United Arab Emirates University

Faculty of Science

Department of Mathematical Sciences

MATH 260: FOUNDATIONS OF GEOMETRY MIDTERM EXAM FALL 2011

Date and time : 02/11/2011, 7-8pm

Student Name: ....

Student ID: ....

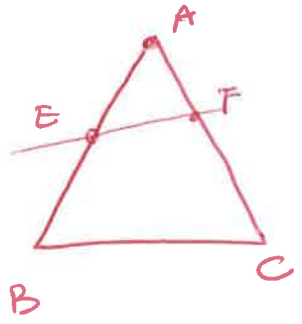
Section: ....

Grade: ..../20

3/ **Exercise 1.** Let ΔABC be a triangle such that $AB = 7$, $AC = 5$, $BC = 8$. Let E a point in segment AB and F a point in segment AC such that $(EF)//(BC)$ and $AE = 3$. Determine AF and EF.

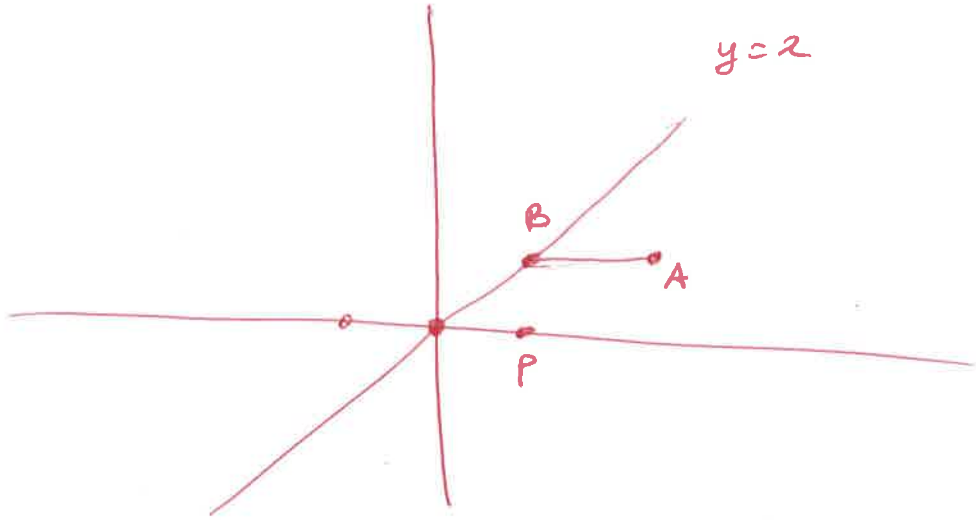
$$\frac{AE}{AB} = \frac{AF}{AC} \Rightarrow AF = \frac{3}{7} \times 5 = \frac{15}{7}$$

$$\frac{EF}{BC} = \frac{AE}{AB} \Rightarrow EF = BC \cdot \frac{AE}{AB} = 8 \cdot \frac{3}{7} = \frac{24}{7}$$



4/ **Exercise 2.** Let $A = (3,1)$, $B = (1,1)$, $O = (0,0)$, $P = (1,0)$ and m the line $y = x$. Find: $t_{AB}(P)$, $\rho_m(P)$, $\rho_m(B)$ and $R_{O,\pi/6}(P)$.

Answer: $t_{AB}(P) = (-1,0)$	$\rho_m(P) = (0,1)$	$\rho_m(B) = B$	$R_{O,\pi/6}(P) = (\frac{\sqrt{3}}{2}, \frac{1}{2})$



3/3

Exercise 3. Let $A = (0,0)$ and $B = (-3,0)$.

(a) Determine the composition $R_{B,\pi} \circ R_{A,\pi/2}$.

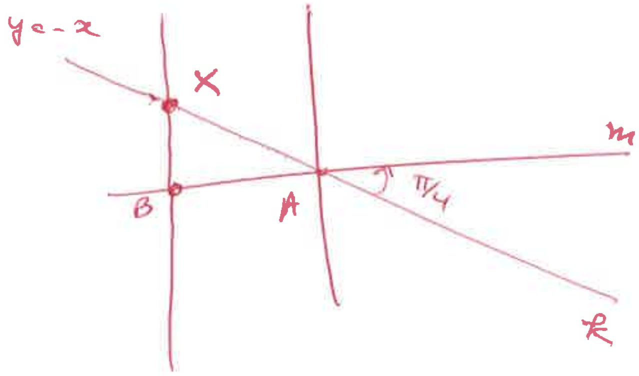
$m = (AB)$;

$R_{A,\frac{\pi}{2}} = \ell_m \circ \ell_k$

k: through A, angle $\frac{\pi}{4}$ from $k \to m$

$R_{B,\pi} = \ell_n \circ \ell_m$

m through B; angle $\frac{\pi}{2}$

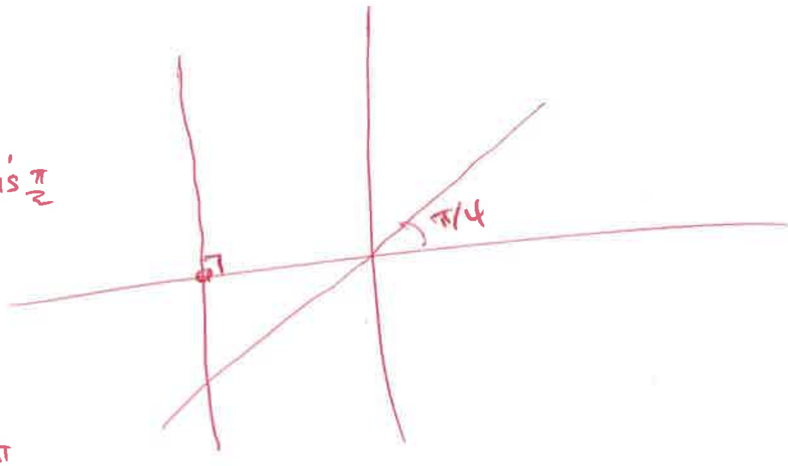


$\boxed{R_{X,\frac{3\pi}{2}} \quad \text{where} \quad X = (-3,3)}$

(b) Determine the composition $R_{A,\pi/2} \circ R_{B,\pi}$.

$R_{B,\pi} = \ell_m \circ \ell_k$

k: through B angle $k \to m$ is $\frac{\pi}{2}$



$R_{A,\frac{\pi}{2}} = \ell_n \circ \ell_m$

n: angle fr m to n is $\frac{\pi}{4}$

$X' = (-3,-3)$

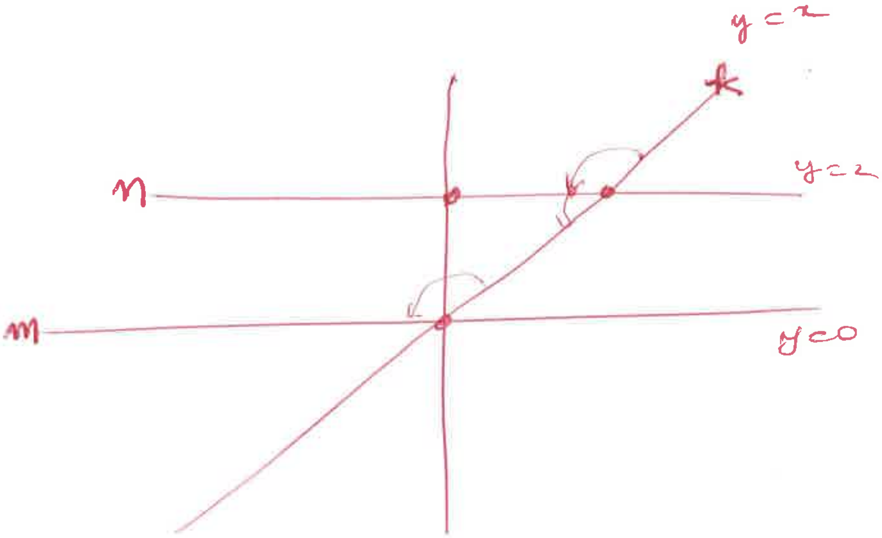
$\boxed{R_{X',\frac{3\pi}{2}}}$

$X' = (-3,-3)$

4.5 **Exercise 4.** Let m be the line of equation $y = 0$, n be the line of equation $y = 2$ and k the line of equation $y = x$.

(a) Determine $\rho_m \circ \rho_k$.

$= R_{O, \frac{3\pi}{2}}$.



(b) Determine $\rho_m \circ \rho_n$.

$t_{2\vec{AB}}$; $\vec{AB} = (0, -2)$

$= t_{\vec{u}}$; $\vec{u} = (0, -4)$

(c) Determine $\rho_n \circ \rho_k$.

$R_{A, \frac{3\pi}{2}}$, $A = (2, 2)$

3.5 **Exercise 5.** For each of the following statements determine whether it is true or false.

(1) The 3 medians of a triangle meet at one point. True ☑ False ☐

(2) If f is a rigid motion such that, $f(0,0) = (0,0)$, $f(1,0) = (1,0)$, and $f(3,0) = (3,0)$, then f is the identity. True ☑ False ☐

(3) If $\rho_m(P) = P'$, then m is the perpendicular bisector of segment PP' True ☑ False ☐

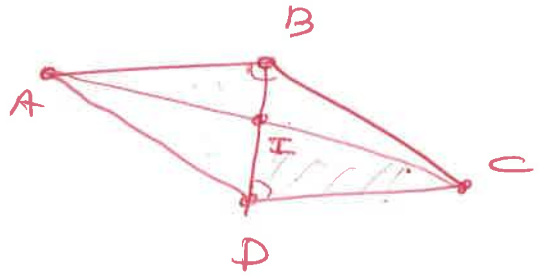
(4) The composition of two rotations $R_{A,\alpha} \circ R_{B,\beta}$ is always a rotation. True ☐ False ☑

(5) For any $n \geq 2$, we have $\rho_m^n = Id$. True ☐ False ☑

`Justify you answer for statement (1).`

Ceva's theorem

2 **Exercise 6.** Let $ABCD$ be a parallelogram. Let I be the intersection of the diagonals AC and BD. Prove that $IA = IC$.



$\triangle ABI \cong \triangle CDI$

~~$\hat{I} \cong \hat{I}$~~

~~$CD \cong AB$~~

$\hat{I} = \hat{I}$, $\hat{C} = \hat{A}$) $\rightarrow$ $\hat{D} = \hat{B}$

$\hat{D} = \hat{B}$, $\hat{C} = \hat{A}$, $CD = AB$.) by ASA.

$ID = IB$ ✓ and $IA = IC$.