United Arab Emirates University

Faculty of Science

Department of Mathematical Sciences

FALL 2022 MATH 2220 TEST 1

Student Name: ....

Student ID: ....

Section: ....

Grade: ..../15

Question 1. [**6 marks**] **(a)** Use the Gauss-Jordan Elimination Method to solve the system

of linear equations:
$$\begin{cases} x_1 - x_2 + x_3 - x_4 &= 0 \\ x_1 + x_3 + x_4 &= 2 \\ 2x_1 + 2x_2 + 2x_4 &= 2 \end{cases}$$

(b) Determine for which values of k is the following system consistent

$$\begin{cases} x_1 + x_2 + x_3 & = & k \\ 2x_1 + 3x_2 + 2x_3 & = & 2k \\ 3x_1 + 4x_2 + 3x_3 & = & k^2 \end{cases}$$

Question 2. [5 mark] Given $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 0 \\ 2 & 0 & 3 \end{bmatrix}$.

(a) Use Gauss-Jordan Method to find A^{-1}.

(b) Write A^{-1} as a product of elementary matrices.

Question 3. [**4 marks**] For each of the questions below, encircle the correct answer.

1. Given $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 3 & 4 \\ 5 & 1 & 2 \end{bmatrix}$, then $tr(3A - B^T - I_3) =$

   (a) 0   (b) 2   (c) 5   (d) 7   (e) None of the above

2. Let A be 3×4 matrix, B be a 3×5 matrix and C a 3×4 matrix, then $AC^T B + 2BB^T$ is

   (a) a 3×5 matrix.

   (b) a 3×4 matrix.

   (c) a 3×3 matrix.

   (d) a 4×3 matrix.

   (e) Not defined.

3. Determine which of the following matrices is in Reduced Row Echelon Form:

   $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   (a) A, B and C.

   (b) A and C.

   (c) A, C and D.

   (d) A, B, C and D.

   (e) C and D.

4. Given A and B two invertible $n \times n$ matrices, then $A(AB^{-1}A^{-1})^{-1}AB^{-1} =$

   (a) I_n (b) A   (c) B   (d) A^2 (e) None of the above