UAEU College of Science كلية العلوم

جامعة الإمارات العربية المتحدة
United Arab Emirates University

Department of Mathematical Sciences

قسم علـوم الرياضيــــات

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MATH 105, Calculus I, Spring 2016

Test 2

Date: 14/3/2016, Time: 45mn

Student Name: ______________________

Student ID: Solution Key.

Attendance Number: ______________________

Question	1	2	3	4	Grade
Student Learning Outcome	2	2	2	2	....../30=....../15
Marks	....../12	....../5	.../7	....../6	

- Show all the steps of your solution for each question.
- Use only Blue or Black pen, neither pencil nor colored.
- Graphics and Programming calculators are not allowed.

Question 1. [12 marks]

(a) Find $\frac{d}{dx}(x^2 tan^{-1}(x+1))$.

Answer: $2x\tan^{-1}(x+1) + \frac{x^2}{1+(x+1)^2}$

$$\frac{d}{dx}\left(x^2\tan^{-1}(x+1)\right) = 2x\tan^{-1}(x+1) + x^2\frac{1}{1+(x+1)^2}$$

$$= 2x\tan^{-1}(x+1) + \frac{x^2}{1+(x+1)^2}.$$

(b) Find $\frac{d}{dx}(ln(2 + sin(2x))$.

Answer: $\frac{2\cos(2x)}{2+\sin(2x)}$

(c) Let $h(x) = xe^x + x^7$. Find the following derivatives: $h'(x)$, $h"(x)$ then $h^{(100)}(x)$.

Answer: $h'(x) = (x+1)e^x + 7x^6$	$h"(x) = (x+2)e^x + 42x^5$	$h^{(100)}(x) = (x+100)e^x$

$$h'(x) = 1 \cdot e^x + xe^x + 7x^6$$
$$= (x+1)e^x + 7x^6.$$
$$h''(x) = \cancel{1} \cdot e^x + (x+1)e^x + 7 \cdot 6 \cdot x^5$$
$$= (x+2)e^x + 42x^5.$$

$h^{(100)}(x) = (x+100)e^x$ because the 100^{th} derivative of x^7 is 0.

Question 2. [5 marks] Use logarithmic differentiation to find the derivative of $\dfrac{\sqrt{x+1}}{(x-1)^{1/3}}$.

Let $y = \dfrac{\sqrt{x+1}}{(x-1)^{1/3}}$

$$\ln(y) = \ln\left(\frac{\sqrt{x+1}}{(x-1)^{1/3}}\right) = \ln\left(\sqrt{x+1}\right) - \ln\left((x-1)^{1/3}\right)$$
$$= \frac{1}{2}\ln(x+1) - \frac{1}{3}\ln(x-1).$$

$$\frac{y'}{y} = \frac{1}{2} \cdot \frac{1}{x+1} - \frac{1}{3} \cdot \frac{1}{x-1} = \frac{3(x-1) - 2(x+1)}{6(x-1)(x+1)} = \frac{x-5}{6(x-1)(x+1)}$$

Then: $y' = \left(\dfrac{x-5}{6(x-1)(x+1)}\right) y$

$$= \frac{x-5}{6(x-1)(x+1)} \cdot \frac{\sqrt{x+1}}{(x-1)^{1/3}}$$
$$= \frac{x-5}{6\sqrt{x+1}\,(x-1)^{4/3}}$$

Question 3. [7 marks] Use implicit differentiation to find an equation of the tangent line to the curve: $x^2 - 2x + y^3 + 2xy = 5$ at the point $(2, 1)$.

Answer: The equation of the tangent line at $(2,1)$ is ....$y = -\frac{4}{7}x + \frac{15}{7}$....

$$\frac{d}{dx}\left[x^2 - 2x + y^3 + 2xy\right] = \frac{d}{dx}[5]$$

$$\Rightarrow \quad 2x - 2 + 3y'y^2 + 2y + 2xy' = 0$$

$$\Rightarrow \quad y'(3y^2 + 2x) = 2 - 2x - 2y$$

$$\Rightarrow \quad y' = \frac{2 - 2x - 2y}{3y^2 + 2x}$$

at $x = 2$ and $y = 1$

$$y' = \frac{2 - 4 - 2}{3 + 4} = \frac{-4}{7}$$

Equation of the tangent Line:

$$y - 1 = -\frac{4}{7}(x - 2)$$

$$\Leftrightarrow \quad y = -\frac{4x}{7} + \frac{15}{7}$$

$$\boxed{y = -\frac{4}{7}x + \frac{15}{7}}$$

Question 4. [6 marks] (a) Find the linear approximation of the function $f(x) = \sqrt{2x+1}$ at $a = 4$, then use it to approximate the number $\sqrt{9.2}$.

$$f'(x) = \frac{2}{2\sqrt{2x+1}} = \frac{1}{\cancel{2}\sqrt{2x+1}}$$

$$f'(4) = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

$$L(x) = f(a) + f'(a)(x-a) = f(4) + f'(4)(x-4)$$

$$= 3 + \frac{1}{3}(x-4) = \frac{1}{3}x + \frac{5}{3}.$$

$f(x) \approx \frac{1}{3}x + \frac{5}{3}$. for all x near 4.

$$\sqrt{9.2} = \sqrt{2\cdot(4.1)+1} = f(4.1) \approx \frac{1}{3}(4.1) + \frac{5}{3}$$

$$\approx 3.033$$

Then $\sqrt{9.2} \approx 3.033$

(b) For what values of x is the linear approximation (found in the previous question) accurate to within $\frac{1}{3}$?

$$|f(x) - L(x)| < 1/3$$

$$\Rightarrow -\frac{1}{3} < \sqrt{2x+1} - \left(\frac{1}{3}x + \frac{5}{3}\right) < 1/3$$

$$\Rightarrow \frac{1}{3}x + \frac{4}{3} < \sqrt{2x+1} < \frac{1}{3}x + \frac{6}{3} =$$

(*) $\sqrt{2x+1} < \frac{1}{3}x + \frac{6}{3} \Rightarrow 3\sqrt{2x+1} < x+6 \Rightarrow 9(2x+1) < (x+6)^2$

$\Rightarrow 18x + 9 < x^2 + 12x + 36 \Rightarrow 0 < x^2 - 6x + 27$

$\Rightarrow x$ any real number.

$\sqrt{2x+1} > \frac{x+4}{3} \Rightarrow 9(2x+1) > (x+4)^2 \Rightarrow 18x + 9 > x^2 + 8x + 16$

$\Rightarrow x^2 - 10x + 7 > 0 \Rightarrow 0.75 < x < 9.25$

so $\boxed{x \in [0.75, 9.25)}$