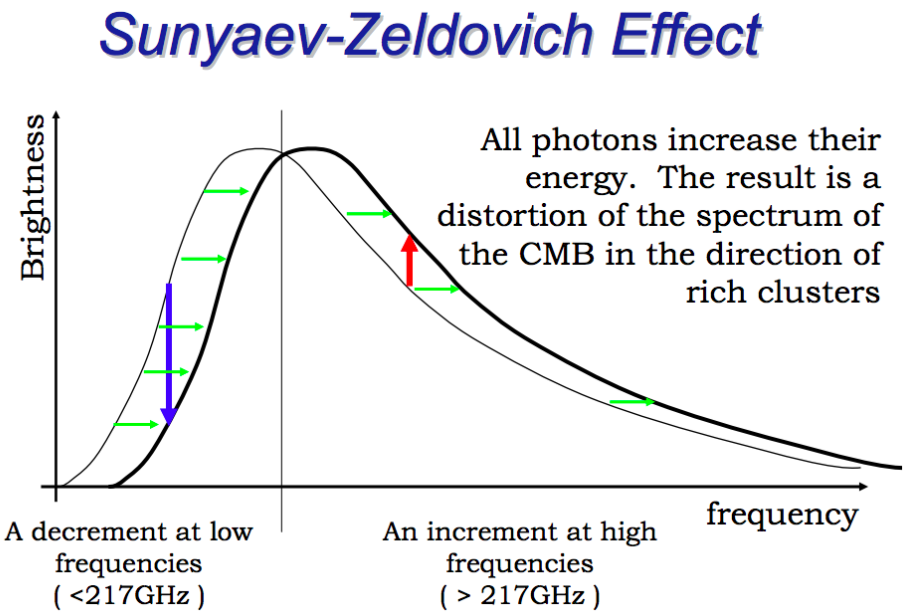

Figure 16: Sunyaev-Zel'dovich effect on the CMB spectrum.

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