Number Theory: Review Problems 1

Exercise 1. Use Mathematical Induction to prove that for all $n \geq 1$, we have: $8|5^{2n}+7$.

Exercise 2. Prove that for any integer a, one of the integers a, $a+1$, $a+2$ is divisible par 3.

Exercise 3. (i) Prove that for any positive integer a and any integer n, we have $gcd(a,n) = gcd(a,a+n)$.

(ii) Determine $gcd(a,a+1)$.

Exercise 4. Prove that if a and b are both odd integers, then $16|a^4+b^4-2$.

Exercise 5. Prove that :

(i) If a is an odd integer then $24|a(a^2-1)$.

(ii) If $gcd(a,b)=1$ and $gcd(a,c)=1$, then $gcd(a,bc)=1$.

(iii) If $gcd(a,b)=1$ then $gcd(ac,b)=gcd(c,b)=1$.

(vi) If $gcd(a,b)=1$ then $gcd(a^2,b^2)=1$.

Exercise 6. (i) Use the Euclidean Algorithm to obtain integers x and y such that: $gcd(120,438) = 120x+438y$.

(ii) Find $lcm(120,438)$.