UAEU College of Science كلية العلوم
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Department of Mathematical Sciences
قسم علـوم الرياضيـــــات
Dr. Nafaa Chbili

MATH 105, Calculus I, Spring 2016

Test 3

Date: 18/4/2016, Time: 45mn

Student Name: ~~NA~~ ______________________

Student ID: Solution Key ______________________

Attendance Number: ______________________

Question	1,2,3 and 5	4	Grade
Student Learning Outcome	3	2	
Marks	12/12	3/3	15/15

- Show all the steps of your solution for each question.
- Use only Blue or Black pen, neither pencil nor colored.
- Graphics and Programming calculators are not allowed.

Question 1. [**3 marks**] Use the Closed Interval Method to find the absolute maximum and the absolute minimum of the function $g(x) = x^3 - 3x + 1$ on the interval $[-2, 2]$.

- Critical points

$g'(x) = 3x^2 - 3$

$g'(x) = 0 \iff 3x^2 - 3 = 0$

$\iff 3(x^2 - 1) = 0$

$\iff x = 1$ or $x = -1$.

- $g(1) = 1^3 - 3 + 1 = -1$
- $g(-1) = (-1)^3 - 3\times(-1) + 1 = -1 + 3 + 1 = 3$
- $g(2) = 2^3 - 3\times 2 + 1 = 3$
- $g(-2) = (-2)^3 - 3\times(-2) + 1 = -8 + 6 + 1 = -1$

Absolute maximum : $g(-1) = g(2) = 3$

Absolute minimum $g(1) = g(-2) = -1$

Question 2. [**4 marks**] Let $f(x) = xe^x$.

(**a**) Find the intervals where the function $f(x)$ is increasing and where it is decreasing.

(**b**) Find all local Extrema (maximum and minimum) of $f(x)$.

(a) $f'(x) = e^x + x e^x$

$= (x+1)\, e^x$

$f'(x) = 0 \iff x+1 = 0 \iff x = -1$

Since $e^x > 0$ for all x

then: $f'(x) > 0$ if $x > -1$

$f'(x) < 0$ if $x < -1$

		-1	
$f'(x)$	$-$	0	$+$
$f(x)$	$\searrow$		$\nearrow$

f is increasing on $(-1, \infty)$

f is decreasing on $(-\infty, -1)$

(b) $(-1, \frac{-1}{e})$ is a Local minimum of $f(x)$.

Question 3. [**2.5 marks**] Determine the intervals where the graph of $f(x) = x^4 + x^3 + x$ is concave upwards and where it is concave downwards, and identify the inflection point(s).

$f'(x) = 4x^3 + 3x^2 + 1$

$f''(x) = 12x^2 + 6x = x(12x+6)$

$f''(x) = 0 \iff x = 0$ or $x = -1/2$

	$-1/2$	0	
$f''(x)$	$+$	$-$	$+$
	concave up	concave down	concave up

inflection points are $\left(\frac{-1}{2}, \frac{-e^{-1/2}}{2}\right)$ and $(0,0)$.

Question 4. [3 marks] Evaluate the following limits.

(a) $\lim_{x\to\infty} \frac{ln(x^2+1)}{x}$.

" $\frac{\infty}{\infty}$ "

using L'Hospital's Rule:

$$\lim_{x\to\infty} \frac{\ln(x^2+1)}{x} = \lim_{x\to\infty} \frac{\frac{2x}{x^2+1}}{1} = \lim_{x\to\infty} \frac{2x}{x^2+1} = 0.$$

$$= \cancel{\lim_{x\to\infty} \frac{2/x}{x\left[\frac{x^2+1}{x}\right]}}$$

(b) $\lim_{x\to 0^+} (1+sin(x))^{\frac{1}{x}}$.

" $(1)^{\infty}$ " : IF

Let $y = (1+\sin(x))^{1/x}$

$$\ln(y) = \frac{1}{x}\ln(1+\sin(x)) = \frac{\ln(1+\sin(x))}{x}$$

by L'Hospital's Rule:

$$\lim_{x\to 0^+} \frac{\ln(1+\sin x)}{x} = \lim_{x\to 0} \frac{\frac{\cos x}{1+\sin x}}{1} = \lim_{x\to 0} \frac{\cos x}{1+\sin x} = \frac{1}{1+0} = 1$$

$$\lim_{x\to 0^+} y = 1 \Rightarrow \lim_{x\to 0^+} (1+\sin(x))^{1/x} = \underline{\underline{e}}$$

Question 5. [**2.5 marks**] Find the dimensions x and y of a rectangle of perimeter 1500m, whose area is the largest.

Area: $A = xy$ $\quad 0 \leq x \leq 750$

$2x + 2y = 1500 \iff x + y = 750$

$\iff y = 750 - x$

Then: $A = (x)(750 - x)$

$= -x^2 + 750x$

$= \cancel{x}($

$A'(x) = -2x + 750$

$A'(x) = 0 \iff x = 375$

x	0		375		750
$A'(x)$		$+$		$-$	
$A(x)$		$\nearrow$		$\searrow$	

A(x) has Absolute maximum at $x = 375$

Answer: Largest Area: $x = 375\text{m}$

$y = 750 - 375$

$= 375\text{m}$