UAEU College of Science كلية العلوم

Department of Mathematical Sciences

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قسم علـوم الرياضيـــــات

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Math 105, Calculus I, Spring 2015

Test 1

Date: 4/3/2015, Time: 45mn

Student name: Solution Key

Student ID: ______________________

Student Major: ______________________

Question	1,2 and 3	4,5 and 6	Grade: ....../15
Student Learning Outcome	2	1	
Marks	....../7.5	....../7.5	

IMPORTANT: Show all the steps of your solution for each question.

Question 1. [4 marks] Find the horizontal and vertical asymptotes of $f(x) = \dfrac{2x^2+x+1}{x^2-1}$.

Vertical asymptotes: The line $x=1$ and The line $x=-1$
Horizontal asymptotes: The line $y=2$

Vertical asymptotes

$x^2-1=0 \iff x=1$ or $x=-1$

Let's check the Limits at 1 and -1

- $\lim\limits_{x\mapsto 1^+} \dfrac{2x^2+x+1}{x^2-1} = \dfrac{4}{0^+} = \infty$
- $\lim\limits_{x\mapsto 1^-} \dfrac{2x^2+x+1}{x^2-1} = \dfrac{4}{0^-} = -\infty$

Then the line $x=1$ is a ~~Hori~~ Vertical asymptote

$\lim\limits_{x\mapsto -1^+} \dfrac{2x^2+x+1}{x^2-1} = \dfrac{4}{0^-} = -\infty$

$\lim\limits_{x\mapsto -1^-} \dfrac{2x^2+x+1}{x^2-1} = \dfrac{4}{0^+} = +\infty$

Then the line $x=-1$ is a vertical asymptote

Horizontal asymptotes

- $\lim\limits_{x\mapsto \infty} \dfrac{2x^2+x+1}{x^2-1} = \lim\limits_{x\mapsto\infty} \dfrac{2x^2}{x^2} = 2 \Rightarrow$ the line $y=2$ is a Horizontal asymptote
- $\lim\limits_{x\mapsto -\infty} \dfrac{2x^2+x+1}{x^2-1} = \lim\limits_{x\mapsto\infty} \dfrac{2x^2}{x^2} = 2 \Rightarrow$ " " "

Question 2. [1.5 marks] Use the Squeeze Theorem to find $\lim_{x\to 0^+} \sqrt{x}\sin(\frac{1}{x^2})$.

Answer: $\lim_{x\to 0^+} \sqrt{x}\sin(\frac{1}{x^2}) = 0$

For all $x \neq 0$, we have $-1 \leq \sin\left(\frac{1}{x^2}\right) \leq 1$

Then $\sqrt{x} \times (-1) \leq \sqrt{x}\sin\left(\frac{1}{x^2}\right) \leq \sqrt{x} \times (1)$

$\Rightarrow -\sqrt{x} \leq \sqrt{x}\sin\left(\frac{1}{x^2}\right) \leq \sqrt{x}$

Since $\lim_{x\to 0^+}(-\sqrt{x}) = 0$ and $\lim_{x\to 0^+}\sqrt{x} = 0$

Then By The Squeeze Theorem, we get:

$$\lim_{x\to 0^+}\sqrt{x}\sin\left(\frac{1}{x^2}\right) = 0$$

Question 3. [2 marks] Evaluate the following limit: $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$.

Answer: $\frac{1}{4}$

$$\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x} = \lim_{x\to 0}\frac{(\sqrt{x+4}-2)}{x}\cdot\frac{(\sqrt{x+4}+2)}{(\sqrt{x+4}+2)}$$

$$= \lim_{x\to 0}\frac{(\sqrt{x+4})^2-4}{x(\sqrt{x+4}+2)} = \lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}(\sqrt{x+4}+2)}$$

$$= \lim_{x\to 0}\frac{1}{\sqrt{x+4}+2} = \frac{1}{\sqrt{0+4}+2} = \frac{1}{4}$$

Question 4. [4 marks] (a) Find the derivative of $f(x) = x\cos x + 3e^{x^2}$.

Answer: $\cos x - x\sin x + 6x\,e^{x^2}$

$\frac{d}{dx}(x\cos x) = 1.\cos x + x.(-\sin x) = \cos x - x\sin x$

$\frac{d}{dx}(3e^{x^2}) = 3.\frac{d}{dx}(e^{x^2}) = 3.2xe^{x^2} = 6xe^{x^2}$

Then $f'(x) = \cos x - x\sin x + 6xe^{x^2}$

(b) Write an equation of the tangent line to the curve $y = x\cos x + 3e^{x^2}$ at the point $(0, 3)$.

Equation of the tangent line: $y = x + 3$

slope of tangent line at $(0,3)$ is $f'(0)$

$f'(0) = \cos(0) - 0.\sin(0) + 6.0.e^0$

$= 1$.

$y - 3 = 1.(x - 0) \Rightarrow y = x + 3$

Question 5. [2 marks] For which values of the constant c is the function h continuous everywhere:

$$g(x) = \begin{cases} cx^2 + 2 \text{ if } x < 1 \\ 2x + 4 \text{ if } x \geq 1 \end{cases}$$

Answer: .... $c = 4$ ....

$$\lim_{x \mapsto 1^-} g(x) = \lim_{x \mapsto 1^-} cx^2 + 2 = c \cdot 1^2 + 2 = c + 2$$

$$\lim_{x \mapsto 1^+} g(x) = \lim_{x \mapsto 1^+} 2x + 4 = 6$$

We should have, $\lim_{x \mapsto 1^-} g(x) = \lim_{x \mapsto 1^+} g(x) = g(1) = 6$

$$\Rightarrow \quad c + 2 = 6$$

$$\Rightarrow \quad c = 4$$

(Note that f is then continuous everywhere)

Question 6. [**1.5 marks**] Show that the following function is not differentiable at 3.

$$f(x) = \begin{cases} -x+3 \text{ if } x \leq 3 \\ x-3 \text{ if } x > 3 \end{cases}$$

(Hint: use the definition of the derivative).

~~we have~~

$f(3+h)$ because if $h>0$, $3+h$ is >3

$$\lim_{h\to 0^+} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{\overbrace{(3+h)-3}^{f(3+h)} - 0}{h} = \lim_{h\to 0} \frac{h}{h} = 1.$$

~~we have~~

$f(3+h)$ because if $h<0$, $3+h<3$

$$\lim_{h\to 0^-} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{\overbrace{-(3+h)+3}^{f(3+h)} - 0}{h} = \lim_{h\to 0} \frac{-h}{h} = -1$$

Thus, $\lim\limits_{h\to 0} \frac{f(3+h)-f(3)}{h}$ Does not exist

$\Rightarrow$ f is Not diff. at 3