On the control of an escaping robot

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Abstract

This paper deals with the elaboration of a feedback control algorithm for the control of an escaping robot engaged in a pursuit-evasion game. The pursuer is a smart robot moving with a control strategy that is unknown and unpredictable to the evader. Our control strategy is based on the use of geometric rules, where the equilibrium point characterizing the distance pursuer-evader is kept unstable. The designed control law acts mainly on the orientation angle. However when the pursuer is maneuvering in the speed, another feedback control law for the linear velocity is designed for the evader in order to keep a minimum distance from the pursuer.

1 Introduction

Wheeled mobile robot motion control is one of the most challenging problems in robot control theory. This is due to the fact that wheeled mobile robots present a typical example of the nonholonomic mechanism, which reduces the space of the control inputs. The control of a wheeled mobile robot from a given initial configuration to a final configuration is not a trivial task. For example it is not possible to rigorously control the robot using linear inputs. Furthermore it is stated by Brockett theorem ([1], [2]) that mobile wheeled robots cannot be stabilized using smooth static feedback law. To overcome this problem, three types of controllers were suggested.

1. The first type uses smooth but time-varying controllers.

2. The second type uses time-invariant but non-smooth controllers

3. The third type is a combination between the first and the second type.

This paper deals with the control of a wheeled mobile robot engaged in a pursuit-evasion game in a obstacle-free open area. The problem of controlling an escaping robot is more difficult than the problem of controlling a robot from an initial to a final configuration, since the robot is continuously chased by a smart pursuer and furthermore the strategy of the pursuer is completely unknown to the escaping robot. Moreover, the nonholonomic constraint renders the problem much harder to solve.

One of the earliest works dealing with the evasion problem is presented in [4], where the players are a robot and a rabbit. The robot moves with a constant speed in a predetermined path around a room. The rabbit tries to avoid collision with the robot using a Min-Max strategy. This problem is simpler than the problem considered here since the robot is not smartly chasing the rabbit. Guidas et al. [3] suggested a complete algorithm based on the search of a finite graph for planning the motion of one or more pursuers to eventually see the evader. A game-theoretical approach framework for robot motion planning is suggested by LaValle [5], [6]. Our approach is based on the use of the geometry and the kinematics equations form the pursuer to the evader rather than game theory. The resulting control strategy is simple and can be implemented with minimum cost. Our control law falls in the first category.

2 Robot model

The escaping robot is modeled as a wheeled mobile robot of the unicycle type, with the following kinematics equations:

$$\begin{aligned} \dot{x}_e &= u_e \cos\phi_e \\ \dot{y}_e &= u_e \sin\phi_e \\ \dot{\phi}_e &= \omega_e \end{aligned} \tag{1}$$

where (x_e, y_e) are the escaping robot coordinates in the Cartesian frame of coordinates, ϕ_e is the robot orientation with respect to positive x-axis. A configuration of the robot is given by the triple $q_e = [x_e, y_e, \phi_e]^T$. The control variables for the mobile robot are u_e and ω_e which are the linear and angular velocities, respectively. As we mentioned previously, wheeled mobile robots present a non-integrable constraint on the velocities. This constraint is called the nonholonomic constraint. From equation (1), the nonholonomic constraint states that:

$$\dot{x}_e \sin\phi_e = \dot{y}_e \cos\phi_e \tag{2}$$

This constraint restricts the space of the possible velocities. From a physical point of view equation (2) means that the robot rolls freely without slipping in the perpendicular direction.

The pursuer is also modeled as a wheeled mobile robot for which the kinematics model is given by:

$$\begin{aligned} \dot{x}_p &= u_p \cos\phi_p \\ \dot{y}_p &= u_p \sin\phi_p \\ \dot{\phi}_p &= \omega_p \end{aligned} \tag{3}$$

Similarly to the escaping robot, (x_p, y_p) are the pursuer coordinates in the Cartesian plane, ϕ_p is the pursuer's robot orientation angle with respect to the positive x-axis. The control inputs for the pursuer are u_p and ω_p, which are the linear and angular velocities also. We assume that the robot pursuer aims to catch the escaping robot by using some closed loop control law. This control law is unknown to the evader. Let us denote by $u_{p,\max}$ the maximum linear velocity for the pursuer and $u_{e,\max}$ the maximum velocity for the escaping robot. We also assume that the maximum linear velocity is the same for both robots, i.e.,

$$u_{p,\max} = u_{e,\max} \tag{4}$$

In a similar way we assume that the minimum turning radius is the same for both robots. Of course, we assume that both robots have an external vision system allowing them to see the other robot and determine some important quantities such as the velocity and the orientation angle. Furthermore both robots have a local on-board intelligence system allowing them to process the data obtained from the sensors.

Mobile robots are also underactuated (underactuated systems occur typically from the nonholonomic constraint), for underactuated systems, the control space is smaller than the configuration space. This is the situation for mobile robots where there are only two inputs (linear velocity and steering angle) to control the robot's three degrees-of-freedom. In this paper, our aim is to design a closed loop control law which allows the escaping robot to escape from its predators. Our control strategy is based on the use of the pursuer-evader kinematics equations combined with some geometric rules. Figure 1 shows the escape-pursuit geometry. The pursuer is denoted by the letter P and the evader by E.

With reference to figure 1, we define the following quantities:
(a) r is the relative distance between the escaping robot and the pursuer reference points. We define the line joining E and P by the line of sight.
(b) The angle of the line of sight σ represents the angle between the positive x-axis and the line of sight. This angle is given by

$$\sigma = \tan^{-1}\left(\frac{y_e - y_p}{x_e - x_p}\right) \tag{5}$$

It is clear that σ depends on the pursuer and the evader coordinates. We also define the angle δ for both the pursuer and the escaping robots, where δ is the angle between the line of sight and the velocity direction. In this case we have:

$$\sigma = \phi + \delta \tag{6}$$

This relation is valid for both the pursuer and the evader.

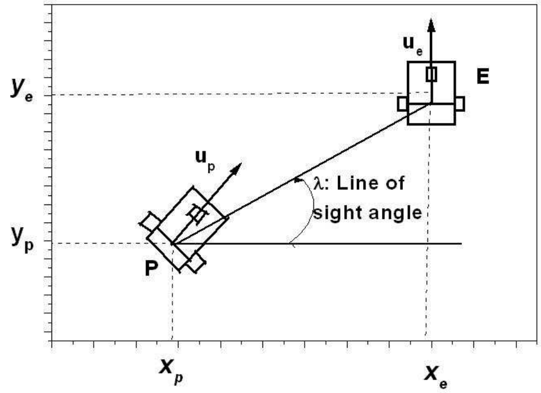

Figure 1: Pursuit-evasion geometry

Let us consider the distance in the Cartesian plane between the escaping robot to the pursuer

$$\begin{aligned} x_{ep} &= x_p - x_e \\ y_{ep} &= y_p - y_e \end{aligned} \tag{7}$$

with

$$x_{ep}^2 + y_{ep}^2 = r_{ep}^2 \tag{8}$$

The aim of the pursuer is to drive x_{ep} and y_{ep} to zero at the same time, while the aim of the escaping robot is keep at least $x_{ep} \neq 0$ or $y_{ep} \neq 0$.

We consider the velocity from the pursuer to the target with respect to the x-axis and the y-axis. By taking the derivative of (7) with respect to time, we get

$$\begin{aligned} \dot{x}_{ep} &= \dot{x}_p - \dot{x}_e \\ \dot{y}_{ep} &= \dot{y}_p - \dot{y}_e \end{aligned} \tag{9}$$

By taking into account the kinematics equations for the pursuer and the target we get

$$\begin{aligned} \dot{x}_{ep} &= u_p \cos\phi_p - u_e \cos\phi_e \\ \dot{y}_{ep} &= u_p \sin\phi_p - u_e \sin\phi_e \end{aligned} \tag{10}$$

The system of equations (10) presents a system of differential equations. Recall that the control variables are u_e and ϕ_e. In the same way u_p and ϕ_p are used by the smart pursuer in order to intercept the escaping robot. The pairs (u_p, ϕ_p) and (u_e, ϕ_e) depend mutually on each other.

3 Control strategy for the escaping robot

Our control strategy for the robot angular velocity is based on the use of the kinematics equations and geometrical rules. The principle of our control strategy is to make the velocity of the escaping robot lying on the line joining the escaping robot and the pursuer. We suggest to use the following control strategy

$$\phi_e = \sigma(t) \pm \pi \tag{11}$$

This control strategy is illustrated in figure 2. According to equation (11), the velocity of the evader lies on the line of sight joining the escaping robot with the pursuer, but it is directed in the opposite direction. Recall that the angle σ depends explicitly on the pursuer and the evader control strategies.

Under the control law (11), the kinematics equations for the escaping robot are given by

$$\begin{aligned} \dot{x}_e &= -u_e \cos\sigma \\ \dot{y}_e &= -u_e \sin\sigma \\ \dot{\phi}_e &= \omega_e = \dot{\sigma} \end{aligned} \tag{12}$$

It is worth noting the control strategy given by (11) depends explicitly on the pursuer's maneuvers, since the line of sight angle depends on the pursuer's steering angle (and coordinates). Similarly to the escaping robot, the pursuer has two control variables, namely, the linear and angular velocities. The control law (11) considers that the pursuer is maneuvering only in the angular velocity. In the next section, we take into account the pursuer linear velocity. We consider two cases, namely when the pursuer is moving with a constant and time-variant velocity.

3.1 Pursuer moving with a constant velocity

If the pursuer is moving with a constant linear velocity, then the control strategy requires the robot also to move with a constant linear velocity, i.e.,

$$v_e = constant$$

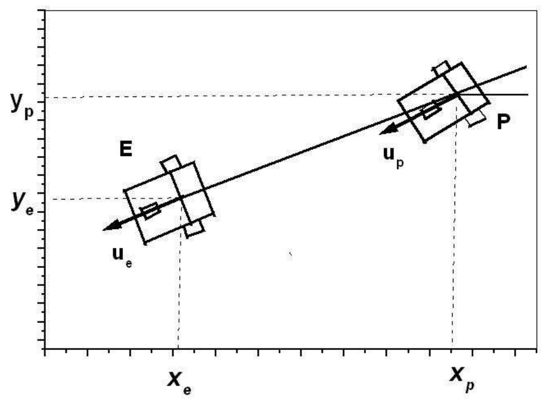

Figure 2: Evasion strategy

where v_e satisfies the following relationship

$$v_e = kv_p \tag{13}$$

with $|k| \geq 1$. Here the only control variable is the robot's orientation angle.

3.2 Pursuer moving with time varying velocity

In the case where the pursuer is maneuvering with time varying velocity, the control law given by (11) is not enough to guarantee the escape. In this case it is necessary for the escaping robot to keep a given distance from the pursuer. To accomplish this task, the escaping robot will move with time varying velocity. So it is necessary to design a closed loop control law for v_e. To accomplish this task, we suggest three different approaches.

1. The escaping robot keeps a constant distance from the pursuer.

2. The escaping robot keeps a constant distance from the pursuer with respect to the x-axis.

3. The escaping robot keeps a constant distance from the pursuer with respect to the y-axis.

These three approaches will be discussed in the following paragraphs.

3.2.1 Constant distance from the pursuer

The relative range between the pursuer and the evader is given by

$$\dot{r} = u_p \cos(\phi_p - \lambda) - u_e \cos(\phi_e - \lambda)$$

This can be obtained using a simple change of variable (for details see [7]). Under the suggested control law, we have

$$\dot{r} = u_p \cos(\phi_p - \lambda) + u_e$$

In order to keep a constant distance from the pursuer, the escaping robot must be controlled in order to satisfy $\dot{r} = 0$. This is accomplished by choosing the following closed loop for the escaping robot's velocity

$$u_e = -u_p \cos(\phi_p - \lambda) \tag{14}$$

Observe that in this case, the evader velocity is always smaller than the pursuer velocity in absolute value.

3.2.2 Constant distance with respect to the x-axis

In this case $\dot{x}_d = 0$. The closed loop control law for the escaping robot's linear velocity is given as follows

$$u_e = u_p \frac{\cos\phi_p}{\cos\phi_e} \tag{15}$$

By considering this equation with the control law for the steering angle, we get

$$u_e = -u_p \frac{\cos\phi_p}{\cos\sigma} \tag{16}$$

with $\sigma \neq \frac{\pi}{2} + k\pi$, $k = 1, 2...$

3.2.3 Constant distance with respect to the y-axis

In this case $\dot{y}_d = 0$, which results in the following closed loop control law for the escaping robot linear velocity

$$u_e = u_p \frac{\sin\phi_p}{\sin\phi_e} \tag{17}$$

Similarly to the previous case, by considering this equation with the control law for the steering angle, we get

$$u_e = -u_p \frac{\sin \phi_p}{\sin \sigma} \tag{18}$$

with $\sigma \neq 0 + k\pi$, $k = 1, 2...$

In both equations (15) and (17), the escaping robot linear velocity depends on the pursuer's linear velocity and steering angle and of course the line of sight angle. Equation (14) which guarentees a constant range is implemetable without any problem, since it states that the evader velocity is smaller than the pursuer's velocity. However equations (16) and (18) pose problem when the line of sight angle is close to $\frac{\pi}{2}$ and zero, respectively.

In general the velocity of the escaping robot can be chosen to vary proportionally to the pursuer velocity, $v_e(t) = kv_p(t)$ with k =constant and $|k| \geq 1$.
By considering equation (10) and equation (11), we get for $\dot{x}_{ep}$ and $\dot{y}_{ep}$

$$\begin{aligned} \dot{x}_{ep} &= u_p \cos \phi_p + u_e \cos \sigma \\ \dot{y}_{ep} &= u_p \sin \phi_p + u_e \sin \sigma \end{aligned} \tag{19}$$

By using equation (5) for the line of sight angle we get

$$\begin{aligned} \dot{x}_{ep} &= u_p \cos \phi_p + u_e \cos \left(\tan^{-1} \tfrac{y_{ep}}{x_{ep}}\right) \\ \dot{y}_{ep} &= u_p \sin \phi_p + u_e \sin \left(\tan^{-1} \tfrac{y_{ep}}{x_{ep}}\right) \\ x_{ep}(t_0) &= x_{ep0} \\ y_{ep}(t_0) &= y_{ep0} \end{aligned} \tag{20}$$

where x_{ep0} and y_{ep0} the initial state. Equation (20) is a system of nonlinear ordinary differential equation. The solution for this system provides the trajectory of the escaping robot according to the pursuer maneuvers. Unfortunately this system is highly nonlinear and no closed form solution can be obtained in the general case. It is clear that our control strategy aims to keep the solution for system (20) far away from its equilibrium point situated at the origin, i.e., keep the equilibrium point (x_{ep}, y_{ep})=$(0, 0)$ unstable.

4 About the nonholonomic constraint

The nonholonomic constraint states that the wheeled mobile robot cannot move perpendicularly to its main axis. This constraint renders the robot control more difficult and challenging. If at the initial state we have $\phi_e(t_0) = \pi \pm \sigma(t_0)$, then the application of the control law (11) is straightforward. Otherwise, when $\phi_e(t_0) \neq \pi \pm \sigma(t_0)$ the control law (11) cannot be implemented directly, in this case it is necessary to drive the escaping robot to an orientation angle that is equal to

$$\phi_e(t) = \sigma(t) \pm \pi \tag{21}$$

at a given time. The application of the control law (11) becomes simple in this case. So in the case where $\phi_{e0}(t) \neq \sigma_0(t) \pm \pi$, it is necessary to use a heading regulation phase in order to drive the escaping robot orientation to its desired orientation $(\sigma(t) \pm \pi)$. Any control algorithm can be used for this purpose. Note that if the escaping robot is not caught during this phase, then it will never be caught during the second phase.

5 Simulation

In order to test our control algorithm, we consider a simulation example. We assume that initially equation (21) is satisfied. The pursuer is using a smart pursuit law, and the evader is using the control strategy described above. The initial coordinates are the following

$$\begin{aligned} x_p = 0, y_p = 0 \\ x_e = 10, y_e = 10 \end{aligned} \tag{22}$$

From (22) the initial line of sight angle is $\lambda = \frac{\pi}{4} rd$. The pursuer initial orientation is $\frac{\pi}{3} rd$. The linear velocities for the pursuer and the evader equal.

Simulation of the pursuit-evasion is shown in figure 3. The pursuer changes its strategy at positions P_1, P_2 and P_3. The relative distance pursuer-evader is shown in figure 4.

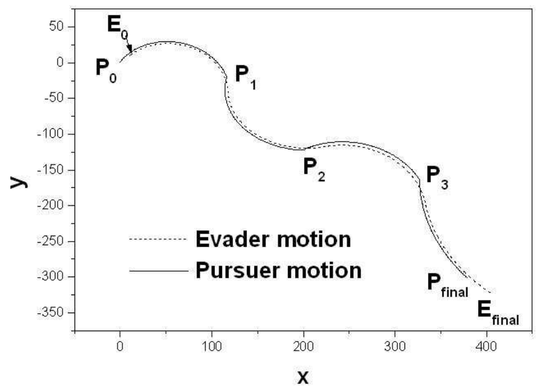

Figure 3: Path traveled by the pursuer and the evader

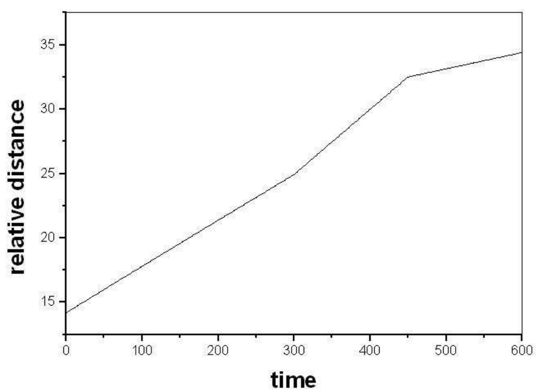

Figure 4: Relative range between the pursuer and the evader during the pursuit

6 Conclusion

We presented an algorithm for the control of an escaping robot. The escaping robot consists of a simple wheeled mobile robot of the unicycle type. The escaping robot is continuously chased by a smart robot which aims to accomplish the interception in minimum time. Our controll startegy which consits of a closed loop control system is based on the use of geometrical rules, where the equilibrium point characterizes the distance between the evader and the pursuer is kept asymptotically unstable (asymptotically stable when considering the backward evolution of time). The control law is simple and can be implemented easily in practice. Using simulation, we show that the control strategy allows to the evader to maneuver and escape successfully to its predator.

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