UAEU College of Science كلية العلوم
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Department of Mathematical Sciences
قسم علوم الرياضيات
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MATH 105, Calculus I, Spring 2016

Test 1

Date: 15/2/2016, Time: 45mn

Student Name: ______________________________

Student ID: Solution key.

Attendance Number: ______________________________

Question	1	2	3	4	Grade
Student Learning Outcome	2	1	2	1	……/30=……/15
Marks	……/6	……/9	…/12	……/3	

- Show all the steps of your solution for each question.
- Use only Blue or Black pen, neither pencil nor colored.
- Graphics and Programming calculators are not allowed.

Question 1. [6 marks] Let $f(x) = \dfrac{\sqrt{x^2+3}}{x-4}$. Determine all the horizontal and vertical asymptotes of $f(x)$. Show all your steps.

Vertical asymtote(s):	line $x = 4$
Horizontal asymtote(s):	line $y = 1$ and line $y = -1$

Vertical asymptotes.

$$\lim_{x \mapsto 4^+} \frac{\sqrt{x^2+3}}{x-4} = \text{“}\frac{\sqrt{19}}{0^+}\text{”} = \infty$$

$$\lim_{x \mapsto 4^-} \frac{\sqrt{x^2+3}}{x-4} = \text{“}\frac{\sqrt{19}}{0^-}\text{”} = -\infty$$

Then the line $x = 4$ is a vertical asymptote.

Horizontal asymptotes

- $$\lim_{x \mapsto \infty} \frac{\sqrt{x^2+3}}{x-4} = \lim_{x \mapsto \infty} \frac{\frac{\sqrt{x^2+3}}{x}}{\frac{x}{x} - \frac{4}{x}} = \lim \frac{\frac{\sqrt{x^2+3}}{\sqrt{x^2}}}{1 - \frac{4}{x}}$$

$$= \lim_{x \mapsto \infty} \frac{\sqrt{1 + 3/x^2}}{1 - 4/x} = 1. \quad \text{since } \lim_{x \mapsto \infty} \frac{4}{x} = 0 \text{ and } \lim_{x \mapsto \infty} \frac{3}{x^2} = 0$$

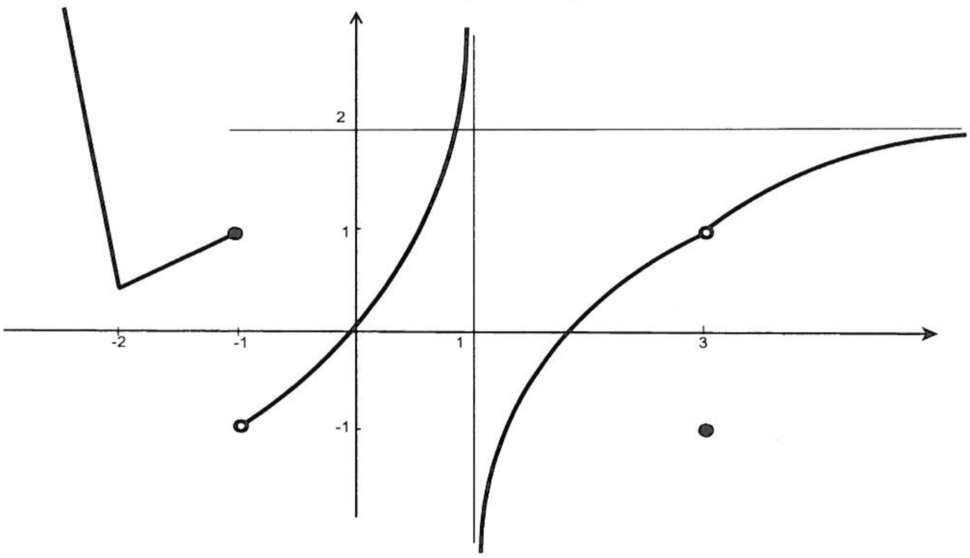
Then the line $y = 1$ is a Horizontal asympote.

- $$\lim_{x \mapsto -\infty} \frac{\sqrt{x^2+3}}{x-4} = \lim_{x \mapsto -\infty} \frac{\frac{\sqrt{x^2+3}}{x}}{\frac{x}{x} - \frac{4}{x}} = \lim_{x \mapsto -\infty} \frac{\frac{\sqrt{x^2+3}}{-\sqrt{x^2}}}{1 - \frac{4}{x}}$$

$$= \lim_{x \mapsto -\infty} \frac{-\sqrt{1 + 3/x^2}}{1 - 4/x} = -1$$

Then the line $y = -1$ is a horizontal asymptote

Question 3. [9 marks] Let $f(x)$ be the function graphed below. Find the following values if they exist. If the limit does not exist then explain why.



(a) $\lim_{x\to 3} f(x)$ $= 1$

(b) $\lim_{x\to -1} f(x)$ DNE because $\lim_{x\mapsto -1^-} f(x) = 1 \neq \lim_{x\mapsto -1^+} f(x) = -1$

(c) $\lim_{x\to 1^-} f(x)$ $= +\infty$

(d) $\lim_{x\to 1^+} f(x)$ $= -\infty$

(e) $\lim_{x\to \infty} f(x)$ $= 2$

(f) Determine all points at which $f(x)$ is NOT continuous.

$f(x)$ is Not Continuous at -1, 1 and 3

(g) Determine all points at which $f(x)$ is NOT differentiable.

$f(x)$ is Not differentiable at $-2, -1, 1$ and 3

Question 3. [12 marks] Evaluate the following limits:

(a) $\lim_{x\to\infty} \frac{2x^3+2x^2-1}{3x^3+x+1}$.

Answer: $2/3$

$$\lim_{x\to\infty} \frac{2x^3+2x^2-1}{3x^3+x+1} = \lim_{x\to\infty} \frac{\frac{2x^3}{x^3}+\frac{2x^2}{x^3}-1/x^3}{\frac{3x^3}{x^3}+\frac{x}{x^3}+\frac{1}{x^3}}$$

$$= \lim_{x\to\infty} \frac{2+\frac{2}{x}-\frac{1}{x^3}}{3+\frac{1}{x^2}+\frac{1}{x^3}} = 2/3.$$

(b) $\lim_{x\to\infty} \frac{\sqrt{x^2+1}-x}{x+1}$.

Answer: 0

$$\lim_{x\to\infty} \frac{\sqrt{x^2+1}-x}{x+1} = \lim_{x\to\infty} \frac{1}{(x+1)(\sqrt{x^2+1}+x)} = \frac{1}{\infty} = 0$$

(c) $\lim_{x\to 2} \dfrac{x^2+x-6}{x-2}$.

Answer: 5

$$\lim_{x\mapsto 2} \frac{x^2+x-6}{x-2} = \lim_{x\mapsto 2} \frac{(x-2)(x+3)}{x-2} = \lim_{x\mapsto 2} \frac{\cancel{(x-2)}(x+3)}{\cancel{x-2}}$$

$$= \lim_{x\mapsto 2} x+3 = 5.$$

(d) $\lim_{x\to 0} x^2[\sin(\frac{1}{x}) + \cos(\frac{1}{x^2})]$. (Hint: use the Squeeze Theorem.)

Answer: 0

for all $x \neq 0$ we have $-1 \leq \sin\frac{1}{x} \leq 1$

and $-1 \leq \cos\left(\frac{1}{x^2}\right) \leq 1$

Then $-2 \leq \sin\left(\frac{1}{x}\right) + \cos\left(\frac{1}{x^2}\right) \leq 2$

~~we get~~:

if we multiply by x^2 we get: $-2x^2 \leq x^2\left(\sin(\frac{1}{x}) + \cos(1/x^2)\right) \leq 2x^2$

since $\lim_{x\mapsto 0} 2x^2 = \lim_{x\mapsto 0} -2x^2 = 0$

Then: by the Squeeze theorem we conclude that: $\lim_{x\mapsto 0} x^2\left(\sin 1/x + \cos(1/x^2)\right) = 0$

Question 4. [3 marks] Let $f(x) = x^3 + x^2 - 3$. Show that there exists a number c between 1 and 2 such that $f(c) = 0$.

$f(x)$ is continuous everywhere.

$f(1) = 1^3 + 1^2 - 3 = -1 < 0$

$f(2) = 2^3 + 2^2 - 3 = 9 > 0$

By the Intermediate theorem there exists c in $(1,2)$ such that $f(c) = 0$.