COLUMBIA | ENGINEERING
The Fu Foundation School of Engineering and Applied Science

Linear Quadratic Gaussian Control of Wind Turbines

Abdulrahman Kalbat

Columbia University in the City of New York

(Teaching Assistant at UAE University "Currently on leave")

October 3rd, 2013

Paper Structure

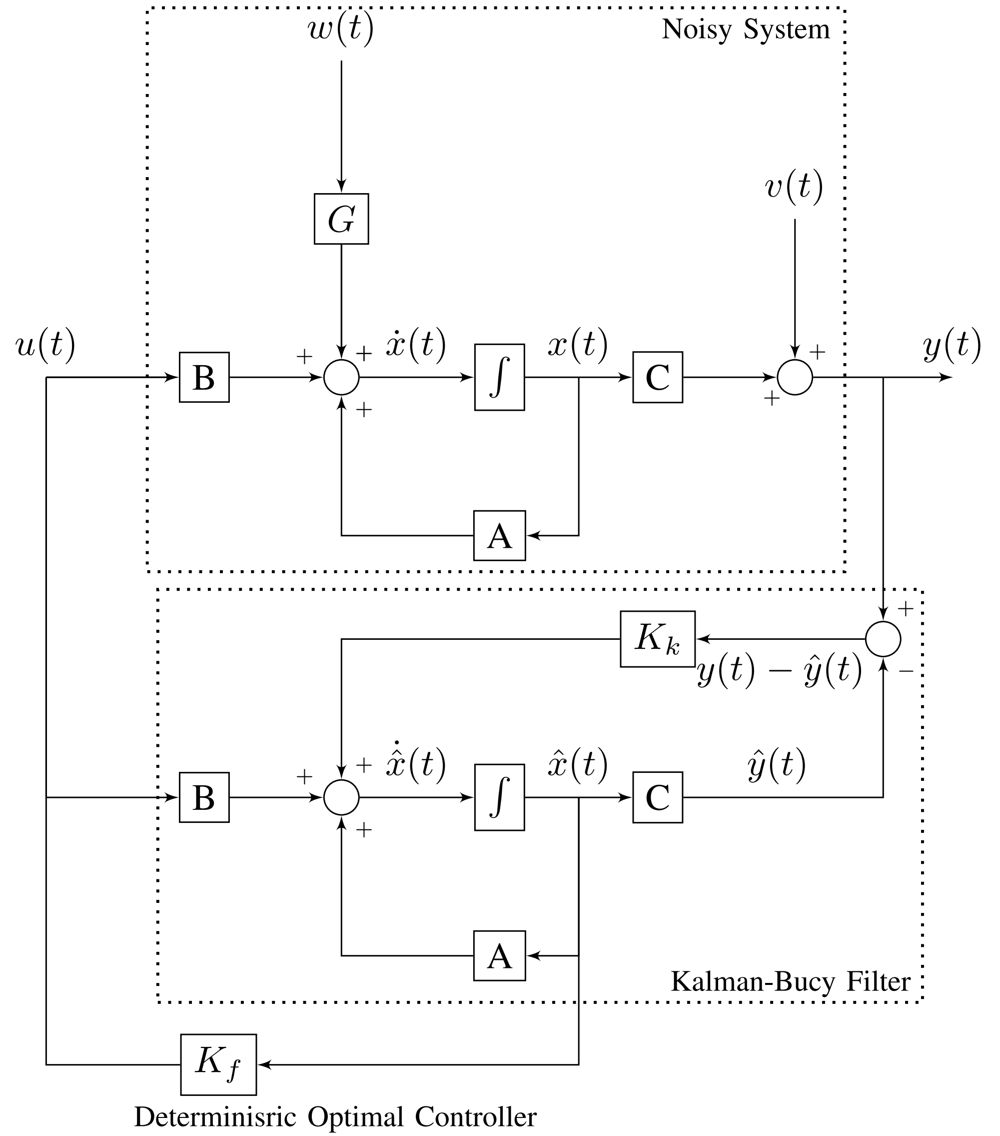
- **Linear Quadratic Gaussian (LQG) Control**
  - General problem formulation
  - Assumptions
  - Design procedures
- **Real system simulation**
  - Controls Advanced Research Turbine (CART)
  - State space model of CART
- **Results and Discussion**
  - Simulation results
  - Limitation of LQG

Linear Quadratic Gaussian (LQG) Control

- LQG is rooted in optimal stochastic control theory
- It is simply a combination of:
  - **Linear Quadratic Regulator (LQR):** for full state feedback
  - **Kalman Filter:** for state estimation
- Linear Quadratic Gaussian (LQG)
  - **Linear** system model
  - **Quadratic** cost function
  - **Gaussian** noises

Linear Quadratic Gaussian (LQG) Control

- LQG block diagram



LQG General Form

- The state space equations of the open loop plant for a standard LQG problem:

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t)$$
$$y(t) = Cx(t) + v(t)$$

where

$x(t)$: state vector

$u(t)$: control input vector

$y(t)$: measured output vector

$w(t)$: process noise

$v(t)$: measurement noise

A : state matrix

B : control input gain matrix

G : plant noise gain matrix

C : measured state matrix

LQG General Form

- The state space equations of the Kalman Filter:

$$\dot{\hat{x}}(t) = (A - K_k C)\hat{x}(t) + Bu(t) + K_k y(t)$$
$$\hat{y}(t) = C\hat{x}(t)$$

where

$\hat{x}(t)$: estimated state vector

$\hat{y}(t)$: estimated output vector

K_k : optimal state estimation gain vector

LQG General Form

- Assumptions:

White Gaussian noises

$$\begin{aligned} E[x(0)] &= \hat{x}_o \\ E[w(t)] &= 0 \\ E[v(t)] &= 0 \end{aligned}$$

Covariance

$$E[w(t)w(\tau)^T] = \begin{cases} W, & \text{if } t = \tau \\ 0, & \text{if } t \neq \tau \end{cases}$$

$$E[v(t)v(\tau)^T] = \begin{cases} V, & \text{if } t = \tau \\ 0, & \text{if } t \neq \tau \end{cases}$$

$$E[w(t)v(\tau)^T] = \begin{cases} R_{12}, & \text{if } t = \tau \\ 0, & \text{if } t \neq \tau \end{cases}$$

Uncorrelation of initial states with the noises

$$\begin{aligned} E[x(0)w(t)^T] &= 0 \\ E[x(0)v(t)^T] &= 0 \end{aligned}$$

LQG Design Procedure

Step 1: Check optimal gain existance criteria:

$$(A, B) \ is \ Controllable$$
$$(A, C) \ is \ Observable$$

Step 2: Optimal state estimation gain calculation:

$$J_k = E\left\{(x - \hat{x})^T (x - \hat{x})\right\}$$
$$AP_k + P_k A^T + GWG^T - P_k C^T V^{-1} C P_k = 0$$ Filter Algebraic Riccati Equation (FARE)
$$K_k = P_k C^T V^{-1}$$

LQG Design Procedure

Step 3

- Optimal State Feedback Gain Calculation:

$$J_f = \int_0^T (z^T Q_f z + u^T R_f u) dt$$

$$A^T P_f + P_f A - P_f B R_f^{-1} B^T P_f + Q_f = 0$$ Control Algebraic Riccati Equation (CARE)

$$K_f = R_f^{-1} B^T P_f$$

- Weighing Matrices Selection:

$$Q = C^T C \qquad R = \rho I$$

or

$$Q_{ii} = \frac{1}{Max\ (x_{ii}^2)} \qquad R_{ii} = \frac{1}{Max\ (u_{ii}^2)}$$

(Bryson's Rule)

LQG Design Procedure

Step 4: Linear Quadratic Gaussian Regulator by combining

Optimal State Estimation and Optimal State Feedback:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A - BK_f & BK_f \\ 0 & A - K_k C \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} G & 0 \\ G & -K_k \end{bmatrix} \begin{bmatrix} w(t) \\ v(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} w(t) \\ v(t) \end{bmatrix}$$

where

$$e(t) = x(t) - \hat{x}(t) \text{ (state estimation error)}$$

Controls Advanced Research Turbine (CART)

- Used by the National Wind Technology Center (NWTC) and operated by the National Renewable Energy Laboratory (NREL) and located at Boulder, Colorado
- Used for:
  - Exploring potential control innovations
  - Field test advanced control systems.
- Very flexible:
  - More than 80 sensors
  - Collective Blade Pitching + Individual Blade Pitching

Controls Advanced Research Turbine (CART)

Numerical Example using MATLAB

- The model was created by linearizing the motion equations at a control design point of:
  - 18 m/s for wind speed
  - 12 degress for rotor collective pitch
  - 42 RPM for rotor speed.
- The main objective is to operate the machine as a variable speed wind turbine in region 3
  - by applying constant torque to the generator
  - by maintaining a constant rotor speed
  - through the collective rotor blade pitching.

Numerical Example using MATLAB

- CART's state space model:

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t)$$
$$y(t) = Cx(t) + v(t)$$

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \text{ where } \begin{cases} x_1(t) : \text{is rotor speed} \\ x_2(t) : \text{is drive train torsion} \\ x_3(t) : \text{is generator speed} \end{cases}$$

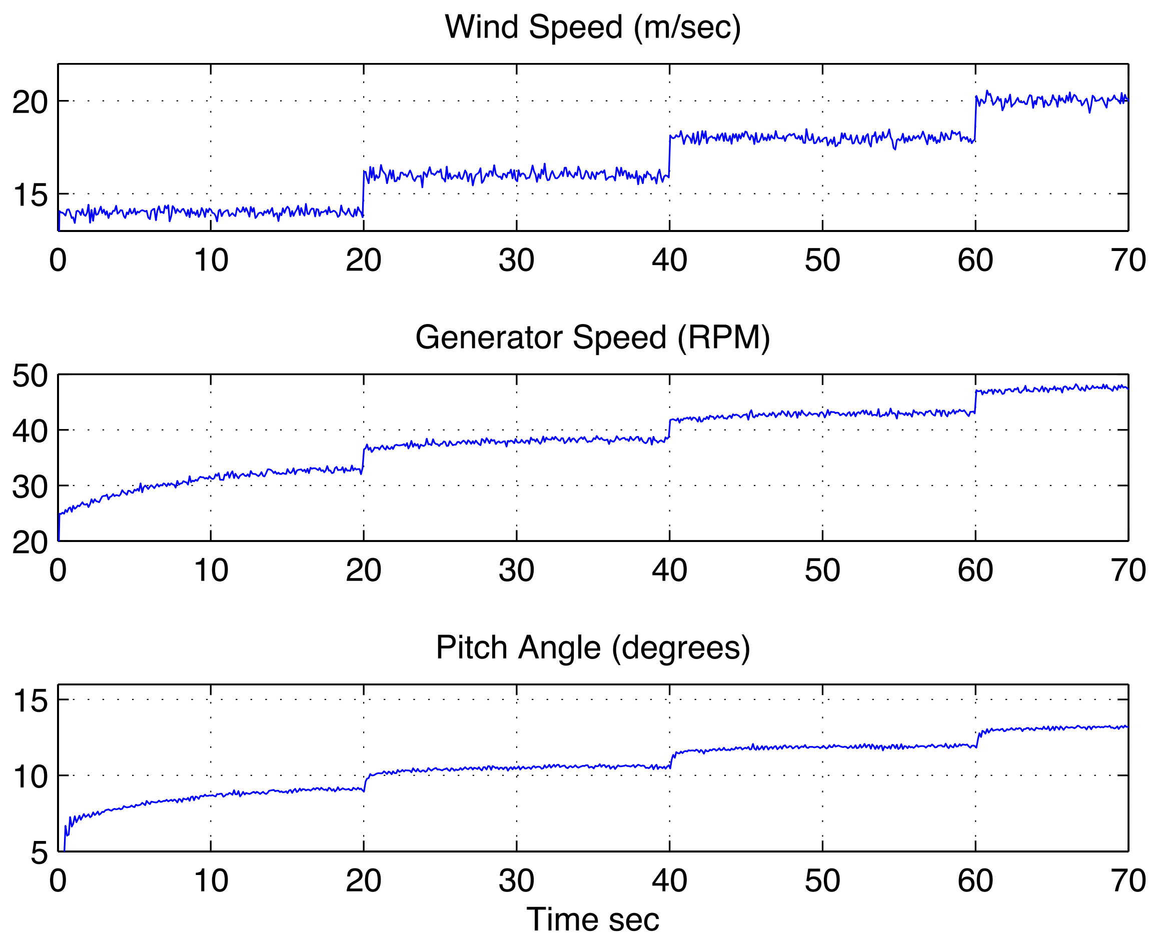
$$u(t) = \begin{bmatrix} w(t) \\ v(t) \end{bmatrix} \text{ where } \begin{cases} w(t) : \text{is turbine system noise} \\ v(t) : \text{is measurement noise} \end{cases}$$

$$A = \begin{bmatrix} -1.4454\text{x}10^{-1} & -3.1078\text{x}10^{-6} & 0.0 \\ 2.6910\text{x}10^{7} & 0.0 & -2.6910\text{x}10^{7} \\ 0.0 & 1.5601\text{x}10^{-5} & 0.0 \end{bmatrix}$$
$$B = \begin{bmatrix} -3.4559 & 0.0 & 0.0 \end{bmatrix}^T$$
$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$
$$G = \begin{bmatrix} 7.8938\text{x}10^{-2} & 0.0 & 0.0 \end{bmatrix}^T$$
$$W = E[w(t)w(\tau)^T] = 0.1 \text{ (Turbine system noise covariance)}$$
$$V = E[v(t)v(\tau)^T] = 0.1 \text{ (Measurement noise covariance)}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1\text{x}10^{-13} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R = 1$$
$$K_k = \begin{bmatrix} 7.6282\text{x}10^{-3} & 1.2663\text{x}10^{2} & 6.2859\text{x}10^{-2} \end{bmatrix}^T$$
$$K_f = \begin{bmatrix} -2.0336 & -2.1225\text{x}10^{-7} & 6.6055\text{x}10^{-1} \end{bmatrix}$$

Results

- Wind turbine’s response to different wind speeds



Operation point

18 m/s

42 RPM

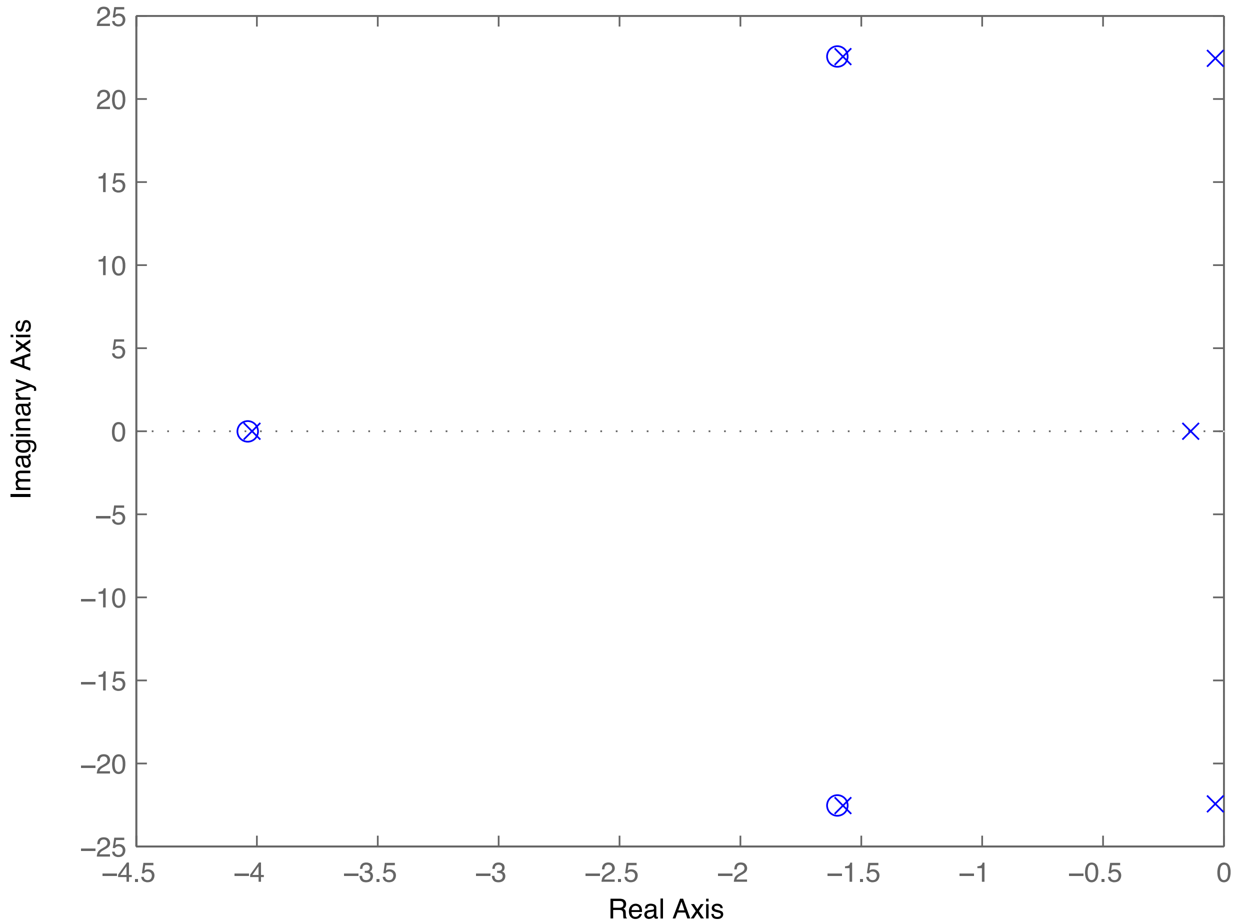
12 degress

Discussion

- The simulated control system is Not Robust
  - robust control system works not only for the linear system which serves as the plant model but it also works for the real physical system with minor performance degradation
- LQR → (Robust)
- Kalman Filter → (Robust)
- LQR + Kalman Filter → (Robustness not guaranteed)
- To recover the robustness of LQR and Kalman Filter:
  - Loop Transfer Recovery (LTR)

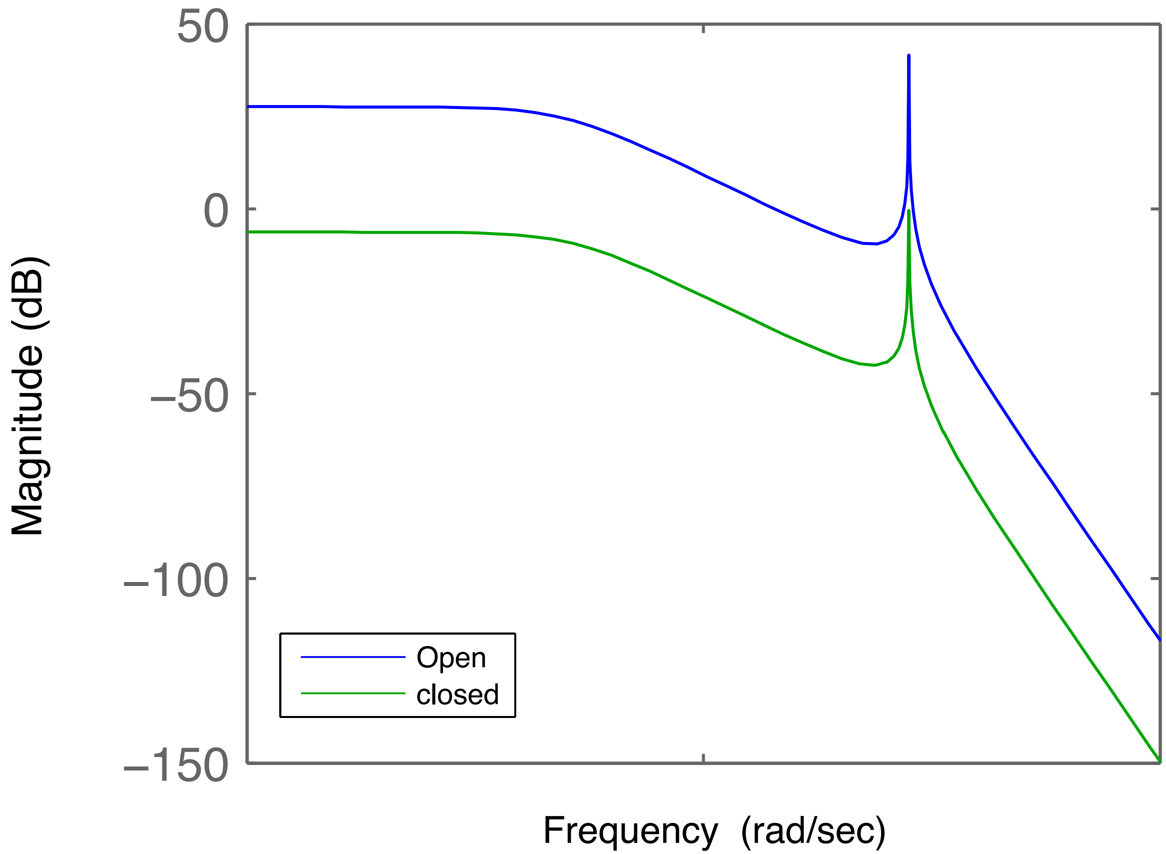
Results

- Poles/Zeros plot of the closed loop system



Results

- Bode plot of the open and closed loop systems



Conclusion

- LQG general formulation, assumptions and design procedure were stated.
- LQG regulator for CART was simulated
- Robustness problem of LQG was shown

Thank You

Contact Information

Abdulrahman Kalbat

ak3369@columbia.edu

akalbat@uaeu.ac.ae