
Fethi Belkhouche
Electrical Engineering and Computer Science Department
Tulane University, New Orleans
LA 70118, USA
Phone: +1 (504) 865 5871
Fax: +1 (504) 862 3293
Email: belkhouf@eecs.tulane.edu

Parviz Rastgoufard
Electrical Engineering and Computer Science Department
Tulane University, New Orleans
LA 70118, USA
Phone: +1 (504) 865 3296
Fax: +1 (504) 862 3293
Email: pr@eecs.tulane.edu

Boumediene Belkhouche
Corresponding Author
Electrical Engineering and Computer Science Department
Tulane University, New Orleans
LA 70118, USA
Phone: +1 (504) 862 3372
Fax: +1 (504) 862 3293
Email: bb@eecs.tulane.edu

Abstract
We consider the problem of robot navigation and tracking of moving objects using a wheeled mobile robot. Our control algorithm uses the proportional navigation law. This approach combines geometrical rules with the kinematics equations of the robot and the moving goal. We derive a relative kinematics model describing the motion of the goal seen by the robot. The aim of our control strategy is to make the robot’s angular velocity proportional to the rate of turn of the imaginary straight line that joins the robot and the goal. Different paths are obtained for different navigation constants. In the presence of obstacles, an obstacle avoidance mode is considered. This mode takes precedence over the navigation-tracking mode. The control strategy allows the robot to reach unpredictably-moving goals under certain assumptions. An extensive simulation is carried out to illustrate our strategy.

Index Terms
Robot navigation–tracking, relative kinematics equations, proportional navigation law.

Type of the paper: Regular paper

F. Belkhouche, P. Rastgoufard and B. Belkhouche
Electrical Engineering and Computer Science Department
Tulane University, New Orleans
LA 70118, USA
Email: belkhouf@eecs.tulane.edu
pr@eecs.tulane.edu
bb@eecs.tulane.edu

Abstract— We consider the problem of robot navigation and tracking of moving objects using a wheeled mobile robot. Our control algorithm uses the proportional navigation law. This approach combines geometrical rules with the kinematics equations of the robot and the moving goal. We derive a relative kinematics model describing the motion of the goal seen by the robot. The aim of our control strategy is to make the robot’s angular velocity proportional to the rate of turn of the imaginary straight line that joins the robot and the goal. Different paths are obtained for different navigation constants. In the presence of obstacles, an obstacle avoidance mode is considered. This mode takes precedence over the navigation-tracking mode. The control strategy allows the robot to reach unpredictably-moving goals under certain assumptions. An extensive simulation is carried out to illustrate our strategy.

Index Terms— Robot navigation–tracking, relative kinematics equations, proportional navigation law.

I. INTRODUCTION

Real-time robot navigation towards an object moving with unknown maneuvers and tracking control are fundamentally important and difficult issues in robotics. Various applications can benefit from this topic, such as surveillance, for example, where the robot aims to track a moving object and keep it in the camera scope. This problem is a real-time problem combining different aspects. There exist mainly two families of methods used to solve this problem [1]; namely feature-based methods and model-based methods. Model-based methods require building a model of the tracked object. Feature-based methods track features such as color, shape, etc. Visual servoing control for tracking moving objects is among the most important feature-based methods [1]. Visual servoing methods are widely used, where different problems and aspects are considered, such as robust real-time visual tracking ([1], [2]), complex objects tracking [3], positioning and localization of the robot with respect to the moving object ([4], [5]), and position visual control in order to keep the moving object in the field of view of the robot [6]. Compensation for abrupt changes in the target motion using the generalized likelihood ratio algorithm was addressed in [7]. Tracking multiple moving targets was discussed in [8], where the proposed tracker uses a two-stage visual tracking method. Car detection and tracking [9] and human detection and tracking ([10], [11]) using visual sensors were also addressed.
The problem of high speed target tracking using a method which integrates Lyapunov theory and potential field method was addressed in [12]. Lyapunov theory is used in [13] in order to design a stable target-tracking control for unicycle mobile robots. An optimal control law for tracking a target moving in a straight line was also addressed in [14].

Fuzzy logic controllers were also suggested for tracking moving objects. Fuzzy logic controllers may simplify the sensory systems, since they do not require precise information on the tracked object. A two-dimensional target tracking system using fuzzy sliding mode control was considered in [15]. Hierarchical Grey-fuzzy motion decision making method [16] and Grey prediction theory fuzzy logic controller [17] were also used. In [18] the authors consider fuzzy logic controllers for multiple mobile robots solving a continuous pursuit problem.

In this paper, we consider the problem of robot tracking and navigation towards a moving object using a model-based strategy. The method used for this purpose consists of the proportional navigation law, which is a closed-loop strategy based on the kinematics equations. This method is well-known in the aerospace community for its simplicity and effectiveness. To the best of our knowledge, the paper by Piccardo and Honderd [19] is among the first attempts to apply the proportional navigation law to solve a robotics problem. There, the authors suggest the use of the standard proportional navigation law for grasping moving objects using a robotic manipulator. The papers by Mehrandezh et al. ([20], [21]) use two different variants of the proportional navigation law for robotic arm interception of fast-maneuvering targets. The results concerning the effectiveness and robustness of the method are rigorously proven. In [22] and [23], the velocity and deviated pursuit, which are special cases of the proportional navigation were used for robot navigation and tracking unpredictably-moving targets. Our aim in this paper is to suggest a solution to the problem of robot navigation towards moving objects in the absence and presence of obstacles based on the proportional navigation law. Our theoretical study is illustrated using an extensive simulation.

This paper is organized as follows: In section II, we describe the problem. In section III, we introduce the kinematics equations for the robot and the moving goal. In section IV, we derive our relative kinematics model of the navigation–tracking problem. The closed loop control law is introduced in section V. The particular case of navigation towards a fixed point is discussed in section VI, where different results are proven. The tracking problem in the presence of obstacles is addressed in section VII. Finally, an extensive simulation illustrating the method is carried out in the last section.

II. PROBLEM STATEMENT

The robot and the moving goal move in the horizontal plane \( W \subset \mathbb{R}^2 \). The goal maneuvers are not a priori known to the robot. Thus, the problem of navigation towards the moving goal is a real-time problem, and off-line strategies are not effective. The aim is to design a closed-loop control law for the robot angular velocity, which insures reaching the moving goal. Closed-loop systems are more effective than open-loop systems, since in closed-loop systems the control input is a function of the state. The navigation problem is considered in both
obstacle-free workspace and in the presence of obstacles. We assume that the following conditions are satisfied

(A1) The robot is faster than the moving goal.
(A2) The minimum turning radius of the robot is smaller than the minimum turning radius of the moving goal.
(A3) The goal maintains a positive velocity and the smoothness of the path.
(A4) The robot has a sensory system, which provides the control system with the necessary information.

The influence of the sensory system on the control strategy is beyond the scope of this paper. The path of the robot in the two-dimensional workspace is denoted by \( P_R(t) \) and the path of the moving goal by \( P_G(t) \). The robot reaches the moving goal when \( P_R(t_f) \approx P_G(t_f) \), where \( t_f \) is the interception or execution time. Our objective in this paper is to suggest a solution to the problems of tracking and navigation towards a moving object.

III. KINEMATICS EQUATIONS

Let point \( O \) be the origin of an inertial reference frame of coordinates. The robot is a simple wheeled mobile robot of the unicycle type. Its kinematics equations are as follows

\[
\begin{align*}
\dot{x}_R &= v_R \cos \theta_R \\
\dot{y}_R &= v_R \sin \theta_R \\
\dot{\theta}_R &= \omega_R
\end{align*}
\]

where \((x_R, y_R)\) is the position of the robot reference point in the inertial frame of reference, \(\theta_R\) is the robot orientation angle with respect to the positive x-axis. \(v_R\) and \(\omega_R\) are the robot control inputs; they represent the linear and angular velocities of the robot, respectively.

The goal moves according to the following kinematics equations

\[
\begin{align*}
\dot{x}_G &= v_G \cos \theta_G \\
\dot{y}_G &= v_G \sin \theta_G
\end{align*}
\]

where \((x_G, y_G)\) is the position of the goal in the inertial frame of reference, \(\theta_G\) is the goal orientation angle with respect to the positive x-axis. \(v_G\) is the goal linear velocity.

In the next section, we define important geometric quantities, and we introduce a kinematics system that models the navigation–tracking problem.

IV. GEOMETRY AND KINEMATICS MODELING OF THE NAVIGATION–TRACKING PROBLEM

The position of the robot's reference point in the inertial reference frame of coordinates is given by the vector \( r_R \). In a similar way, the position of the goal in the inertial reference frame of coordinates is given by the vector \( r_G \). The relative range between the robot and the moving goal is given by

\[
r_{GR} = r_G - r_R
\]  

and the Euclidean distance is

\[
r_{GR} = \sqrt{(x_G - x_R)^2 + (y_G - y_R)^2}
\]  

It is clear that

\[
\begin{align*}
\mathbf{r}_R &= x_R \mathbf{u}_x + y_R \mathbf{u}_y, \quad r_R = \sqrt{x_R^2 + y_R^2} \\
\mathbf{r}_G &= x_G \mathbf{u}_x + y_G \mathbf{u}_y, \quad r_G = \sqrt{x_G^2 + y_G^2}
\end{align*}
\]
where \( \mathbf{u}_x \) and \( \mathbf{u}_y \) are the unit vectors. The relative velocity between the robot and the target is given by

\[
_\mathbf{r} \mathbf{G} \mathbf{R} = _\mathbf{r} \mathbf{G} - _\mathbf{r} \mathbf{R} \tag{7}
\]

where \( \mathbf{v}_G \) and \( \mathbf{v}_R \) are the velocity vectors of the goal and the robot, respectively. Figure 1 represents the geometry of the navigation problem. We define the following quantities:

1) The imaginary straight line that starts at the robot reference point and is directed at the goal along the positive direction of \( _\mathbf{r} \mathbf{G} \mathbf{R} \) is the line of sight \( \mathbf{L}_{GR} \).

2) The line of sight angle \( \sigma_{GR} \) is the angle between the positive x-axis and the line of sight.

3) \( \mathbf{L}_R \) and \( \mathbf{L}_G \) are the lines that start at the origin of the inertial frame of reference and are directed towards the robot and the goal, respectively.

4) \( \sigma_R \) and \( \sigma_G \) are the line of sight angles for the robot and the goal, they represent the angles from the reference line to the lines of sight \( \mathbf{L}_R \) and \( \mathbf{L}_G \), respectively.

The angle of the line of sight robot-goal is expressed as follows

\[
\tan \sigma_{GR} = \frac{y_G - y_R}{x_G - x_R} \tag{9}
\]

Note that \( \sigma_{GR} \) and \( \dot{\sigma}_{GR} \) are not defined when \( _\mathbf{r} \mathbf{G} \mathbf{R} = 0 \), i.e., when the robot reaches the goal. In our approach, a polar coordinates representation is used to model the navigation problem. Polar coordinates were used by many authors to design control laws for wheeled mobile robots ([13], [24]) as well as underwater vehicles. Let us consider the following change of variable

\[
x = r \cos \sigma \tag{10}
\]

\[
y = r \sin \sigma
\]

where \( r \) is the radial variable and \( \sigma \) is the angular variable. The time derivatives of \( r \) and \( \sigma \) are given by

\[
\dot{r} = \frac{\dot{x} x + \dot{y} y}{r} \tag{11}
\]

\[
\dot{\sigma} = \frac{\dot{x} y - \dot{y} x}{r^2} \tag{12}
\]

respectively. By considering the change of variable given in (10), the time derivatives of \( r \) and \( \sigma \) and the kinematics equations of the robot and the target, we get

\[
\dot{r}_R = v_R \cos (\theta_R - \sigma_R) \tag{13}
\]

\[
r_R \dot{\sigma}_R = v_R \sin (\theta_R - \sigma_R)
\]

and

\[
\dot{r}_G = v_G \cos (\theta_G - \sigma_G) \tag{14}
\]

\[
r_G \dot{\sigma}_G = v_G \sin (\theta_G - \sigma_G)
\]

Equations (13) and (14) represent the kinematics equations of the robot and the goal in polar coordinates. In this new representation, the control input for the mobile robot becomes \( (v_R, \theta_R) \) instead of \( (v_R, \dot{\theta}_R) \). In addition
to the linear velocity and the orientation angle, equations (13) and (14) depend also on the line of sight angles \( \sigma_R \) and \( \sigma_G \). The velocity vectors \( \mathbf{v}_R \) and \( \mathbf{v}_T \) can be decomposed into two components with respect to \( \mathbf{L}_R \) and \( \mathbf{L}_G \), respectively. We have for the robot

\[
\mathbf{v}_R = v^r_R \mathbf{u}_r + v^t_R \mathbf{u}_t \tag{15}
\]

where \( \mathbf{u}_r \) and \( \mathbf{u}_t \) are the unit vectors along and across the line of sight \( L_R \), respectively. We have for the moving goal velocity vector

\[
\mathbf{v}_G = v^r_G \mathbf{u}_r + v^t_G \mathbf{u}_t \tag{16}
\]

where \( \mathbf{u}_r \) and \( \mathbf{u}_t \) are the unit vectors along and across the line of sight \( L_G \), respectively. \( v^r_r \) and \( v^r_G \) are the radial velocities; \( v^t_r \) and \( v^t_G \) are the tangential velocities.

Clearly, by considering equations (13) and (14), we get

\[
v^r_R = r_R \hat{\sigma}_R \quad \text{and} \quad v^r_G = r_G \hat{\sigma}_G \tag{17}
\]

The relative velocity vector \( \mathbf{v}_{GR} \) can be written as follows

\[
\mathbf{v}_{GR} = v^r_{GR} \mathbf{u}_r + v^t_{GR} \mathbf{u}_t \tag{18}
\]

\[
= \dot{r}_{GR} \mathbf{u}_r + r_{GR} \hat{\sigma}_{GR} \mathbf{u}_t \tag{19}
\]

Here, the unit vectors \( \mathbf{u}_r \) and \( \mathbf{u}_t \) are the unit vectors along and across the line of sight robot-goal \( L_{GR} \). By considering equations (13) and (14), the equations for \( v^r_{GR} \) and \( v^t_{GR} \) are expressed as follows

\[
v^r_{GR} = v_G \cos (\theta_G - \sigma_{GR}) - v_R \cos (\theta_R - \sigma_{GR})
\]

\[
v^t_{GR} = v_G \sin (\theta_G - \sigma_{GR}) - v_R \sin (\theta_R - \sigma_{GR})
\]

These equations describe the velocity of the goal seen by the robot along and across the line of sight robot-goal. The first equation in (20) gives the variation of the relative distance between the robot and the target; the second equation gives the turning rate of the goal with respect to the robot.

V. PROPORTIONAL NAVIGATION LAW

The proportional navigation guidance law is among the most discussed guidance strategies in the aerospace community. This guidance law is simple and effective. The literature on the proportional navigation law is substantial ([25], [26], [27], [28]), where various aspects are studied. There exist different ways to define the proportional navigation [19]. In our path planning context, we choose a simple definition, where the aim of the proportional navigation guidance law is to make the robot angular velocity proportional to the rate of turn of the angle of the line of sight robot-goal, which means that

\[
\omega_R = \dot{\theta}_R = N \hat{\sigma}_{GR} \tag{21}
\]

where \( N \) is a positive real number called the proportionality factor or constant, with \( N \geq 1 \). Note that \( N = 1 \) corresponds to the pursuit ([22], [25]). Depending on the initial state, we have the following cases

1) \( \theta_R (t_0) = N \sigma_{GR} (t_0) \); in this particular case, the proportional navigation law is reduced to

\[
\theta_R (t) = N \sigma_{GR} (t) \tag{22}
\]

2) \( \theta_R (t_0) \neq N \sigma_{GR} (t_0) \); the proportional navigation
law is characterized by

\[ \theta_R(t) = N \sigma_{GR}(t) + \theta_R(t_0) - N \sigma_{GR}(t_0) \quad (23) \]

If we put \( c = \theta_R(t_0) - N \sigma_{GR}(t_0) \), the robot’s steering angle is given by

\[ \theta_R(t) = N \sigma_{GR}(t) + c \quad (24) \]

The constant \( c \) has an important influence on the path of the robot. The robot’s kinematics equations under the steering law given by (24) are given by

\[ \begin{align*}
\dot{x}_R &= v_R \cos(N \sigma_{GR} + c) \\
\dot{y}_R &= v_R \sin(N \sigma_{GR} + c) \\
\dot{\theta}_R &= \omega_R = N \dot{\sigma}_{GR}
\end{align*} \quad (25) \]

Recall that \( \sigma_{GR} \) is a function of the robot and the target coordinates in the global reference frame of coordinates. The navigation–tracking kinematics equations under the proportional navigation law are given by

\[ \begin{align*}
\dot{r}_{GR} &= -v_R \cos(K \sigma_{GR} + c) \\
r_{GR} \dot{\sigma}_{GR} &= -v_R \sin(K \sigma_{GR} + c)
\end{align*} \quad (26) \]

with

\[ K = N - 1 \quad (27) \]

The navigation constant \( N \) plays an important role in the proportional navigation. Different values of \( N \) result in different paths; thus \( N \) acts as a control variable. The dynamics limitations of the robot can be considered through \( N \). This point is beyond the scope of this paper. From equations (22) and (24), it is clear that the proportional navigation law has a periodic aspect with \( N \).

In the next section, we discuss the particular case, where the robot is controlled using the proportional navigation law to reach a stationary point.

VI. A SPECIAL CASE: NAVIGATION TOWARDS A STATIONARY GOAL

The control law given by equation (21) can be used to navigate the robot towards a stationary point. In this case, the kinematics equations modeling the navigation problem are the following

\[ \begin{align*}
\dot{r}_{GR} &= -v_R \cos(K \sigma_{GR} + c) \\
r_{GR} \dot{\sigma}_{GR} &= -v_R \sin(K \sigma_{GR} + c)
\end{align*} \quad (28) \]

By dividing the radial velocity by the tangential velocity, we get

\[ \frac{dr_{GR}}{d\sigma_{GR}} = \frac{r_{GR} \cos(K \sigma_{GR} + c)}{\sin(K \sigma_{GR} + c)} \quad (29) \]

By integrating equation (29) we get

\[ r_{GR} = r_{GR0} \exp \left( \int_{\sigma_{GR0}}^{\sigma_{GR}} \frac{d\sigma_{GR}}{\sin(K \sigma_{GR} + c)} \right) \quad (30) \]

where \( r_{GR0} = r_{GR}(t_0) \) and \( \sigma_{GR0} = \sigma_{GR}(t_0) \) are the initial values of the relative range and the line of sight angle, respectively. The solution for the relative distance as a function of the line of sight angle is given by

\[ r_{GR} = r_{GR0} \left( \frac{\sin(K \sigma_{GR} + c)}{\sin(K \sigma_{GR0} + c)} \right)^{\frac{1}{N-1}} \quad (31) \]

for \( N > 1 \). The robot navigating under the proportional navigation law reaches the goal from almost all initial states. Several examples are shown in our simulation. It is worth noting that the robot does not move in a straight line when navigating towards a fixed point using the proportional navigation law, except in the case of the pure pursuit, \( N = 1, c = 0 \). The proof in the case of a moving goal is difficult [25], and requires that \( v_M > v_T \).

For the particular case when \( N = 1 \), the proof can be
found in [22].

A. Heading regulation

Under the robot kinematics constraints, the application of the proportional navigation requires the following equation to be satisfied at the initial time

$$\theta_R(t_0) = N\sigma_{GR}(t_0) + \theta_R(t_0) - N\sigma_{GR}(t_0) \quad (32)$$

In most cases this equation is not satisfied, which means that the application of the proportional navigation law from any initial state is not straightforward. To solve this problem, a heading regulation phase is necessary. During this phase the robot's orientation angle is controlled in order to satisfy equation (32). Various techniques for controlling nonholonomic vehicles can be used to drive the robot to a configuration where equation (32) is satisfied.

VII. Navigation–Tracking in the Presence of Obstacles

The problem of navigation tracking of a moving goal becomes more difficult in the presence of obstacles. When an unexpected obstacle appears in the path of the robot, the robot should have the ability to dynamically avoid the collision with the obstacle, and return to the normal path after the obstacle is passed. For obstacle avoidance, various methods were used such as potential field methods ([29], [30], [31], [32]) and vector field histogram ([33], [34]). The navigation–tracking problem in the presence of obstacles combines local path planning and global path planning. We suggest that the robot moves in two modes:

1) Navigation mode: The robot moves using the proportional navigation law when no obstacle is detected in the active region.

2) Obstacle avoidance mode: This is a local path-planning problem, which has the priority over the navigation mode. Only obstacles in the active region are considered. A histogram, which maps the polar position of the obstacles in the active region is constructed. The histogram provides also the Euclidean distance robot-obstacles.

A. Obstacles modeling

For modeling, we suggest to approximate the obstacles by circles. Let $A_i$ be an arbitrary shaped obstacle detected by the robot. A circle $B_i$ that encloses $A_i$ is constructed as shown in figure 2. In order to compensate for the robot width, another circle enclosing $B_i$ is constructed by extending $B_i$ by the robot radius. This circle gives the final representation of the obstacle and it is denoted by $C_i$. This approach avoids problems which can be posed by certain obstacles such as U shaped obstacles. Under this formulation, the robot can be seen as a point-like vehicle. It is worth noting that in some situations it is more convenient to approximate $A_i$ by an ellipsis instead of a circle.

Let $R_{view}$ be the view range, which is the range for the scanner set where obstacles are considered. $R_{view}$ gives the robot’s active region, and only obstacles within this range are considered, as the obstacle avoidance problem has a local aspect. When obstacle $C_i$ appears in the active region of the robot, the robot sensors return the distance $d_{i1}$ and $d_{i2}$. The angles $\beta_{i1}$ and $\beta_{i2}$ are the angles defined
2. When the robot is in a collision course with the obstacle, the robot deviates from its nominal path in order to avoid the obstacle. The proportional navigation law is applied again after the obstacle is passed.

B. Collision avoidance mode

1) A special case: when the obstacle appears in the line of sight robot-goal: This special case is shown in figure 4. The proportional navigation law allows the robot to deviate from the obstacle, and at the same time reach the goal. This can be achieved by controlling the navigation constant, which is chosen depending on the position and the radius of the obstacle. Consider figure 4. In order to reach the goal, the robot deviates towards point $A$ to the left of the obstacle or point $B$ to the right of the obstacle. Point $A$ and point $B$ are characterized by given values for the range and the line of sight angle, denoted by $(r_A, \sigma_A)$ and $(r_B, \sigma_B)$, respectively. These values are assumed to be known. When the robot passes through point $A$ or $B$, equation (31) is satisfied. However it is necessary to compute the navigation constant, which allows the robot to pass through the transient points $A$ or $B$. Form equation (31), it is possible to derive the following equation

$$K \log \frac{r_{GR}}{r_{GR0}} = \log \left( \frac{\sin (K \sigma_{GR} + c)}{\sin (K \sigma_{GR0} + c)} \right)$$

(33)

This is a nonlinear equation in the navigation constant, which can be solved numerically easily. By solving this equation for the given values of $(r_A, \sigma_A)$, or $(r_B, \sigma_B)$, we get the value of $N$ which allows the robot to pass through point $A$ or point $B$ and reach the goal. The robot can reach point $A$ or point $B$ using the proportional
The robot is in a collision course with the center of the obstacle $C_i$ when $\dot{\sigma}_i = 0$, which means that from the second equation in (35)

$$N\sigma_{GR} + c = \sigma_i$$  \hspace{1cm} (36)

Since $C_i$ is a circle, the collision course is characterized by

$$\theta_R = N\sigma_{GR} + c \in [\beta_1, \beta_2]$$  \hspace{1cm} (37)

From equations (34) and (35), it is easy to determine whether the robot is approaching or moving away from obstacle $C_i$. A polar histogram for the obstacles in the active region is then built as follows

$$H(\gamma) = \bigcup_{i=1}^{m} h_i(\gamma)$$  \hspace{1cm} (38)

where $m$ is the total number of obstacles in the active region and $h_i$ is defined as follows

$$h_i = \frac{r_i}{R_{view}}, \text{if } \gamma \in [\beta_1, \beta_2]$$

$$h_i = 0, \text{if } \gamma \notin [\beta_1, \beta_2]$$  \hspace{1cm} (39)

The instantaneous value of the robot’s orientation angle can also be represented in the polar histogram. The $y$-axis in the polar histogram represents a distance ratio. Figure 5 represent the polar histogram constructed for the configuration of figure 3. The polar histogram given by (38) provides a mapping for the obstacles and their distance with the robot; the histogram is a dynamical histogram which is updated over a time period. In order to avoid colliding with obstacle $C_i$, the robot is controlled in the navigation constant such that $N\sigma_{GR} + c \notin [\beta_1, \beta_2]$. It is possible to choose $N$ such that

$$N\sigma_{GR} + c = \eta + \beta_{i1}$$  \hspace{1cm} (40)
such that Subscript 1 refers to the phase before the detour. Subscript 2 refers to the phase after the detour.

**Algorithm**

The algorithm for the obstacle avoidance mode can be summarized as follows

1. For all obstacles in the active region if \( r_i \leq R_{act} \) and \( N\sigma_{GR} + c \in [\beta_1, \beta_2] \), then modify the navigation constant such that

\[
N\sigma_{GR} + c \notin [\beta_1, \beta_2]
\]

(43)

with

- for a left detour: \( \theta_R = N\sigma_{GR} + c > \beta_2 \).
- for a right detour: \( \theta_R = N\sigma_{GR} + c < \beta_1 \).

2. After the obstacle is passed, the robot returns to the navigation mode using the proportional navigation law.

An extensive simulation is carried out in the next section.

**VIII. SIMULATION**

Our aim here is to illustrate the method using an extensive simulation. We consider several examples and scenarios.

Example 1: Stationary goal, \( c = 0 \)

Our aim is to illustrate the navigation using the proportional navigation law towards a fixed goal for different values of the navigation constant. The goal coordinates in the world coordinates system are given by (20,20). The robot is initially at the origin of the world coordinates system; this gives \( r_{GR0} = \sqrt{800} \), \( \sigma_{GR0} = 45^\circ \). We assume that, at the initial time, equation (32) is satisfied; therefore the heading regulation phase is not necessary. We also assume that the speed, the time and the distance are without units for simplicity. The paths traveled by the robot are shown in figure 6, where the robot starts from the same initial position but with different orientation angles. Different paths are obtained for different values of the navigation constant. The robot can reach the goal by turning to the left \( (N = 2, 3, 4) \) or to the right \( (N = 5.5, 6, 7) \). When the robot reaches the goal from a left turn, the final value of the line of sight angle is zero. When the robot reaches the goal from a right turn, the final value for the line of sight angle is \( \frac{2\pi}{N-1} \) (the zero and \( \frac{2\pi}{N-1} \) are the asymptotically stable equilibrium positions for the line of sight angle).

Example 2: Stationary goal, \( c \neq 0 \)
Fig. 6. Robot navigation towards a fixed point using the proportional navigation for different values of $N$, $c = 0$. The robot starts from the same initial position, but with different orientation angles.

Here, we maintain the same navigation constant ($N = 3$) and we simulate the robot path for different values of $c$. An illustration is shown in figures 7. Compared to the previous example, the paths present large detours for some values of $c$. This property can be used for obstacle avoidance. Similarly to the previous case, we can distinguish between left detour ($c = \frac{\pi}{3}$, $\frac{\pi}{5}$, $\frac{\pi}{9}$) and right detour ($c = \frac{5\pi}{6}$, $\frac{\pi}{4}$, $\frac{2\pi}{3}$). The robot reaches the goal with different final orientation angles.

Example 3: Obstacle avoidance: Stationary goal

There is an obstacle in the line of sight robot-goal. A navigation constant $N = 3$ with $c = 0$ or $c = \frac{\pi}{4}r$ allows the robot to reach the goal by passing through point $A(5.9, 15)$ or $A(-11.85, 15)$, respectively. The detour obtained for $c = \frac{\pi}{4}r$ is larger, which allows to avoid larger obstacles. This is illustrated in figure 8.

Example 4: Navigation towards a goal moving in a straight line

The goal is moving in a straight line parallel to the y-axis. The robot navigates using the proportional navigation law. The paths traced by the robot are shown in figures 9.

Example 5: Navigation towards a goal moving in a
circle

This scenario is shown in figure 10 where the robot starts uses different values for \( N \) and \( c \). In all cases, the robot reaches the moving goal successfully.

Example 6: Navigation towards a moving goal in the presence of obstacles

Two scenarios are considered to illustrate this situation. In both cases, the robot deviates from its nominal path when the obstacle is detected in the robot’s path.

The robot starts from point \( R_0 \) \((0,0)\), and aims to reach the goal which starts from \( G_0 \) \((20,20)\).

Case 1: This case is shown in figure 11. The robot is initially navigating using \( N = 3, c = 0 \). The robot deviates from its nominal path when the obstacle is detected in the collision course within a given distance. The robot starts deviating at point \( P \) by changing the navigation constant. Both left detour and right detour are possible.

– for a left detour; the new value of the navigation constant is \( N = 1.5 \). Using equation (42), the value of \( c \) which keeps the smoothness of the path is \( c = 45^\circ .729 \).

– for a right detour; the new value of the navigation constant is \( N = 6 \). Using equation (42) we get \( c = -91^\circ .459 \).

Case 2: The goal is moving in a straight line. Similarly to the previous case, both detours are possible

– for a left detour; the new value of the navigation constant is \( N = 1.5 \). Using equation (42) we get \( c = 42^\circ .826 \).

– for a left detour; the new value of the navigation constant is \( N = 6 \). Using equation (42) we get \( c = -85^\circ .652 \).

The paths are shown in figures 11 and 12. The robot path in the case of free-obstacle workspace is represented in a dashed line.

IX. CONCLUSION

In this paper we presented a method for robot navigation and tracking of unpredictably–moving objects.

Our strategy uses the proportional navigation guidance law, where geometrical rules are combined with the
the goal with respect to the robot is derived. The aim of the robot when the robot is in a collision course with an relative kinematics model describing the motion of the goal with respect to the robot is derived. The aim of our control strategy is to make the robot angular velocity proportional to the rate of turn of the line of sight angle. In the presence of obstacles, two modes are used, namely the navigation mode using the proportional navigation law and the obstacle avoidance mode, where an angular histogram for the obstacles is considered. The robot path is different for different navigation constant. This property is used to change the normal path of the robot when the robot is in a collision course with an obstacle. The method is illustrated using an extensive simulation. Important points such as robot navigation using the proportional navigation law under the dynamics and the kinematics limitations, and obstacle avoidance using the dynamic histogram integrated with the proportional navigation law can be considered for future investigations.

Fig. 12. Navigation in the presence of obstacles using the proportional navigation law

REFERENCES


