Reliability Modeling of Circuits with Multi-State Aging Gates

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Abstract

As electronic devices are being scaled towards single digit nanometers, the frequency of errors that such nano-circuits will exhibit is expected to increase sharply. This is both a warning sign and a call to arms for including reliability/yield from the very early circuit design phases as a fourth optimization parameter (besides area, power and speed). In this paper we present an approach for reliability modeling of circuits, where every gate is considered as a discrete stochastic model. These models have (a few) discrete states, including an operating and a faulty state. This allows for treating the aging phenomena of circuits more realistically and obtaining more accurate reliability estimates.

1. INTRODUCTION

With the continuous shrinkage of the sizes of electronic devices, there are growing concerns about the yield and reliability of future nano-circuits [1]. That is why CAD tools which will accurately estimate reliability are bound to get center stage in the EDA industry.

As shown in [2], failures that appear in nano-circuits cannot typically be modeled using exponential distributions. In fact, in [3], it was shown that the lognormal distribution is much more adequate for describing device failures—as opposed to the memoryless alternative, i.e., the exponential distribution (which has been in use for quite some time). This paper presents a reliability modeling approach that handles circuits where each of the gates is represented as a small discrete stochastic model. The gates are modeled using various probability distributions that take into account the aging phenomena. The approach combines the proxel-based method (PBM) [4] with a probability algebra-based exhaustive counting method (presented in [5]).

The experiments conducted show that the PBM constrains the size of the models that it can be applied to. However, these experiments also show that PBM allows for:

- a very high degree of flexibility (e.g., in the description of models); as well as
- very high accuracy (comparable to Monte-Carlo simulations).

Consequently, PBM can be seen as bridging the gap between high-precision Monte-Carlo simulations and less accurate numerical methods. The paper will start by presenting the theory, and will go on demonstrating the PBM on a sample model.

The models that we present here are simple (each gate has two states) in order to demonstrate the approach. In general, the number of states is not limited at all, and as discussed in Section 5, the approach can be extended to model even more complex situations.

For modeling purposes we use state-transition diagrams, where transitions are associated with general probability distribution functions.

2. THE PROXEL-BASED METHOD

The PBM is a supplementary variable-based method [6], which deterministically simulates discrete stochastic models. The method works by converting a non-Markovian model into a Markovian one using additional variables that track the aging of events. These are then used for calculating the probabilities with which the events can happen at any point in time.

The PBM was formalized and thoroughly studied in [4]. Its main strength is that it can be applied to models that contain any probability distribution functions (unlike the probabilistic transfer matrix [7] or probabilistic gate models [8], which work with Markovian models only). Therefore it allows for much more realistic modeling of the phenomena (in our case gates). This is also the reason why we cannot provide comparison of the PBM to these alternative approaches.

In [4] there is comparison of PBM to discrete-event simulation, in which PBM shows as more efficient when rare-events are involved. This is valid especially if we want to reproduce the quality of the transient solutions obtained using PBM. The failures in the gates are rare events, and thus our choice to use the PBM for their simulation. Additionally, PBM achieves higher accuracy than discrete-event simulation because it does not rely on generating random numbers.

The proxel-based method is highly flexible and it can easily be applied to analyze models with transient errors, as well as errors that appear due to wear-out (see [7] for more details). PBM has been successfully applied to simulation problems in the automotive industry [9], however, it was never used by the semiconductor industry.

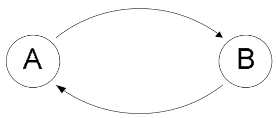

Figure 1. Two-state sample model

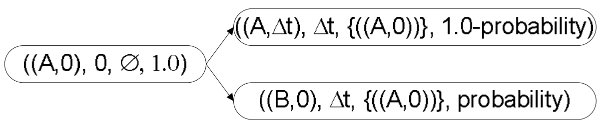

Figure 2. Two-level proxel tree for the sample model from Figure 1

PBM advances by observing all possible developments of a model, probabilistically weighting each of them. At the end, a transient solution is obtained which contains the probability of each state at every point in time. The probabilities are calculated based on the instantaneous rate function (IRF). The IRF denotes the probability that an event will happen within a predetermined elementary time step, given that it has been pending for a certain amount of time (indicated as ‘age intensity’).

Mathematically, the IRF $\mu(t)$ is calculated using the probability density and cumulative distribution functions (correspondingly denoted by $f(t)$ and $F(t)$), as follows:

$$\mu(t) = \frac{f(t)}{1 - F(t)}.$$

In Figure 1 we present a basic two-state model and in Figure 2 the first two levels of its proxel-tree. The latter one describes the advancement of the model in terms of proxels. As mentioned previously, the state vector contains also the ‘age intensity’ (e.g. (A, 0), (A, Δt)). The initial probability is denoted as 1.0 which is further distributed to all possible successor states. The proxel-based method converts the non-Markovian model into a Markovian one by recording the ‘age intensities’ in the state variables. They are used for calculating the probabilities for the corresponding transitions.

The parameters between the state and the probability in the proxel vector are the global simulation time and the set of routes leading to the state presented in the proxel. Each route is defined as a sequence of states.

In the following we present the algorithm of our approach that incorporates the proxel-based method.

3. THE ALGORITHM

The goal of the algorithm that we present here is to calculate the reliability of a give circuit, given the model descriptions of all of the gates. Therefore, the inputs to our algorithm are the state-transition diagrams of all of the gates in the circuit, as well as description of the circuit to be analyzed. Briefly, the algorithm consists of the following steps:

- Compute the probability of having a correct result (in von-Neumann terms [10]) for each combination i, $i = \{1, ..., 2^g\}$ of faulty gates, where g is the number of gates. This results into a set of probability values p_i.

  E.g. If the circuit consists of two gates then the number of faulty gates combinations (i) is 4. We then check the outputs for each faulty combination against those of the fully operational circuit (when all gates are operational). The ratio of matching outputs and the number of outputs yields p_i for the i-th combination. This process is demonstrated in the next section (see also Table 1).

- Use PBM to compute the probability of having each combination of faulty gates at each point in time, given their discrete stochastic models’ descriptions. This results into a set of probability functions f_i, $i = \{1, ..., 2^g\}$.

  Again, in the case of two gates where each gates is described by two discrete states (faulty and operational), this results into a discrete stochastic model of four discrete states (see Fig. 2 and Fig. 3). By knowing the probability distributions that describe failures and recoveries of the gates, the model is simulated using the PBM to obtain a transient solution of the model that consists of the probability functions for all four discrete states..

- Combine the results from the previous two steps to compute the transient reliability of the circuit $r(t)$, as follows:

$$r(t) = \sum_{i=1}^{2^g} f_i(t)\, p_i \,. \quad (1)$$

The result of this algorithm is the probability that the circuit delivers a correct result (in von-Neumann terms), as a function of time. In the following we will demonstrate this algorithm on a sample model.

4. EXPERIMENTS AND RESULTS

4.1. Simple Circuit Model

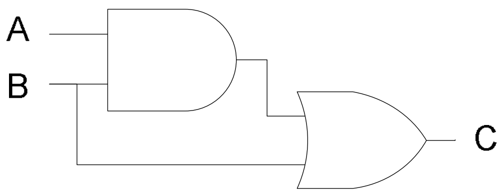

Figure 3. Sample circuit model that we use to demonstrate our approach

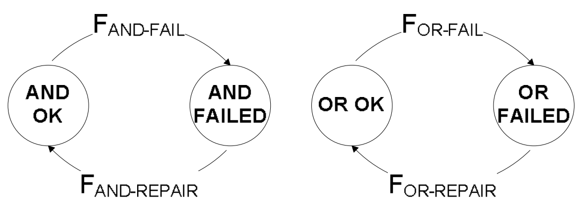

Figure 4. State diagrams of both gates from the circuit in Figure 3

In Figure 3 is shown the model circuit that we use for demonstrating our approach. For simplicity reasons the circuit consists of only two gates, an AND and an OR gate. Additionally, each of the gates is represented by a simple state-transition diagram that represents the different states that each gate can be in. These are also shown in Figure 4.

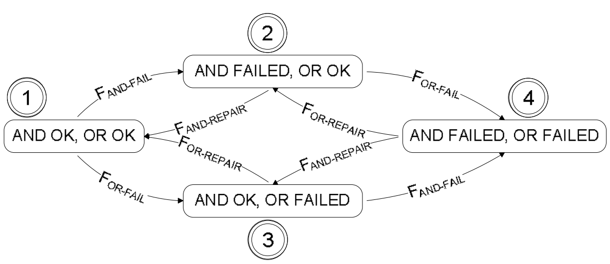

Figure 5. State diagram for the circuit from Figure 3

Based on the gates' models, the circuit can be in only one of four possible discrete states at a time, as follows:

- both gates, AND and OR, are not faulty;
- AND is faulty, and OR is not faulty;
- AND is not faulty, and OR is faulty; and finally
- both AND and OR gates are faulty.

This is represented by the state-transition diagram shown in Figure 5, which also incorporates the transitions among the four states. For brevity we have also numbered the states (from 1 to 4). The diagram consists of 4 discrete states and 8 possible transitions.

To demonstrate the flexibility of our approach, we use 4 different probability distributions for the 8 possible transitions, as follows:

- $F_{AND\text{-}FAIL}$ ~ Lognormal (3, 0.3),
- $F_{AND\text{-}REPAIR}$ ~ Weibull (4, 2),
- $F_{OR\text{-}FAIL}$ ~ Normal (5, 1), and
- $F_{OR\text{-}REPAIR}$ ~ Uniform (5, 7).

We chose 4 different probability distributions to show that the proxel-based method is independent of the types of distribution functions involved in the model. In reality these distribution function would have to be justified by an input modeling process and chosen such that they represent the failures as closely as possible.

Each gate model can contain variable number of states; it is not limited to two. For instance, we might need few intermediate states. However, it is very important that we map each of the gate states to a correct-output probability. We chose only two states for each of the gates for simplicity reasons.

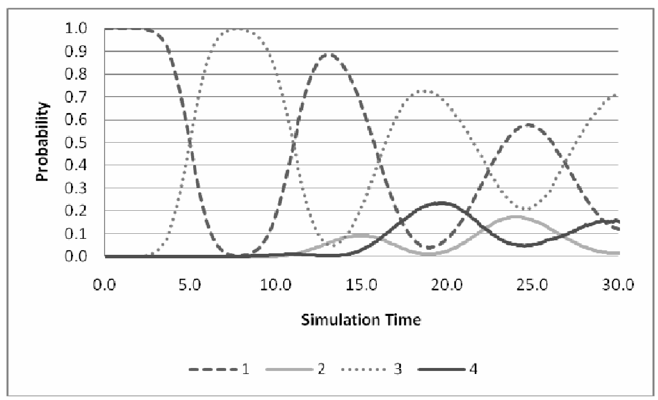

Figure 6. Transient solution of the model from Figure 5

4.2. Demonstration of the Algorithm and Results from Experiments

The probabilities for having a correct output in von-Neumann terms in each of the discrete states of the given circuit (see Fig. 2) have been calculated to be as follows:

$$p_1 = 1.0, p2 = 0.5, p3 = 0.0, p4 = 0.5.$$

In the following we show how we obtained the probability for delivering a correct result of the faulty gate combination 2. Combination 2 describes the circuit state when AND gate has failed and OK gate is operational. In

Table 1 we denote the output of the fully operational circuit (combination 1) and for combination 2. From the table we can observe that in 50% of the cases (i.e. 2 out of 4) the result corresponds to the one when the circuit is fully operational. That is also the value assigned to p_2, i.e. 0.5. In the same way we have calculated the remaining probabilities.

Table 1. Output values for the circuit when in state 1 and 2

A	B	C (state 1)	C (state 2)
0	0	0	1
0	1	1	1
1	0	0	1
1	1	1	1

Next, using the PBM, we obtained a transient solution of the stochastic model presented in Figure 5 that describes the probabilities of the four discrete states (see Figure 6). This combined with the faulty gate combination probabilities yielded the transient reliability, i.e. the reliability of the circuit as a function of time (shown in Figure 7(a)). We used the following formula to calculate the reliability function for this case:

$$r(t) = 1.0\,f_1(t) + 0.5\,f_2(t) + 0.5\,f_4(t),$$

which is the specific form of the Equation (1).

The size of the time step used by PBM was Δt = 0.2, and the run was up to t = 30. We also show a preview of the reliability over a longer period of time (up to t = 300) computed using a larger Δt = 1.0, which shows that the model is slowly approaching a steady state. These simulations took 11.17 and 26.11 seconds correspondingly on a PC with 1 GB RAM running at 2.0 GHz. The fact that we chose a larger Δt to obtain a solution over a longer period of time also shows the flexibility of the PBM. PBM can easily be adapted to provide fast approximate solutions, trading accuracy for speed. Even though the accuracy of the latter solution (displayed in Figure 7(b)) is obviously lower than the ones obtained using a smaller Δt, it still provides a detailed insight into the reliability of the circuit (e.g. the clue that the model goes into a steady state eventually).

5. POTENTIAL OF THE APPROACH

The approach that we present in this paper is demonstrated on a simple model to support the perception of our idea. Nevertheless, we want to emphasize that the models of the gates are not limited in the number of discrete states used for their description. This implies that a gate, besides faulty and operational, can also have a number of intermediate states.

For applying our approach it is only vital to denote which state describes a faulty and which one operational gate. This information can then be translated to obtain the probabilities p_i for delivering a correct result for all of the faulty gate combinations.

In addition, because of the flexibility of the PBM, the approach can be easily extended such as to deal with state-dependent transitions. In practice this would imply e.g. allowing gates' failure probability distributions to depend on the number of times they have failed, as well as on other quantities. This might be a situation worthwhile considering and one that might closer reflect the reality.

With this we would like to highlight the potential that the PBM has in assessing reliability of the future nano-circuits.

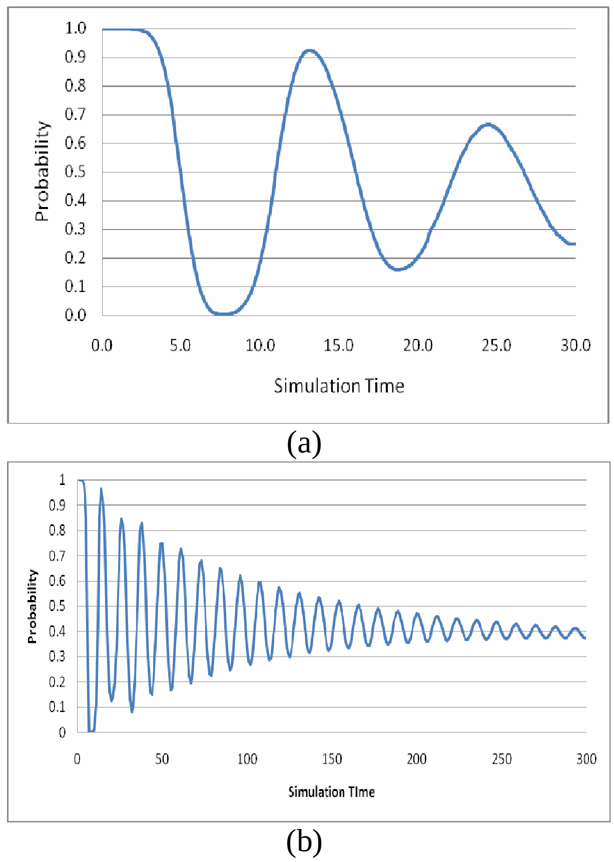

Figure 7. Reliability of the circuit: (a) up to t = 30; and (b) up to t = 300.

5.1. Uncertain States

As we showed, the proxel-based method can easily model reliability of circuits where gates can have a number of states. One interesting situation that arises due to the continuous nature of electricity and device variations is that sometimes states of the gates cannot be solely identified as OK or FAILED. Instead there needs to be a more complex definition such as “under set of conditions A it can be described as 80% OK and 20% FALSE, and under conditions B it can be described as 30% OK and 70% FALSE”. The conditions can be of different nature. They can describe temperature settings, or even lower level quantities. In addition, states can be also described by functions of various relevant parameters, extracted from the device level. This is what we refer to as “uncertain” states.

This might further be interesting as new research results show that analysis on gate level is not sufficient. Circuit models will have to be observed at a lower level, even at level of electrons [11].

This is part of our future interests and we believe that the proxel-based method can be highly suitable for handling situations like this.

6. SUMMARY AND OUTLOOK

In this paper we have presented a new approach for handling non-Markovian gate models in assessing circuits’ reliability. For this purpose, we have used PBM in combination with probability algebra. These have proven to be very suitable for handling models that involve time phenomena. In summary, our approach has the following significant advantages:

- supports general probability distributions, thereby allowing more realistic modeling;
- provides smooth transient solutions with controllable accuracy; and
- allows for modeling the gates as small stochastic models that can have a few states.

As a result, this approach allows for computing reliability functions which provide a detailed insight in the reliability of the circuit with regards to time. Our goal is to provide a tool that would automate this process and based on the description of the circuit and the models of the gates would generate a reliability function.

This paper presents a portion of an initial attempt to apply the proxel-based method to tackle the problem of assessing reliability of the future nano-circuits. Based on our findings, we are confident that our approach can be further developed and integrated into complex EDA (Electronic Design Automation) tools.

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