Probability Bounds of Throughput Performance of Real-Time Traffic in WiMAX Networks

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Abstract—WiMAX is receiving growing attention as a reliable contender for broadband network applications. Therefore, it is important to develop measures and benchmarks for the performance of this relatively new technology. These benchmarks are intended for service providers in order to be able to plan their networks properly to ensure robust and continuous service to their networks’ end users. In this paper, we develop the bounds of the probability of throughput starvation occurring within the network. This is done based on the operating conditions within the network and the level of service that is required by different network customers. We validate the resulting models via simulation experiments. We compare the results obtained through the use of our models to those obtained through experiments.

Index Terms—WiMAX, IEEE 802.16, MAC, Throughput analysis, OFDMA

I. INTRODUCTION

WiMAX networks are becoming an attractive choice for deploying broadband access services. This is due, among other reasons, to their relatively low cost compared to the other broadband technologies such as DSL. WiMAX networks are based on the IEEE 802.16 standard [1] and use OFDMA [13] as its physical layer technology. In order to support the rapid proliferation of the WiMAX technology, it is important to develop performance benchmarks that can be used to evaluate the throughput performance of WiMAX based networks. Several studies have been conducted to analyze the throughput performance in other wireless network technologies e.g. [3], [4]. As far as WiMAX is concerned, some studies have previously provided some insights into the performance trends within this type of networks, see for example [5], [6], [14]. In this study, we derive the upper and lower bounds of an important performance benchmark that can be used by service providers to measure the ability of their WiMAX networks to provide customers with their required levels of service. This performance measure is “the probability of system throughput starvation”. It is defined as the probability of throughput starvation occurring within the system under certain operating conditions. We only consider point-to-multipoint (PMP) networks, see Figure 1. In these networks, a base station (BS) is responsible for managing the traffic for network members which are generally called subscriber stations (SS). The traffic in PMP networks is divided into uplink (UL) traffic and downlink (DL) traffic. We only consider DL traffic in this paper.

The probability bounds that we derive in this study are intended to determine the limits of the starvation probability within WiMAX networks. Therefore, network service providers will be able to calculate the bounds of the probability that there will be throughput deficiency within their networks at given operating parameters and conditions.

The rest of this paper is organized as follows. In Section II, we give some background about WiMAX classes of service and the supporting physical layer (PHY) for the purpose of this study. In Section III, we derive the analytical models of bounds of the probability measure that we defined above, under different operating conditions. In Section IV, we show the results of our simulation experiments and compare these results to the ones obtained through our models. In Section V, we conclude the study.

II. WiMAX QoS Classes and Supporting PHY

WiMAX is a telecommunications technology that targets providing wireless data over long distances in a variety of ways, from point-to-point links to full mobile cellular type access with end-to-end QoS guarantees. It is based on the IEEE 802.16 standard. Based on the various levels of QoS guarantees of the running applications, IEEE 802.16 deploys four service flow classes [1]:

- Unsolicited Grant Service (UGS): Supports real-time services with constant data bit rate, such as Voice over IP (VoIP) without silence suppression. The QoS metrics for
this service class are minimum reserved traffic rate (which is equal to the maximum sustained traffic rate), maximum latency, tolerated jitter, and request transmission policy.

- real time Polling Service (rtPS): Supports real-time services with variable size data on a periodic basis, such as moving picture experts group (MPEG) and VoIP with silence suppression or streaming video. The QoS service flow metrics for this service are minimum reserved traffic rate, maximum sustained traffic rate, maximum latency, and request transmission policy.

- non-real-time Polling Service (nrtPS): Supports non-real-time services that require variable size data grant bursts on a regular basis. Subscriber stations (SSs) contend for bandwidth (for uplink transmission) during contention request opportunities which are granted at regular intervals irrespective of the network load. An example application of this QoS class is FTP. The mandatory QoS service flow parameters for this service are minimum reserved traffic rate, maximum sustained traffic rate, traffic priority, and request transmission policy.

- Best Effort (BE): For data streams for which no minimum service level is required. Subscriber stations (SSs) contend for bandwidth (for uplink transmission) during contention request opportunities, these opportunities, unlike nrtPS, are not guaranteed and depend on the network load. The mandatory QoS service flow parameters for this service are minimum sustained traffic rate, traffic priority, and request transmission policy.

In order to support the diverse QoS requirements mentioned above, WiMAX Forum’s Mobile Task Group has adopted OFDMA as the baseline of the physical layer. OFDMA is a multiple-access scheme that inherits the ability of OFDM to combat the intersymbol interference and frequency selective fading channels [11].

In order to reduce the effect of selective fading channels in wireless networks, OFDMA divides the total frequency bandwidth into a number of narrow band subcarriers in such a way that each subcarrier (SC) will experience a flat fading channel. This is due to the fact that the SC width is selected to be much smaller than the coherence bandwidth of the channel. The OFDMA PHY layer based on OFDM modulation is designed for Non-Line-of-Sight (NLOS) operation in the frequency band from 2 GHz to 11 GHz [2]. It should be noted that the modulation in OFDMA is done in the frequency domain.

In order to transform the data back to the time domain, the Inverse Fourier Transform (IFT) can be used to construct the OFDMA waveform during a symbol time. A small end-portion of the symbol time, which is as long as the maximum delay spread channel, is called a cyclic prefix (CP). This small end-portion is copied at the beginning of the symbol time duration to eliminate the effect of delay spread. Thus, the intersymbol interference is dealt with while maintaining orthogonality among the subcarriers.

Figure 2 shows the behaviour of wireless fading channel gains in relation to the OFDMA symbols which represent the time domain, and the subcarriers which represent the frequency domain. Channel gains, for a given subcarrier, may differ from one of the OFDMA symbol to another. This depends on the multipath time variations of the medium between the BS and SSs. On the other hand, for a fixed OFDMA symbol, the channel gain differs from one subcarrier to another. This is due to the frequency selective fading effect of the wireless channel.

In one OFDMA symbol time interval, the active subcarriers are grouped into subsets of subcarriers; each subset is called a subchannel [2]. On the DL, a subchannel may have data intended for different users. On the uplink (UL), a mobile station (MS) may be assigned one or more subchannels. Hence, the multiple antennas that may be deployed on the MS unit can transmit simultaneously. The subcarriers that form one subchannel are not neighbors, in general. An OFDMA can fully use multiple user channel variations via two-dimensional resource allocation, namely frequency and time domains, as indicated in Figure 2.

As mentioned above, multiple data subcarriers are grouped into subchannels. Each subchannel forms a slot with one or more OFDMA symbols. There are two main types of subcarrier allocation techniques: Distributed Sub Carrier Allocation (DSCA) and Adjacent Sub Carrier Allocation (ASCA). In general, the distributed allocations perform very well in mobile applications, while adjacent subcarrier permutations can be properly used for fixed, portable, or low mobility environments [8]. In this study, we assume that both the BS as well as the SSs are not mobile. Nevertheless, we used DSCA for fair resource distribution and to maximize throughput.

III. THROUGHPUT STARVATION PROBABILITY BOUNDS DERIVATION

In this study, we consider downlink (DL) traffic for real time traffic types (UGS and rtPS) only. We consider two main system situations:

- Equal QoS requirements by all network SSs, and,
- Different QoS requirements by these SSs.

In both cases, we assume that each SS experiences different channel gain conditions than the other SSs within the network, which is the general case that is encountered in WiMAX.
networks. In a previous study [7], we addressed the special case where all SSs experience similar channel gain conditions. We also make the important assumption that network resources can satisfy the throughput requirements of at least any single SS in the network.

A. Equal SS QoS requirements

In this case, each SS requests traffic with QoS requirements that are similar to those of the other SSs within the network. This can be the case in an environment where system users are running similar applications or running applications that have similar bandwidth needs. There are currently many WiMAX scheduling algorithms, see for example [9], [15], [16], [10], [12]. However, we use a simple scheduling scheme that helps demonstrate the true bounds of the sought probability. According to our chosen scheduling scheme, some SS is picked and assigned subcarriers until its QoS requirements are satisfied, and then the next SS is served and so on.

The signal-to-noise ratio (SNR) of the traffic received by SS k at subcarrier n is given by:

$$\gamma_{k,n} = \frac{N|h_{k,n}|^2 \cdot p_{k,n}}{B_w \cdot N_o}, \forall n \in N, \text{and}, \forall k \in K,$$

where the indexing sets of the available subcarriers and active SSs are denoted by $N = \{1, 2, \ldots, N\}$ and $K = \{1, 2, \ldots, K\}$, respectively. $N_o$ is the additive white Gaussian noise (AWGN) power at the receiver, $h_{k,n}$ is the channel fading gain experienced by SS k at the nth subcarrier, $p_{k,n}$ is the transmit power of SS k allocated to the nth subcarrier, and $B_w$ is the channel bandwidth.

We assume that each subcarrier undergoes a Rayleigh flat fading subchannel and that $\{\gamma_{k,n} | \forall n \in N, \forall k \in K\}$ are i.i.d. Thus, $\gamma_{k,n}$ is an exponentially distributed random variable with the probability density function (pdf) of the following formula:

$$f_{\gamma_{k,n}}(w) = \frac{1}{\bar{\gamma}} \exp(-\frac{w}{\bar{\gamma}}), \; w \geq 0,$$

where $\bar{\gamma}$ is the average received SNR, $\bar{\gamma} \triangleq E[\gamma_{k,n}].$ Define $\gamma_{k,n}^{\text{max}} \triangleq \text{max}|\gamma_{k,n}|, 1 \leq k \leq K$. The pdf of $\gamma_{k,n}^{\text{max}}$ can be expressed as follows:

$$f_{\gamma_{k,n}^{\text{max}}}(w) = \frac{K}{\bar{\gamma}} \left(1-\exp\left(-\frac{w}{\bar{\gamma}}\right)\right)^{K-1} \exp\left(-\frac{w}{\bar{\gamma}}\right),$$

$$\forall m \in N, \; w \geq 0$$

(3)

The maximum possible attainable system throughput at a given time frame can be the actual system throughput when the number of both SSs and subcarriers in the system becomes large. Therefore, the probability of two or more SSs seeing the channel gain of a given subcarrier as the best gain for both of them, is negligible. The maximum attainable throughput is given by:

$$T_{h^{\text{max}}} = \frac{T_{DL} \cdot B_w}{T_f} \sum_{n=1}^{N} \hat{n}_b(\gamma_{n}^{\text{max}}) \text{ bits/sec},$$

where:

- $T_{DL} = T_f/2$ is the DL sub-frame, where $T_f$ is the time frame duration in OFDMA symbols,
- $\hat{n}_b(\gamma_{n}^{\text{max}})$ is the payload, with $O_{DL}$ being the medium access control (MAC) and physical layer (PHY) overhead in the downlink (in OFDMA symbols),
- $B_w = B_w/N$ is the width of each subcarrier in Hz, with $B_w$ being the system bandwidth, and

$$\hat{n}_b(\gamma_{n}^{\text{max}}) = n_b(\gamma_{n}^{\text{max}}) \cdot (1 - BE_{c}(\gamma_{n}^{\text{max}})^{N}(\gamma_{n}^{\text{max}}),$$

(5)

where $BE_{c}(\gamma_{n}^{\text{max}})$ is the bit error rate associated with the received SNR, $\gamma_{n}^{\text{max}}$, at the nth subcarrier, with $n_b$ being the number of information bits transmitted on the nth subcarrier per one unit of time (second) per one unit of bandwidth (Hz). It depends on the value of $\gamma_{n}^{\text{max}}$ and the used adaptive modulation and coding (AMC) module, see Table I.

<table>
<thead>
<tr>
<th>Table I</th>
<th>Modulation and Coding Schemes for $BE_{c} = 10^{-6}$ in IEEE 802.16.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation(coding)</td>
<td>$n_b(\gamma_{n}^{\text{max}})$</td>
</tr>
<tr>
<td>BPSK(1/2)</td>
<td>0.5</td>
</tr>
<tr>
<td>QPSK(1/2)</td>
<td>1.0</td>
</tr>
<tr>
<td>QPSK(3/4)</td>
<td>1.5</td>
</tr>
<tr>
<td>16QAM(1/2)</td>
<td>2.0</td>
</tr>
<tr>
<td>16QAM(3/4)</td>
<td>3.0</td>
</tr>
<tr>
<td>64QAM(2/3)</td>
<td>4.0</td>
</tr>
<tr>
<td>64QAM(3/4)</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Therefore, $T_{h^{\text{max}}}$ is a summation of $N$ independent discrete random variables each with a probability mass function (pmf) that completely depends on the channel gain statistics through the SNR, $\gamma_{n}^{\text{max}}$.

The pmf of $n_b(\gamma_{n}^{\text{max}})$ is given by:

$$P_{\gamma_{n}^{\text{max}}} = P_{\gamma_{i}}(n_b(\gamma_{n}^{\text{max}}) = n_i) = P_{\gamma_{i}}(\gamma_{i} \leq \gamma_{n}^{\text{max}} < \gamma_{i+1})$$

$$= \int_{\gamma_i}^{\gamma_{i+1}} f_{\gamma_{n}^{\text{max}}}(w) \; dw$$

$$= \sum_{t=0}^{K-1} \binom{K-1}{t} \frac{K}{K-t} \cdot \frac{(-1)^{K-t-1}}{\gamma_{\gamma_{n}^{\text{max}}}} \exp\left(-\frac{(K-t)\gamma_{\gamma_{n}^{\text{max}}}}{\gamma_{\gamma_{n}^{\text{max}}}}\right) \cdot \exp\left(-\frac{(K-t)\gamma_{\gamma_{n}^{\text{max}}}}{\gamma_{\gamma_{n}^{\text{max}}}}\right),$$

where $\gamma_i, i = 1, 2, \ldots, 7$ are the thresholds of the 7 non-overlapping regions shown in Table I with $\gamma_8 = \infty$ and $\gamma_0 = 0$, and $n_i \in \{0.5, 1.5, 2.5, 3.4, 4.5\}$ is the number of information bits on the M-ary QAM symbol.

Starvation occurs in the system when the total attainable throughput over all subcarriers becomes lower than the total requested throughput by all K active SSs, which is denoted by $T_{h^{\text{req}}}$.

The system starvation probability lower bound, $P_{\text{st-lower}}$, can be calculated as the probability that the maximum attainable throughput, $T_{h^{\text{max}}}$, is less than $T_{h^{\text{req-min}}}$ which is the sum of all minimum requested throughputs of all SSs. That is, $P_{\text{st-lower}}$ at a given time frame is given by:

$$P_{\text{st-lower}} = P_{\gamma_{n}^{\text{max}}} (T_{h^{\text{max}}} < T_{h^{\text{req-min}}})$$

(7)

To find a closed form formula for the probability in (7), we need to find the pdf of $T_{h^{\text{max}}}$. For a large $N$, and according
to the Central Limit Theorem (CLT), the pdf of $T_{h}^{\text{max}}$ can be approximated as a Normal distributed random variable, i.e.

$$
\frac{T_{h}^{\text{max}} - \mu_{T h}}{\sigma_{T h}} \rightarrow \mathcal{N}(0, 1),
$$

(8)

where $\mathcal{N}(0, 1)$ denotes a normal distribution with zero mean and variance equal to 1, with $\mu_{T h}$ being the mean of $T_{h}^{\text{max}}$ and $\sigma_{T h}^2$ is the variance of $T_{h}^{\text{max}}$. Both $\mu_{T h}$ and $\sigma_{T h}^2$ can be found as a function of $\mu_{\hat{x}}^2$ and $\sigma_{\hat{x}}^2$, which are the mean and the variance of $\hat{x}_b(\gamma_{n}^{\text{max}})$, respectively:

$$
\mu_{T h} = \frac{\hat{\lambda}_{DL} B_{sc}}{T_f} \sum_{n=1}^{N} \mu_{\hat{x}}^2
$$

$$
= \frac{\hat{\lambda}_{DL} B_{sc}}{T_f} \sum_{n=1}^{N} \sum_{i=1}^{7} P_{\text{max}} n_i (1 - \text{BER})^{n_i}
$$

$$
= N \frac{\hat{\lambda}_{DL} B_{sc}}{T_f} \sum_{i=1}^{7} \hat{P}_{\text{max}} n_i (1 - \text{BER})^{n_i},
$$

(9)

and

$$
\sigma_{T h}^2 = \left( \frac{\hat{\lambda}_{DL} B_{sc}}{T_f} \right)^2 \sum_{n=1}^{N} \sigma_{\hat{x}}^2
$$

$$
= N \left( \frac{\hat{\lambda}_{DL} B_{sc}}{T_f} \right)^2 \left( \mathbb{E}\{\hat{x}_b^2(\gamma_{n}^{\text{max}})\} - \mu_{\hat{x}}^2 \right),
$$

(10)

where

$$
\mathbb{E}\{\hat{x}_b^2(\gamma_{n}^{\text{max}})\} = \sum_{i=1}^{7} \hat{P}_{\text{max}} n_i (1 - \text{BER})^{2n_i}
$$

and

$$
\mu_{\hat{x}} = \mathbb{E}\{\hat{x}_b(\gamma_{n}^{\text{max}})\} = \sum_{i=1}^{7} \hat{P}_{\text{max}} n_i (1 - \text{BER})^{n_i}
$$

Thus, $P_{s-t}^{\text{low}}$ in (7) can be obtained as follows:

$$
P_{s-t}^{\text{low}} = P_r \left( T_{h}^{\text{max}} < T_{h}^{\text{req-min}} \right)
$$

$$
= P_r \left( \frac{T_{h}^{\text{max}} - \mu_{T h}}{\sigma_{T h}} < \frac{T_{h}^{\text{req-min}} - \mu_{T h}}{\sigma_{T h}} \right)
$$

$$
= \int_{-\infty}^{\frac{T_{h}^{\text{req-min}} - \mu_{T h}}{\sigma_{T h}}} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{u^2}{2} \right) du
$$

$$
= 1 - Q \left( \frac{T_{h}^{\text{req-min}} - \mu_{T h}}{\sigma_{T h}} \right),
$$

(11)

where $Q(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp \left( -t^2/2 \right) dt$ is the Q-function.

The upper bound of the system starvation probability occurs when all active SSs within the network request their maximum sustained throughputs. Thus, $P_{s-t}^{\text{up}}$ can be obtained as follows:

$$
P_{s-t}^{\text{up}} = P_r \left( T_{h}^{\text{max}} < T_{h}^{\text{req-max}} \right)
$$

$$
= \int_{-\infty}^{\frac{T_{h}^{\text{req-max}} - \mu_{T h}}{\sigma_{T h}}} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{u^2}{2} \right) du
$$

$$
= 1 - Q \left( \frac{T_{h}^{\text{req-max}} - \mu_{T h}}{\sigma_{T h}} \right)
$$

(12)

It should be mentioned here that for equal QoS for all SSs, $T_{h}^{\text{req-min}} = \gamma_{min}^{\text{req}}$ and $T_{h}^{\text{req-max}} = \gamma_{max}^{\text{req}}$ in (11) and (12), respectively, where $\gamma_{min}^{\text{req}}$ and $\gamma_{max}^{\text{req}}$ are the minimum requested and maximum sustained throughputs by each SS, respectively.

### B. Different SS QoS requirements

This is the general case in WiMAX networks. In this case, each SS has QoS requirements that are generally different from those of the other SSs within the network. Here also we use a simple scheduling scheme that enables us to obtain the targeted probability bounds. According to the scheduling scheme in this case, the SS with the highest QoS requirements is served first until satisfaction. Then, the SS with the second highest QoS is served and so on.

Following the same argument as in the case of equal SS QoS requirements, we can get the lower and the upper bounds of the system starvation probability when each SS within the system has throughput requirements that are different from those of the other SSs in the system. However, the difference from the case of similar SS QoS requirements is that $T_{h}^{\text{req-min}} = \sum_{k=1}^{K} r_{k}^{\text{min}}$ and $T_{h}^{\text{req-max}} = \sum_{k=1}^{K} r_{k}^{\text{max}}$, where $r_{k}^{\text{min}}$ and $r_{k}^{\text{max}}$ are the minimum requested and maximum sustained throughputs of SS $k$, respectively. Therefore,

$$
P_{s-t}^{\text{low}} = P_r \left( T_{h}^{\text{max}} < T_{h}^{\text{req-min}} \right)
$$

$$
= \int_{-\infty}^{\frac{\sum_{k=1}^{K} r_{k}^{\text{min}} - \mu_{T h}}{\sigma_{T h}}} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{w^2}{2} \right) dw
$$

$$
= 1 - Q \left( \frac{\sum_{k=1}^{K} r_{k}^{\text{min}} - \mu_{T h}}{\sigma_{T h}} \right),
$$

and,

$$
P_{s-t}^{\text{up}} = P_r \left( T_{h}^{\text{max}} < T_{h}^{\text{req-max}} \right)
$$

$$
= \int_{-\infty}^{\frac{\sum_{k=1}^{K} r_{k}^{\text{max}} - \mu_{T h}}{\sigma_{T h}}} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{w^2}{2} \right) dw
$$

$$
= 1 - Q \left( \frac{\sum_{k=1}^{K} r_{k}^{\text{max}} - \mu_{T h}}{\sigma_{T h}} \right)
$$

### IV. Model Evaluation

The models that we have derived in the previous section are intended to be used in evaluating WiMAX networks to obtain the lower and upper bounds of the probability of starvation within these networks. That is, using our models will enable network operators to calculate the bounds of the probability that there will be shortages of service within their networks at given operating parameters and conditions. Our goal in this evaluation is to ensure that the trends that result from using our models are similar to those obtained through experiments.

In this section, we validate these models using Matlab simulation experiments. We assume that each SS always has one thousand channel samples and we calculate the average result over these samples.
Channel fluctuations are assumed to be slow enough in such a way that does not cause this channel to change during one time frame duration. In our simulations, we only consider two service classes, namely UGS and rtPS, where traffic arrives at the BS at the beginning of each time frame. Packet arrival and departure is assumed to follow Poisson distribution while packet inter-arrival time is assumed to follow exponential distribution. Traffic is assumed to arrive at the beginning of each frame and so is the assignment of subcarriers to the SSs. UGS traffic flows are generated as constant bit rate (CBR) traffic. On the other hand, rtPS traffic flows are generated as variable bit rate (VBR) traffic with a minimum requested throughput of 64 kbps and a maximum sustained throughput of 512 kbps. Because of the similarity in the results and to avoid redundancy, we only show the results of the rtPS experiments.

In figures 3 and 4 we see that as the number of SSs, $K$, increases with fixed number of total available subcarriers ($N = 256$), the starvation probability increases. This results from the increase of the gross requested throughput by all active subscriber stations. If we increase the time frame from 10 ms (as in Figure 3) to 20 ms (as in Figure 4), the system resources increase and more SSs (5 additional SSs in this case) can be served in the network with system starvation probability that is close to zero.

Simulation results that are shown in Figure 5 indicate that with a fixed number of SSs ($K = 50$), the system starvation probability decreases as the channel bandwidth increases. This stems from the fact that the system resources available to the SSs increase thus increasing the possibility of serving more users.

Figure 6 shows that the starvation probability decreases as the time frame duration, $T_f$, increases. The explanation of this is similar to the case of increasing the channel bandwidth that we discussed above.

It should be noted that the results in the case of different QoS requirements by SSs are similar to the case of system throughput starvation with equal QoS requirements of all SSs. Therefore, we did not include these results to avoid redundancy.

It is worth mentioning that in all cases above the experimental results are quite close to those obtained by the analytical model.

### V. Conclusions

WiMAX is a growing technology that supports different classes of service within broadband wireless access networks. Therefore, it is necessary to develop performance measures and benchmarks that enable the assessment of the ability of networks based on this technology to provide the required levels of service.

In this paper, we derived the bounds of an important performance measure for point-to-multipoint WiMAX networks. This measure deals with the bounds of the probability of throughput starvation occurring within the system under certain operating conditions and levels of service that are required by network customers. Specifically, we investigated two main customer requirements situations: when all customers have equal QoS requirements and when they have different QoS requirements. We derived the models for the lower and upper bounds of the probability of system throughput starvation in these two cases. In our analysis, we assumed that each SS within the network experiences different channel gain conditions than the other SSs within the network, which is the general operating condition within WiMAX networks. Using simulation experiments, we showed that our analytical models provide results that are very close to those obtained through experiments. These models can be very useful to WiMAX service providers as they can be used as planning tools for network establishment and expansion.

### TABLE II

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$, number of subcarriers</td>
<td>256</td>
</tr>
<tr>
<td>$B_{w}$, channel bandwidth</td>
<td>1.25 MHz</td>
</tr>
<tr>
<td>$N_0$, AWGN power at the SS receiver</td>
<td>$1.565 \times 10^{-6}$</td>
</tr>
<tr>
<td>$K$, number of SSs in the network</td>
<td>50</td>
</tr>
<tr>
<td>BS total transmit power, $P_{tot}$</td>
<td>1 Watts</td>
</tr>
<tr>
<td>$T_f$, time frame duration</td>
<td>10 ms</td>
</tr>
<tr>
<td>$r_{min}$, minimum required throughput</td>
<td>64 kbps</td>
</tr>
<tr>
<td>$r_{max}$, maximum required throughput</td>
<td>512 kbps</td>
</tr>
<tr>
<td>OFDM symbol duration</td>
<td>12.5 µs</td>
</tr>
<tr>
<td>DL overhear, $O_{DL}$ (in OFDMA symbols)</td>
<td>20</td>
</tr>
</tbody>
</table>

The SSs in the simulation experiments are stationary and uniformly distributed in a circle and are served by one BS that is located at the center of the circle.

We measure the effect on the probability of throughput starvation while varying the following parameters:

- Number of SSs,
- Channel bandwidth, and,
- Frame duration.

We conduct a separate experiment for each of the above cases. Unless otherwise indicated in connection to each experiment, system parameters are as given in Table II.

In all cases, we compare the starvation probability bounds that we obtained through experiments to those obtained by calculations using the analytical model. All simulation results are obtained with a 95% confidence interval that is lower than 0.0031.

### REFERENCES


Fig. 3. System starvation probability versus the number of active subscriber stations in the network, \( T_f = 10 \) ms, \( T_{th}^{req-min} = 10 Mbps \) and \( T_{th}^{req-max} = 13.5 Mbps \).

Fig. 4. System starvation probability versus the number of active subscriber stations in the network, \( T_f = 20 \) ms, \( T_{th}^{req-min} = 10 Mbps \) and \( T_{th}^{req-max} = 13.5 Mbps \).

Fig. 5. System starvation probability versus the channel bandwidth \( B_w \) with 50 active SSs.

Fig. 6. System starvation probability versus the duration of the time frame, \( T_f \) (m sec), \( T_{th}^{req-min} = 5Mbps \) and \( T_{th}^{req-max} = 6.5Mbps \).