**United Arab Emirates University**

**Faculty of Science**

**Department of Mathematical Sciences**

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# FALL 2022 MATH 2220 TEST 2

**Student Name**: .....

**Student ID**: .....

**Section**: .....

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Grade: ...../15

**Question 1.** **[3 marks]** Determine whether the 3 vectors $v_1 = (1, 1, 0, 0)$, $v_2 = (2, 1, 0, 1)$ and $v_3 = (0, 1, 2, 1)$ are linearly independent. Show all the details of your work.

**Question 2.** **[3 marks]** Determine whether the vector $w = (1, 2, 0)$ lies in the subspace of $\mathbb{R}^3$ spanned by the vectors $v_1 = (1, 2, 3)$, $v_2 = (0, 2, 2)$ and $v_3 = (2, 3, 5)$.

**Question 3.** **[3 marks]** Determine whether the 3 vectors $v_1 = (1, 1, 3)$, $v_2 = (0, 1, 2)$ and $v_3 = (1, 0, 1)$ form a basis of $\mathbb{R}^3$.

**Question 4.** **[3 marks]** Given $p_1(x) = x$, $p_2(x) = 2 + x$ and $p_3(x) = 1 + x + x^2$. Compute the Wronskian of $p_1, p_2, p_3$ and show that the 3 polynomials are linearly independent.

**Question 5. [3 mark]** Given $A = \begin{bmatrix} 0 & 0 & 3 \\ 2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

Use the determinant formula to find the inverse of $A$, if it exists. Show all the details of your work.

**Question 6.** [**5 marks**] For each of the questions below, encircle the correct answer.

**1.** Which of the following is a subspace of $\mathbb{R}^3$?

**(A)** $\{(x, y, x^2); x, y \text{ real numbers }\}$

**(B)** $\{(x, 0, 2); x \text{ real number }\}$

**(C)** $\{(x, x+1, x-1); x \text{ real number }\}$

**(D)** $\{(x, y, y); x, y \text{ real number }\}$

**2.** For which values of $a$ is the matrix $A = \begin{bmatrix} a & 1 & 1 \\ 0 & 3 & 0 \\ a & 1 & a \end{bmatrix}$ non-invertible?

**(A)** 0 and 2 **(B)** 0 and 1 **(C)**2 **(D)** $-1$ **(E)** None of the above

**3.** Given $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$, then $\begin{vmatrix} a & d & g \\ b+a & e+d & h+g \\ 2c & 2f & 2i \end{vmatrix} =$

**(A)** 0 **(B)** 4 **(C)** 8 **(D)** $-8$ **(E)** None of the above

**4.** Let $A, B$ and $C$, $(3 \times 3)$-matrices such that $|A| = 2, |B| = 2$ and $|C| = 4$ then $|2ABA^tC^{-1}| =$

**(A)** 2 **(B)** 4 **(C)** 8 **(D)** 16 **(E)** None of the above

**5.** Let $W$ be the subspace of $\mathbb{R}^3$ defined by $x - 2y - z = 0$. Which of the following is a basis for $W$?

**(A)** $(1, 0, 1)$ and $(0, 2, 1)$

**(B)** $(1, 1, -1)$

**(C)** $(1, 0, 1)$ and $(0, 1, -2)$

**(D)** $(0, -1, 2)$

**(E)** None of the above