UAEU College of Science كلية العلوم

Department of Mathematical Sciences

جامعة الإمارات العربيــة المتحدة
United Arab Emirates University

قسم علــوم الرياضيـــــات

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Math 105, Calculus I, Spring 2015

Test 2

Date: 22/4/2015, Time: 45mn

Student name: ______________________________

Student ID: Solution- Key

Student Major: ______________________________

Question	1 and 2	3	4	
Student Learning Outcome	2	3	5	Grade: ....../15
Marks	....../8	....../4	....../3	

IMPORTANT: Show all the steps of your solution for each question.

Question 1. [4 marks] (a) Find the derivative of the function $f(x) = (\sqrt{4x^2+1})tan^{-1}(2x)$.

$$f'(x) = \frac{d}{dx}\left(\sqrt{4x^2+1}\right)\tan^{-1}(2x) + \sqrt{4x^2+1}\cdot\frac{d}{dx}\tan^{-1}(2x) \quad \text{(Product Rule)}$$

$$\frac{d}{dx}\left(\sqrt{4x^2+1}\right) = 8x\cdot\frac{1}{2}(4x^2+1)^{-\frac{1}{2}} = 4x(4x^2+1)^{-\frac{1}{2}} \quad \text{(Chain Rule)}$$

$$\frac{d}{dx}\left(\tan^{-1}(2x)\right) = 2\cdot\frac{1}{1+(2x)^2} = \frac{2}{1+4x^2} \quad \text{(Chain Rule)}$$

so; $$f'(x) = \frac{4x}{\sqrt{4x^2+1}}\cdot\tan^{-1}(2x) + \sqrt{4x^2+1}\cdot\frac{2}{1+4x^2}$$

$$= \frac{4x}{\sqrt{4x^2+1}}\tan^{-1}(2x) + \frac{2}{\sqrt{4x^2+1}}$$

(b) Let $g(x) = ln(e^x + e^{-x})$. Find $g'(x)$ and $g''(x)$.

$$g'(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad \text{(Chain Rule)}$$

$$g''(x) = \frac{(e^x+e^{-x})(e^x+e^{-x}) - (e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2} \quad ; \text{(Quotient Rule)}$$

$$= \frac{e^{2x} + e^{-2x} + 2 - (e^{2x} + e^{-2x} - 2)}{(e^x+e^{-x})^2} = \frac{4}{(e^x+e^{-x})^2}$$

Question 2. **[4 marks] (a)** Use implicit differentiation to find an equation of the tangent line to the curve: $x^2 + y^2 + 2y + 1 = 10$ at the point $(1, 2)$.

$x^2 + y^2 + 2y + 1 = 10 \Rightarrow \frac{d}{dx}\left[x^2 + y^2 + 2y + 1\right] = \frac{d}{dx}[10]$

$\Rightarrow 2x + 2y'y + 2y' = 0$

$\Rightarrow 2x + y'(2y+2) = 0 \Rightarrow y'(2y+2) = -2x \Rightarrow \boxed{y' = \frac{-2x}{2y+2}}$

At the point $(1,2)$; $x=1$ and $y=2$

Then $y' = \frac{-2}{4+2} = -\frac{1}{3}$

The tangent line at $(1,2)$ has Equation

$y - 2 = -\frac{1}{3}(x-1) \Rightarrow \boxed{y = -\frac{1}{3}x + \frac{7}{3}}$

(b) At which points is the tangent line to the curve $x^2 + y^2 + 2y + 1 = 10$ horizontal?

The tangent is Horizontal if $y' = 0$. Let's first find y' by implicit differentiation. From the previous question

~~$\frac{d}{dx}[x^2+y^2+2y+$~~ $y' = \frac{-2x}{2y+2}$

Then, $y' = 0 \iff x = 0$.

if $x = 0$, then: $0^2 + y^2 + 2y + 1 = 10 \Rightarrow y^2 + 2y - 9 = 0$

Thus, $y = \frac{-2+\sqrt{40}}{2} \simeq 2.16$ or $y = \frac{-2-\sqrt{40}}{2} \simeq -4.16$

So the tangent is Horizontal at: $\left(0, \frac{-2+\sqrt{40}}{2}\right)$ and $\left(0, \frac{-2-\sqrt{40}}{2}\right)$

Question 3. [4 marks]

(a) Find the critical points of the function $f(x) = x^3 - 6x^2 + 9x + 1$.

$f'(x) = 3x^2 - 12x + 9$

$f'(x) = 0 \iff 3x^2 - 12x + 9 = 0 \iff x = 1$ or $x = 3$

The critical point are $(1, 5)$ and $(3, 1)$

(here $f(1) = 1^3 - 6 \times 1^2 + 9 \times 1 + 1 = 5$
$f(3) = 3^3 - 6 \times 3^2 + 9 \times 3 + 1 = 1$)

(b) Find the absolute maximum of the function $f(x) = x^3 - 6x^2 + 9x + 1$ in the closed interval $[0, 4]$.

$f(1) = 5$

$f(3) = 1$

$f(0) = 1$

$f(4) = 4^3 - 6 \times 4^2 + 9 \times 4 + 1 = 64 - 96 + 36 + 1 = 5$

then
$f(1) = f(4) = 5$ is an absolute maximum in the closed interval $[0, 4]$

Question 4. [**3 marks**] Find the linear approximation of the function $f(x) = (x+1)^{\frac{4}{3}}$ at $a = 0$, then use it to approximate the number $(1.1)^{\frac{4}{3}}$.

$$f'(x) = \frac{4}{3}(x+1)^{4/3-1} = \frac{4}{3}(x+1)^{\frac{1}{3}}$$

At $a = 0$; $$L(x) = f'(0)(x-0) + f(0)$$

$$= \frac{4}{3}x + 1.$$

L $\cancel{f(0)\approx}$ $(1.1)^{4/3} = f(0.1)$

$$\approx L(0.1)$$

$$\approx \frac{4}{3} \cdot 0.1 + 1$$

$$\approx 1.133$$