

MATH 105, Calculus I, Spring 2016

Test 3

Date: 18/4/2016, Time: 45mn

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Student ID: Solution Key

Attendance Number: _____

Grade 102.../15	Marks	10.../12	30.../3
	Student Learning Outcome	3	2
	Question	1,2,3 and 5	4

- Show all the steps of your solution for each question.
- Use only Blue or Black pen, neither pencil nor colored.
- Graphics and Programming calculators are not allowed.

Question 1. [3 marks] Use the Closed Interval Method to find the absolute maximum and the absolute minimum of the function $g(x) = x^3 - 3x + 1$ on the interval $[-2, 2]$.

• Critical points

$$g'(x) = 3x^2 - 3$$

$$\Leftrightarrow 3x^2 - 3 = 0$$

$$\Leftrightarrow 3(x^2 - 1) = 0$$

$$\Leftrightarrow x = 1 \text{ or } x = -1.$$

• $g(1) = 1^3 - 3 + 1 = -1$

• $g(-1) = (-1)^3 - 3(-1) + 1 = -1 + 3 + 1 = 3$

• $g(2) = 2^3 - 3 \times 2 + 1 = 3$

• $g(-2) = (-2)^3 - 3 \times (-2) + 1 = -8 + 6 + 1 = -1$

Absolute maximum: $g(-1) = g(2) = 3$

Absolute minimum: $g(1) = g(-2) = -1$

Question 2. [4 marks] Let $f(x) = xe^x$.

- (a) Find the intervals where the function $f(x)$ is increasing and where it is decreasing.
 (b) Find all local Extrema (maximum and minimum) of $f(x)$.

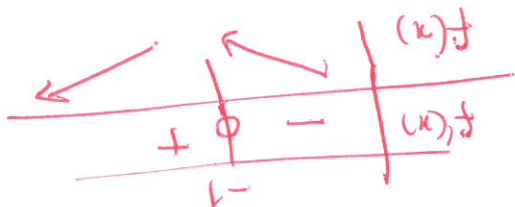
$$(a) \quad f'(x) = e^x + xe^x = (x+1)e^x$$

$$f'(x) = 0 \Leftrightarrow x+1 = 0 \Leftrightarrow x = -1$$

Since $e^x > 0$ for all x

Then: $f'(x) > 0$ if $x > -1$

$f'(x) < 0$ if $x < -1$



f is increasing on $(-1, \infty)$

f is decreasing on $(-\infty, -1)$

(b) $(-1, \frac{1}{e})$ is a local minimum of $f(x)$.

Question 3. [2.5 marks] Determine the intervals where the graph of $f(x) = x^4 + x^3 + x$ is concave upwards and where it is concave downwards, and identify the inflection point(s).

$$f'(x) = 4x^3 + 3x^2 + 1$$

$$f''(x) = 12x^2 + 6x = x(12x + 6)$$

$$f''(x) = 0 \Leftrightarrow x = 0 \text{ or } x = -\frac{1}{2}$$

			$f''(x)$
	+	+	concave up
	-	+	concave down
	+	-	concave up
		0	

in flexion points are $(-\frac{1}{2}, \frac{1}{2})$ and $(0,0)$.

Question 4. [3 marks] Evaluate the following limits.

(a) $\lim_{x \rightarrow \infty} \frac{x}{\ln(x^2 + 1)}$

$\frac{\infty}{\infty}$

using L'Hospital's Rule:

$$\lim_{x \rightarrow \infty} \frac{x}{\ln(x^2 + 1)} = \lim_{x \rightarrow \infty} \frac{x}{\ln(x^2 + 1)}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{\frac{2x}{x^2 + 1}}$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 + 1}{2x} = 0$$

~~$\lim_{x \rightarrow \infty} \frac{x}{\ln(x^2 + 1)}$~~

(b) $\lim_{x \rightarrow 0^+} (1 + \sin(x))^{1/x}$

Let $y = (1 + \sin(x))^{1/x}$

$$\ln(y) = \frac{1}{x} \ln(1 + \sin(x))$$

IF ∞ IF

by L'Hospital's Rule:

$$\lim_{x \rightarrow 0^+} \ln(y) = \lim_{x \rightarrow 0^+} \frac{\ln(1 + \sin(x))}{x}$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{1 + \sin(x)}}{\cos(x)}$$

$$= \lim_{x \rightarrow 0^+} \frac{1}{\cos(x)(1 + \sin(x))} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow 0^+} y = 1$$

$$\Rightarrow \lim_{x \rightarrow 0^+} (1 + \sin(x))^{1/x} = e$$

Question 5. [2.5 marks] Find the dimensions x and y of a rectangle of perimeter 1500m, whose area is the largest.

Area: $A = xy$

$$2x + 2y = 1500 \Rightarrow x + y = 750$$

$$\Leftrightarrow y = 750 - x$$

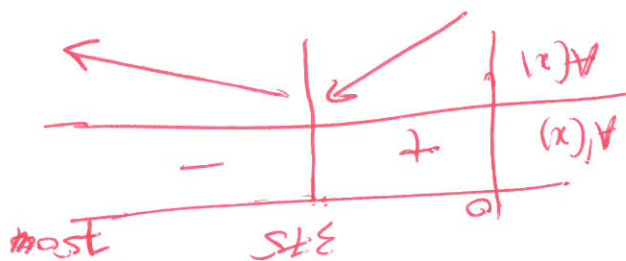
$$0 \leq x \leq 750$$

Then: $A = (x)(750 - x)$

$$= -x^2 + 750x$$

$$A'(x) = -2x + 750$$

$$A'(x) = 0 \Leftrightarrow x = 375$$



$A(x)$ has Absolute maximum at $x = 375$

Answer: Largest Area: $x = 375\text{m}$
 $y = 750 - 375$
 $= 375\text{m}$