Application of Daubechies Wavelets for Image Compression
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ABSTRACT
We use the capacity of Daubechies wavelet for image compression. The main subject of wavelet theory is reconstructing an arbitrary function using simple functions in interval such that as we want to reconstruct in scales or vary transforms of interval, we obtain the same primary simple function again. Fourier transforms have many weaknesses; we show how wavelet solves these problems. Wavelets with compact support have many interesting properties and we explain about Daubechies wavelet. Finally, by using MatLab software and wavelet we have examined the results. Keywords: Wavelet Transforms, Image processing, Image compression, Daubechies Wavelet

1 INTRODUCTION
For many decades, scientists wanted more appropriate functions than the sines and cosines which comprise the bases of Fourier analysis, to approximate choppy signals. Sines and cosines are bases of $L^2([0,2\pi])$ while they don’t belong $L^2(\mathbb{R})$. For solving this problem we should find a base that produce $L^2(\mathbb{R})$. We prefer a function that belongs to $L^2(\mathbb{R})$. In Fourier Transforms, building blocks Sines and Cosines are always alternative, so this method might be appropriate for signals filters or signals compressing, they are like a wave and independent of time. The most interesting dissimilarity between Fourier and Wavelet transforms is that individual wavelet functions are localized in space. This localization feature, along with wavelets localization of frequency, makes many functions and operations using wavelets sparse as transformed into the wavelet domain.

This sparseness, in turn, obtains a number of useful applications such as data compression, detecting features in images and removing noise from time series. Someone worked on Wavelets Theory and searched about family of function with compact support or orthogonally. Finally Daubechies could find a family of function with both properties. The most important application of wavelets is processing, transforming information and especially image compressing. Until now, scientists have applied Fourier transforms for these actions but now Wavelet Transforms are in competition with them and even better for some cases.

Wavelets theory is based on Fourier series and Fourier transforms. In early 20th century, scientists realized that Fourier series and Fourier Transforms have some weaknesses in solving mathematical problems. After that researchers worked on different wavelet theories until that becomes a strong field of mathematical analyses. Any discussion of wavelets begins with the first and the simplest; Haar wavelet. Haar wavelet is discontinuous, and resembles a step function.

At first step Daubechies developed the theory and construction of orthonormal wavelets with compact support. Wavelets with compact support have many interesting properties. They can be constructed to have a given number of derivatives and a given number of vanishing moments. To generate an image we need some data. The amount of these data might be big, as well as process and transmit them aren’t possible. To do this we use image compressing that operates based on reduce and omit redundant data. The decompression processes the inverse of the compression process. One can use image compression to reduce the size of an image before you transmit it. The compressed image retains many of the original image’s features but requires less bandwidth.

2 WAVELET TRANSFORMS
Definition1. Wavelet is a function $\Psi$ in $L^2(\mathbb{R})$ if $\Psi_{jk}(x) = (D^j(\tau_k \Psi))(x)$ $\forall j,k \in \mathbb{Z}$ is an orthonormal basis for $L^2(\mathbb{R})$. Such that $(\tau_k f)(x) = f(x-k)$ $\forall k \in \mathbb{Z}$
$D^j f(x) = 2^{j/2} f(2^j x)$ $\forall j \in \mathbb{Z}$
$\tau$ is a Translation operator and $D$ is Dilation operator.

Definition2. A multiresolution analysis (MRA) consists of a sequence of closed subspaces $\{v_j \in \mathbb{Z}\}$ of $L^2(\mathbb{R})$ that satisfy
- $v_j \subseteq v_{j+1}$
- $f(x) \in v_j \Rightarrow f(2x) \in v_{j+1}$
There exists a function \( \phi \in V_0 \) such that \( \{\phi(x-n), n \in \mathbb{Z}\} \) is an orthonormal basis for \( V_0 \). The function \( \phi \) is called a scaling function of the giving MRA.

The main aim to be finding the function \( \Psi \) such that family of \( \Psi_{m,n}, m,n \in \mathbb{Z} \) is an orthonormal basis for \( \mathbb{L}^2(\mathbb{R}) \). We assume \( \{V_i\}_{i \in \mathbb{Z}} \) is an MRA for \( \mathbb{L}^2(\mathbb{R}) \) and \( w_i = \{f \in V_{i+1} \mid f \perp V_i\} \) so we have

\[
v_{i+1} = v_i \oplus w_i
\]

It means that for all of \( f \in V_{i+1} \), we can write uniquely \( f = f_i + g_i \rightarrow f_i \in V_i \), \( f \in V_{i+1} \)

According to \( w_i \)'s we have

\[
w_k \subset V_{i+1} \subset V_i \quad , \quad w_k \perp V_i \quad \forall k < j
\]

Consequently \( w_i \perp w_k \), \( k \neq j \)

\[
v_{i+1} = v_i \oplus w_i
\]

\[
= v_{i-1} \oplus w_{i-1} \oplus w_i
\]

\[
= \ldots \oplus w_{i+1} \oplus v_i \oplus w_i
\]

\[
v_{i+1} = \bigoplus_{k=i}^{i+1} w_k
\]

\[
\bigcup_{i \in \mathbb{Z}} V_i \subset \bigoplus_{i \in \mathbb{Z}} w_i
\]

And we know \( \bigcup_{i \in \mathbb{Z}} V_i = \mathbb{L}^2(\mathbb{R}) \) so we have

\[
\mathbb{L}^2(\mathbb{R}) = \bigoplus_{i \in \mathbb{Z}} w_i
\]

Last equality shows that finding an orthonormal basis for \( \mathbb{L}^2(\mathbb{R}) \) is equivalent with finding function \( \Psi \) such that \( \Psi_{m,n}, m,n \in \mathbb{Z} \) is an orthonormal basis for \( w_i \).

### 3 DAUBECHIES WAVELET

Because of the compact support and orthogonality, Daubechies wavelets can describe the details of the problem conveniently and accurately. The other advantage of compact support is that the Daubechies wavelet-based element has fewer degrees of freedom than others, so Daubechies wavelet-based element has enormous potential in analysis of the problem with local high gradient, the properties of Daubechies wavelet will be presented briefly. We assume that the scaling function \( \phi \) satisfies the dilation equation

\[
\phi(x) = \sqrt{2} \sum_{n \in \mathbb{Z}} c_n \phi(2x-n)
\]

\[
\phi_{0,n} = \sqrt{2} \phi(2x-n)
\]

\[
c_n = \left( \phi, \phi_{0,n} \right), \quad \sum_{n=-\infty}^{\infty} |c_n|^2 \leq \infty
\]

If the scaling function \( \phi \) has compact support, only a finite \( c_n \)'s have nonzero values. The associated generating function \( \tilde{m}(\omega) = \frac{1}{\pi} \sum_{n} c_n e^{-i\omega n} \) is a trigonometric polynomial and it satisfies the orthogonality condition

\[
|\tilde{m}(\omega)|^2 + |\tilde{m}(\omega + \pi)|^2 = 1
\]

\[
\tilde{m}(0) = 1, \tilde{m}(\pi) = 0
\]

If coefficients \( c_n \)'s are real, then the corresponding scaling function as well as the mother wavelet \( \Psi \) will also be real-valued.

\[
\Psi(x) = \sqrt{2} \sum_{n=-\infty}^{\infty} d_n \phi(2x-n)
\]

where the coefficients \( d_n \) are given by

\[
d_n = (-1)^{n-1} c_{-n-1}
\]

### 4 IMAGE COMPRESSING

An image identified by an integer matrix with domain \([0, 255]\), 0 shows absolute black and 255 shows absolute white. Image compression can be performed in the original spatial domain or in a transform domain. In the latter case, first, the image is transformed, and a subsequent compression operation is applied in the transform domain. The usage of the wavelet transform for image compression has drawn significant attention.

Figure 1 show compression and decompression process. The primary advantage of the wavelet transform compared with the cosine transform is that the wavelet transform is localized in both spatial and frequency domains. Therefore, the transformation of a given signal will contain both spatial and frequency information of that signal. In other words, the cosine transformations extend indefinitely in all areas of spatial frequency information that is scattered. This property of image compression by wavelet shows promising results in medicine.

The selection of wavelet filters is a crucial step which indicates the performance of image compression. A good filter bank should be reasonably fast, providing a transform where most of the energy is packed in a small number of coefficients, and should not introduce distortions. The second step of compression is quantization. The purpose of quantization is to map a large number of input values into a smaller set of output values by reducing the precision of the data. In this step the information may be lost. In third step, the quantized coefficients are subjected to run-length coding followed by Huffman coding. Run-length coding examines consecutive quantized coefficients and determines the sequences made of coefficients that have the same value. Each sequence is represented by the common value and the number of coefficients in the run, results in considerably more compact information compared to the entire sequence of coefficients.

There are many methods for image compression. More sophisticated methods are available which combine wavelet decomposition and quantization. This is the
basic principle of progressive methods. On one hand, during decoding progressivity makes it possible to obtain an image whose resolution increases gradually. In addition, it is possible to obtain a set of compression ratios based on the length of the preserved code. This compression usually involves a loss of information, but this kind of algorithm enables also lossless compression. Such methods are based on three ideas.

Ideas include using wavelet decomposition to ensure sparsely (a large number of zero coefficients), classical encoding methods and decisive for the use of wavelets in image compression, is that make the tree structure of the wavelet decomposition fundamentally. The EZW coding algorithm introduced by Shapiro. EZW combines stepwise thresholding and progressive quantization, focus on more efficient way to encode the image coefficients, in order to minimize the compression ratio.

5 IMAGE COMPRESSION BY USING MATLAB, HAAR WAVELET AND DAUBECHIES WAVELET

We use Haar wavelet, Daubechies wavelet and MatLab, for compression arbitrary image. We use

![Compression process](image)

![Decompression process](image)

**Figure 1.** (a) Compression process, (b) Decompression process

Progressive methods of compression, starting with EZW algorithm and Haar wavelet then tried to improve the results by using the Daubechies wavelet.

A measure of achieved compression is given by the compression ratio (CR) and the Bit-Per-Pixel (BPP) ratio. CR and BPP represent equivalent information. CR indicates that the compressed image is stored using CR% of the initial storage size while BPP is the number of bits used to store one pixel of the image. For a grayscale image the initial BPP is 8. For a true color image the initial BPP is 24, because 8 bits are used to encode each of the three colors (RGB color space).

The challenge of compression methods is to find the best compromise between a low compression ratio and a good perceptual result. The key parameter is the number of loops. By increasing it, we will have better recovery but a worse compression ratio.

We implement the Haar and the Daubechies wavelet compression and reconstruction. For comparing the visual effectiveness of image compression, this implementation is applied for image compressing on 4 test images. You can see figure 2-figure 5. At first, we used 6 steps only, which produce a very coarse decompressed image. After that, we examined a slightly better result by 9 steps and finally a satisfactory result using 12 steps. At the end, we compare the results by Haar and Daubechies wavelet.

Daubechies wavelet gives better results in all cases images. It has better recovery and compression ratio. It shows the percentage of missing of image.

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Fourier transforms are based on sines and cosines and they are appropriate for smooth cases but most of the cases aren’t smooth. One of the most important applications of wavelets is image compression. The first and the simplest wavelet is Haar that appropriate for image processing and image compressing on images that have a lot of angular points. Most of the images aren’t smooth so this kind of wavelet isn’t useful. We compress image with Daubechies wavelet and understand that this wavelet is better than previous. In the family of Daubechies wavelet for image compressing, the key parameter is the number of loops; increasing it, leads to better recovery, but to a worse compression ratio so we suggest using appropriate number of loops to get a better result.

REFERENCES


Figure 2. Mask Image

At first, we used 6 steps only, images aren’t clear for both. After that, we examined by 9 steps. Compression ratio with Haar wavelet is 0.48% but with Daubechies wavelet is 0.37%. Finally we gain a satisfactory result using 12 steps with Daubechies wavelet because it has better recovery and compression ratio is 2.42% but Compression ratio with Haar wavelet is 3.31%.
Compression ratio by 6 steps for both is equal. By 12 steps, compression ratio with Haar wavelet is 19.58% but it is 15.38% for Daubechies wavelet.
Compression ratio by 6 steps for both is nearly equal. By 12 steps, compression ratio with Haar wavelet is 4.66% but it is 3.23% for Daubechies wavelet.
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Compression ratio by 6 steps for both is equal. By 12 steps, compression ratio with Haar wavelet is 2.47% but it is 2.13% for Daubechies wavelet.