

MATH 2220, Linear Algebra for Eng., Fall 2022

Test 3

Date: 23/11/2022, Time: 45mn

Student Name: _____

Student ID: _____

Attendance Number: _____

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|-------------------------|-------------------|-------------------|-------|
| Question | 1-2 | 3-4 | Grade |
| Course Learning Outcome | 3 | 1 | |
| Marks | $\frac{\dots}{9}$ | $\frac{\dots}{6}$ | |

- Show all the steps of your solution for each question.
- Use only Blue or Black pen, neither pencil nor colored.
- Graphics and Programming calculators are not allowed.

Question 1. [4 marks] Given the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$.

(a) Determine the eigenvalues of A .

(b) Determine a basis and the dimension of the eigenspace corresponding to the eigenvalue $\lambda = 2$.

(c) Determine whether A is diagonalizable. Justify your answer.

Question 2. [5 marks] Given the matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$.

(a) Find a matrix P that diagonalizes A .

(b) Use your answer in question (a) to compute A^{10} .

Question 3. [2 marks] Given $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 2 & 2 & 1 & 3 \\ 3 & 3 & 2 & 5 \end{bmatrix}$.

(a) Determine a basis for the row space of A .

(b) Find the rank of A .

Question 4. [4 marks] For each of the questions below, encircle the correct answer.

1. Given $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Then $v = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector of A corresponding to the eigenvalue:

- (A) 2 (B) 3 (C) -1 (D) 1 (E) None of the above

2. Let A and B be two similar matrices. Which of the following is not correct?

- (A) $\text{Nullity}(A) = \text{Nullity}(B)$.
 (B) $\text{Rank}(A) = \text{Rank}(B)$.
 (C) $\det(A) = \det(B)$.
 (D) A and B have the same eigenspaces.
 (E) A and B have the same characteristic polynomial.

3. Let A be a (4×7) -matrix whose rank is 3, then the nullity of A^T is:

- (A) 0 (B) 1 (C) 2 (D) 3 (E) None of the above

4. Given $A = \begin{bmatrix} 1 & 0 & 1 & 2 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 2 & 1 & 4 & 5 \end{bmatrix}$. Then $\text{nullity}(A)$ is

- (A) 0 (B) 1 (C) 2 (D) 3 (E) None of the above