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Full paper

Modified parallel navigation for ball interception by a wheeled mobile robot goalkeeper

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Abstract—This paper deals with the design of a control strategy for a wheeled mobile robot goalkeeper whose task is to intercept the ball before it goes inside the goal. The control law is based on the parallel navigation guidance law where the goalkeeper moves on lines which are parallel to the initial line of sight that joins the robot and the ball. A relative kinematics model in polar coordinates is derived. Two approaches based on two different versions of parallel navigation are used. In the first approach, we introduce a new version for parallel navigation which is adapted to the case of the goalkeeper. In this formulation, the robot is controlled in the linear velocity and moves along a predefined path that covers the goal. The particular case where the goalkeeper moves on the goal line is considered in more detail and important quantities such as the interception time are derived in closed form in some particular cases. In the second approach the robot is controlled in the orientation angle, giving more flexibility for the robot motion. The robot path depends on the ball path, the linear velocities of the robot and the ball, and their initial positions. Ball interception by the goalkeeper is proven in this case also. Our control strategies are illustrated using an extensive simulation.

Keywords: Robotic soccer; wheeled mobile robot goalkeeper; ball interception; parallel navigation; kinematics models.

1. INTRODUCTION

Robotic soccer presents an important example of complex behavior, where the various tasks such as ball tracking and interception, moving obstacle avoidance, and team collaboration are combined together. In fact, robotic soccer has attracted the attention of researchers from various fields such as control theory, artificial intelligence and computer artificial vision. RoboCup is a popular research domain in robotic soccer. According to Ref. [1], the aim of RoboCup is the following: ‘By mid-21st century, a team of fully autonomous humanoid robot soccer players

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shall win the soccer game, comply with the official rules of the FIFA, against the winner of the most recent World Cup'. During the last decade, the literature dealing with soccer robotics has shown important developments. However, the problem of robot soccer players is very complex and includes various tasks. As a result of this complexity, most papers deal with one specific problem. The paper by Burkhard *et al.* [2] gives a description of the recent developments and the future challenges for soccer robotics.

This paper deals with the design of a control law for a wheeled mobile robot goalkeeper. The control design of wheeled mobile robots soccer players has been considered by many authors and various techniques were suggested [3, 4].

The task of the goalkeeper is to intercept the ball before it goes inside the goal. Of course this task is different from other players' tasks and requires a special strategy. The problem of ball tracking and interception is a real-time problem, since the ball motion is not known *a priori*. This renders the problem more difficult. The problem of ball interception by a mobile robot is considered by using different approaches. In Ref. [5], the authors used the concept of qualitative and relative velocity for ball interception in a dynamic environment. Another approach based on Lyapunov theory was suggested in Ref. [6] for general target tracking. These methods can be used by an ordinary soccer player. However, the problem of goalkeeping requires a special treatment.

Our control strategy for the goalkeeper is designed based on geometrical rules combined with the kinematics equations. We use polar coordinates for the state space representation of the kinematics equations.

Here, the control law for the wheeled mobile robot goalkeeper is achieved using a variant of parallel navigation. The principle of parallel navigation [7, 12] is to make the goalkeeper move on lines which are parallel to the initial line of sight that joins the goalkeeper and the ball. To achieve this task, the wheeled mobile robot is controlled in the linear velocity or the orientation angle. The idea of using parallel navigation for ball interception by the goalkeeper is recent [8]. Here, the method used is a new variant of parallel navigation adapted to the case of the goalkeeper. According to Refs. [9, 10], quite similar strategies to parallel navigation are used by baseball outfielders in order to catch the ball. The advantage of parallel navigation is the zero miss distance; i.e. the interception is guaranteed under certain conditions, as will be proven. Also, methods based on geometrical rules are known for their robustness [7].

This paper is organized as follows. We discuss the geometry of the interception in Section 3. In Section 4, we discuss the interception course using parallel navigation. In Section 5, we introduce the first approach, where the robot is controlled in the linear velocity. In Section 6, the particular case where the goalkeeper moves in a straight line is discussed. Our second approach is discussed in Section 7. An extensive simulation is carried out in Section 8.

2. ROBOT GOALKEEPER MODEL

The goalkeeper is modeled as a wheeled mobile robot of the unicycle type. The kinematics equations for this type of robots are given by:

$$\begin{aligned}\dot{x}_g &= v_g \cos \theta_g, \\ \dot{y}_g &= v_g \sin \theta_g, \\ \dot{\theta}_g &= w_g,\end{aligned}\tag{1}$$

where v_g and w_g are, respectively, the linear velocity of the wheel and its angular velocity around the vertical axis. These velocities are taken as the control inputs. The triple $q = (x_g, y_g, \theta_g) \in R^2 \times S^1$ represents the generalized coordinates, where (x_g, y_g) represents the robot's coordinates in the Cartesian plane of reference and θ_g represents the robot's orientation angle with respect to the positive x -axis. It is worth noting that the control strategy developed in this paper is valid for other types of robots such as omnidirectional robots. We choose to model the goalkeeper motion by using the unicycle kinematics model because of its simplicity and because it captures the main features of wheeled mobile robots motion.

The wheeled mobile robot goalkeeper is assumed to satisfy the following conditions:

- (i) The robot goalkeeper can move forward and backward.
- (ii) The robot can measure in real-time the ball's linear and angular velocities.
- (iii) The robot keeps the line of sight view with the ball most of the time (short-time occlusions are allowed), and can measure the angle between the reference line and the line of sight joining the robot and ball.

The last two assumptions mean that the robot has a sensory system which allows it to continuously measure important quantities such as the ball orientation with respect to a reference line and the ball linear velocity. The continuous measurement of the ball parameters is necessary, since the control strategy must be elaborated in real-time. The influence of the sensory system on the control loop is beyond the scope of this paper.

3. GEOMETRY OF THE INTERCEPTION

The geometry of the ball interception is shown in Fig. 1. The wheeled mobile robot goalkeeper is denoted by G and the ball by B. The ball is modeled as a geometrical point. Important geometric quantities are shown in Fig. 1. We define the following terms. (i) The straight line that starts at G and is directed at B is called the line of sight. This line is denoted by L . (ii) The relative distance between the goalkeeper and the ball is denoted by r . (iii) The angle from the positive x -axis to the line of sight is called the line of sight angle. This angle is denoted by σ . The initial line of sight at time t_0 when the ball is launched is denoted by L_0 . The initial line of sight angle $\sigma(t_0)$ is denoted by σ_0 .

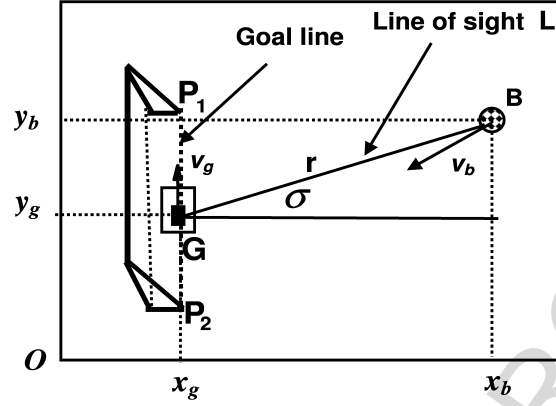


Figure 1. Goalkeeper-ball interception geometry.

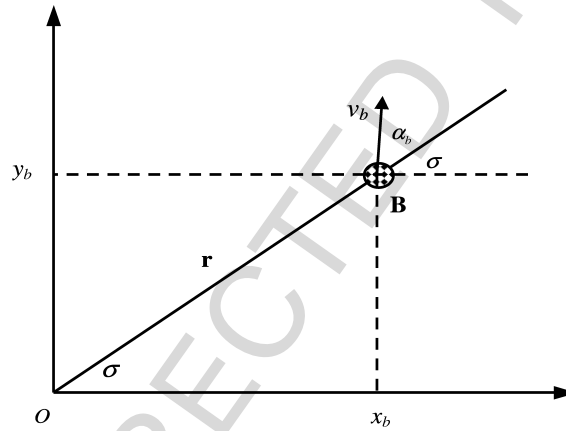


Figure 2. Kinematics of the ball's motion.

The goalkeeper aims to catch the ball before it goes inside the goal. Of course this task is not achieved by tracking the ball in the soccer field, but by intercepting the ball when the ball is in a certain neighborhood of the goal. Hence, the robot will stay within a given distance from the goal. In this paper the robot goalkeeper moves along a predefined path that joins point P_1 to point P_2 . This path covers the entire goal. If the ball goes beyond points P_1 or P_2 then the goalkeeper does not have to intercept it. It is worth noting that any configuration of the soccer field and the goal can be obtained from Fig. 1 by a simple rotation or coordinates change.

Important quantities for the ball kinematics modeling are shown in Fig. 2. The velocity of the ball is denoted by v_b . The angle from the positive x -axis to the ball's velocity vector is called the path angle or the ball's orientation angle, denoted by θ_b . We also define the angle α_b by:

$$\alpha_b = \theta_b - \sigma, \quad (2)$$

where α_b is the angle between the line of sight and the velocity vector. The aim of the ball is to go inside the goal, thus restricting the values of θ_b . For example, for Fig. 1, where the ball is to the right side of the goal, the ball can reach the goal only if $\theta_b \in (\pi/2, 3\pi/2)$.

The ball's linear velocity can be resolved into two components along and across L . The velocity component along L is the radial velocity and is denoted by $v_{b\parallel}$. In a similar way, the velocity component across L represents the tangential velocity which is denoted by $v_{b\perp}$. The values of $v_{b\parallel}$ and $v_{b\perp}$ are given by the following equations

$$\begin{aligned} v_{b\parallel} &= v_b \cos \alpha_b, \\ v_{b\perp} &= v_b \sin \alpha_b. \end{aligned} \quad (3)$$

As we mentioned previously, our control strategy is based on the use of the kinematics equations combined with geometrical rules; hence, the geometry of the interception is of a particular importance.

The ball can perform two types of motion, i.e. accelerating and non-accelerating motion. For an accelerating ball, either the orientation angle θ_b or the linear velocity v_b varies as a function of time. For a non-accelerating ball, both the orientation angle and the linear velocity are constant. Of course the case of a non-accelerating ball is simpler for analysis, and many quantities such as the interception time and position can be found in closed form. Also, the interception is easier than the case of an accelerating ball and does not require a highly maneuvering goalkeeper.

The use of polar coordinates for the state space representation of the kinematics equations of wheeled mobile robots of the unicycle type is not recent. In fact, this representation was used by many authors to design control laws. For example, in Ref. [11], polar representation allowed the design of a closed loop control law using a simple Lyapunov function. This control law is suitable for steering, path following and navigation. In Ref. [8], polar coordinates combined with geometrical rules are used for the design of simple and effective control laws for the robot. In this paper also, our control strategy is designed based on polar representation. Consider the robot representation in Fig. 3 and consider the following variable change:

$$\begin{aligned} x &= r \cos \sigma, \\ y &= r \sin \sigma, \end{aligned} \quad (4)$$

where r and σ are as shown in Fig. 3. By considering the kinematics equations for the unicycle wheeled robot and system (4), we get the following equations:

$$\begin{aligned} \dot{r} &= v_g \cos(\theta_g - \sigma), \\ r\dot{\sigma} &= v_g \sin(\theta_g - \sigma). \end{aligned} \quad (5)$$

This system shows the time evolution of the distance r and the angle σ as a function of the robot linear velocity and orientation angle.

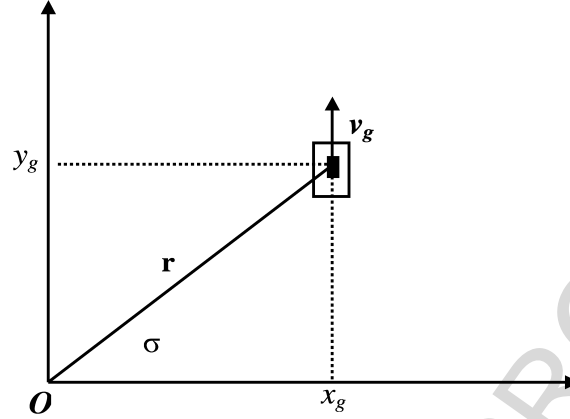


Figure 3. Kinematics of the goalkeeper's motion.

4. INTERCEPTION COURSE USING PARALLEL NAVIGATION

Parallel navigation is a closed-loop control law which is used for the interception of moving objects [7]. In Ref. [9], the authors suggested that baseball outfielders use a control strategy similar to parallel navigation in order to catch the ball.

An important advantage of parallel navigation as a control law for the goalkeeper is the fact that parallel navigation presents a closed-loop control system, i.e. the control inputs depend on the state variables of the system. Closed-loop systems present better performance than open loops. For example, they are more robust to any external disturbance.

We consider that the reference frame of coordinates is attached to the ball. The relative distance between the ball and the goalkeeper varies as follows:

$$\dot{r} = v_{b\parallel} - v_g \cos(\theta_g - \sigma). \quad (6)$$

In a similar way, the line of sight angle varies as follows:

$$r\dot{\sigma} = v_{b\perp} - v_g \sin(\theta_g - \sigma). \quad (7)$$

Parallel navigation states that the goalkeeper will move on lines L_1, L_2, \dots, L_n which are parallel to the initial line of sight L_0 . An illustration is shown in Fig. 4. As a result, the line of sight angle is constant. This simply means that the line of sight rate will be equal to zero:

$$\dot{\sigma} = 0, \quad (8)$$

which means that (7) becomes:

$$v_g \sin(\theta_g - \sigma) = v_{b\perp}. \quad (9)$$

If we consider the system of equations (3) which gives the formulae for the radial and tangential velocities for the ball, then we get for the relative distance:

$$\dot{r} = v_b \cos(\theta_b - \sigma) - v_g \cos(\theta_g - \sigma) \quad (10)$$

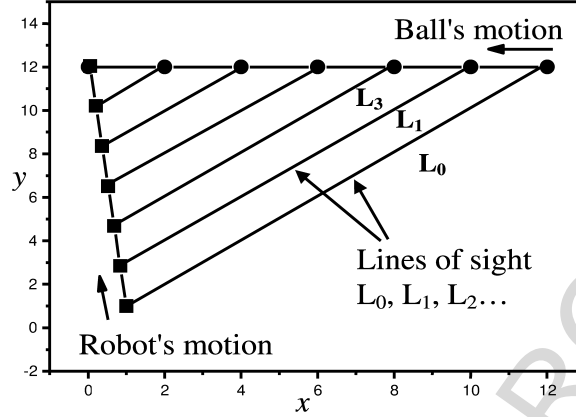


Figure 4. An illustration of parallel navigation.

and similarly for the line of sight angle rate:

$$r\dot{\sigma} = v_b \sin(\theta_b - \sigma) - v_g \sin(\theta_g - \sigma). \quad (11)$$

By replacing the tangential velocity by its value for parallel navigation, (9) becomes:

$$v_b \sin(\theta_b - \sigma) = v_g \sin(\theta_g - \sigma). \quad (12)$$

Since the line of sight angle $\sigma(t)$ is constant and equal to its initial value, we can write:

$$v_b \sin(\theta_b - \sigma_0) = v_g \sin(\theta_g - \sigma_0). \quad (13)$$

This equation gives the relationship between the robot goalkeeper control inputs and the ball's variables. As it is well known, the robot goalkeeper has two control variables, i.e. linear and angular velocities. In this paper, we suggest two solutions for the robot control input. In the first approach the robot moves in a predefined path with a predefined function for the orientation angle. The robot is controlled in the linear velocity to stay in the interception course. In the second approach the robot has more freedom in terms of the path, where it moves with constant linear velocity and it is controlled in the orientation angle to stay in the interception course.

5. GOALKEEPER CONTROLLED IN THE LINEAR VELOCITY

Here, we assume that the robot goalkeeper will move in a predefined path that joins the points P_1 and P_2 (the poles). This path can be a straight line, half a circle or an arc of a circle of a given radius. These cases are illustrated in Fig. 5. Note that other types of paths can be used; however, for simplicity, it is preferred to use linear or circular motions.

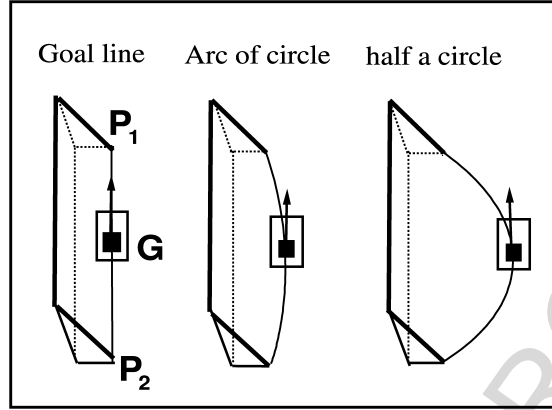


Figure 5. Path types traced by the goalkeeper.

The predefined path of the goalkeeper is characterized by the predefined orientation angle which will be denoted by:

$$\theta_g = \theta_{g\text{def}}, \quad (14)$$

where $\theta_{g\text{def}}$ can be either a constant (in the case of a straight line) or a time-varying function (in the case of circular motion). Equation (14) is equivalent to the following equation in terms of the robot angular velocity:

$$w_g = \dot{\theta}_{g\text{def}}. \quad (15)$$

The function $\theta_{g\text{def}}$ can be chosen according to many factors, such as the ball maneuvers and the initial value of the line of sight angle. Since the orientation is predefined, the only control variable is the linear velocity. The parallel navigation as stated by (13) allows us to derive the control law for the goalkeeper's linear velocity. This allows us to obtain:

$$v_g = \frac{v_b \sin(\theta_b - \sigma_0)}{\sin(\theta_{g\text{def}} - \sigma_0)}. \quad (16)$$

From this equation, it is clear that v_g is not defined when $\theta_{g\text{def}} = \sigma_0$. This case is an important particular case, which corresponds to the pure pursuit where the velocity of the pursuer lies on the line of sight. If the robot's predefined path is the goal line, then we have $\theta_{g\text{def}} \neq \sigma_0$ unless the ball itself is on the goal line. However, it is possible that $\theta_{g\text{def}} = \sigma_0$ when the predefined path is circular (even though this is rare). The problem can be solved by considering a delay in the launch time, where instead of considering $\sigma_0 = \sigma(t_0)$, a new value of the line of sight angle $\sigma_1 = \sigma(t_1)$ which satisfies $\theta_{g\text{def}} \neq \sigma_1$ is used.

Even though the control laws given by (54) and (16) are different, they use the same principle. The main difference between the two approaches is that in the first case the robot will move towards the interception point (of course if the robot is not initially heading towards this point, then a heading regulation is necessary).

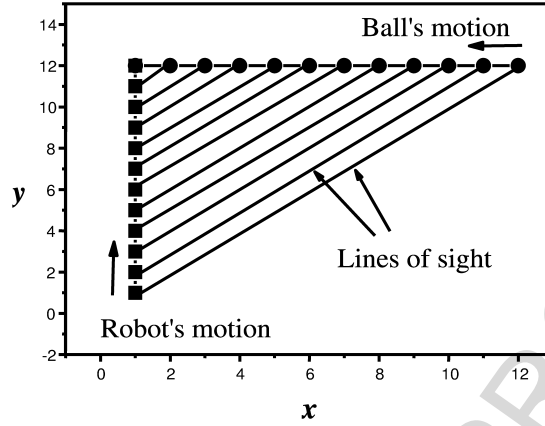


Figure 6. An illustration of modified parallel navigation.

In the second case the robot is on a collision course and it is controlled in v_g in order to arrive to the interception point at the same time as the ball. Unlike the previous case, the predefined path satisfies the non-holonomic constraint; this is an important point. A comparison between the parallel navigation formulated by the control law (54) and the parallel navigation formulated by the control law (16) is shown in Figs 4 and 6, where the ball is performing the same motion, which is linear for simplicity. For the first case (Fig. 4), the goalkeeper's linear velocity is constant. For the second case (Fig. 6), the orientation angle is constant. The interception is achieved successfully for both cases. It is worth noting that for the first case, it is assumed that the initial orientation for the robot satisfies (54), hence no heading regulation is necessary.

Now, consider (16) with $\theta_{g\text{def}} \neq \sigma_0$. It is clear from the control law (16) that the goalkeeper's linear velocity depends on the following parameters and variables:

- (i) The preset value for the robot orientation angle, which can be seen as a preset control input.
- (ii) The initial line of sight angle, which depends on the initial positions of the ball and goalkeeper.
- (iii) The ball's maneuvers, i.e. the ball's orientation angle and linear velocity.

The dependence of the robot goalkeeper's control input on the ball's maneuver is normal, since the interception of the ball requires the goalkeeper to move according to the ball's maneuvers. The relationship between v_g and the ball's orientation angle is non-linear. The robot may slow down or go faster when θ_b changes (we will see an example in the simulation).

Unlike the relationship between v_g and the ball's orientation angle, the relationship between v_g and the ball's velocity is linear:

$$v_g = k(t)v_b, \quad (17)$$

with:

$$k(t) = \frac{\sin(\theta_b - \sigma_0)}{\sin(\theta_{gdef} - \sigma_0)}, \quad (18)$$

where $k(t)$ is a time-varying proportionality factor in general. It is worth noting that $k(t)$ maybe greater or smaller than 1 depending on the values of θ_b and θ_{gdef} . From the dimensions of the goal and the field, the ball will travel longer distances than the goalkeeper, which means that in general $k(t) < 1$. This is an important difference from the other formulation of parallel navigation [8], where the interception requires $v_g > v_b$.

It is also important to note that the sign of $k(t)$ is not necessarily positive — $k(t)$ may change sign during the interception process. If $k(t)$ changes sign, then the robot changes its direction. This case happens usually when the ball suddenly changes its orientation angle, e.g. by performing a piece-wise linear motion. An example illustrating this property is considered in the simulation.

Equation (16) states that v_g is equal to zero (hence the goalkeeper does not move) in the following two cases:

- (i) When $v_b = 0$, the ball is not moving.
- (ii) When $\theta_b = \sigma_0$, with $v_b \neq 0$, in this case, the ball is moving in the line of sight straight in the direction of the goalkeeper; the goalkeeper will intercept the ball without moving if the ball keeps its orientation angle constant.

5.1. Robot limitations

Of course wheeled mobile robots present a physical limitation on the maximum linear velocity. This problem arises in our case when the ball is close to the goal line (hence θ_{gdef} is close to σ_0) and is moving with high speed. However, the further the ball's initial position is from the goal, the smaller is the required linear velocity of the goalkeeper, since the distance traveled by the goalkeeper will be smaller than the distance traveled by the ball. In fact, the further the initial position of the ball is from the goal, the closer θ_b is to σ_0 and hence $k(t)$ becomes smaller. In real soccer this property is satisfied in general, since the goal dimensions are much smaller than the soccer field dimensions. Let v_{gmax} be the maximum value for the linear velocity of the goalkeeper and v_{bmax} be the maximum velocity of the ball. The control input for the robot allows us to write:

$$v_{gmax} = k_T v_{bmax}, \quad (19)$$

where k_T is the maximum tolerable value for the velocity ratio. From (18), we have:

$$\frac{\sin(\theta_b - \sigma_0)}{\sin(\theta_{gdef} - \sigma_0)} < k_T. \quad (20)$$

This equation gives the minimum value for $\sin(\theta_{gdef} - \sigma_0)$ as follows:

$$\sin(\theta_{gdef} - \sigma_0) > \frac{\sin(\theta_b - \sigma_0)}{k_T}. \quad (21)$$

The goalkeeper maximum speed constraint is satisfied when the predefined angle θ_{gdef} and the line of sight angle σ_0 satisfy (21).

For the radial acceleration given by $a_{\text{g}}^{\text{rad}} = v_{\text{g}}^2/r$, where r is the curvature radius of the path, it is always possible to choose r so that the radial acceleration is smaller than its maximum value $a_{\text{g}}^{\text{rad}} < a_{\text{gmax}}^{\text{rad}}$. In the case where the goalkeeper moves on the goal line, we simply have $a_{\text{g}}^{\text{rad}} = 0$.

For the tangential acceleration (defined as $a_{\text{g}}^{\text{tan}} = dv_{\text{g}}/dt$), the maximum value is given by:

$$a_{\text{gmax}}^{\text{tan}} = k_{\text{T}}\dot{v}_{\text{bmax}} + \dot{k}_{\text{T}}v_{\text{bmax}}. \quad (22)$$

If the ball is not accelerating ($\dot{v}_{\text{b}} = 0$ and $\dot{\theta}_{\text{b}} = 0$), and the predefined path is a straight line (from which it results $\dot{k} = 0$), then $a_{\text{g}}^{\text{tan}} = 0$. If the ball is moving in a straight line and accelerating in the velocity (this is the case in general in real soccer robotics), and the goalkeeper is moving in a straight line, then $k = 0$, and thus $a_{\text{gmax}}^{\text{tan}} = k\dot{v}_{\text{bmax}}$, from which the maximum tolerable value for k and the appropriate values of θ_{gdef} can be obtained.

5.2. Positive versus negative velocities

Earlier, we made the assumption that the robot can move forward and backward. In many situations, the interception requires the robot to move forward and backward.

Initially the robot goalkeeper is at a middle distance between points P_1 and P_2 . When the ball is launched the robot has to make a decision in which direction to move (towards point P_1 or point P_2). For positive values of v_{g} the robot moves towards P_1 and for negative values of v_{g} the robot moves towards P_2 . The sign of v_{g} is determined by θ_{b} , θ_{gdef} and σ_0 .

If we consider the configuration of Fig. 1, where $\theta_{\text{gdef}} = \theta_{\text{g0}} = \pi/2$, then the goalkeeper will move towards point P_1 when:

$$\theta_{\text{b}} - \sigma_0 \in (0, \pi), \quad (23)$$

and towards P_2 when:

$$\theta_{\text{b}} - \sigma_0 \in (\pi, 0). \quad (24)$$

If θ_{b} suddenly changes its interval, then the robot changes its direction.

5.3. Robot kinematics equations under the control law

The wheeled mobile robot under the control law (16) moves in the Cartesian frame of coordinates according to the following kinematics equations:

$$\begin{aligned} \dot{x}_{\text{g}} &= [v_{\text{b}}k(t)] \cos \theta_{\text{gdef}}, \\ \dot{y}_{\text{g}} &= [v_{\text{b}}k(t)] \sin \theta_{\text{gdef}}, \\ \dot{\theta}_{\text{gdef}} &= w_{\text{g}}. \end{aligned} \quad (25)$$

It is clear that in the case where the robot moves on the goal line, we have $w_g = 0$. For system (25), the solution for the robot position can be obtained easily for a non-accelerating ball when $w_g = 0$.

5.4. Time variation of the relative distance

Assume that the robot orientation angle is chosen such as that $\theta_{gdef}(t) \neq \sigma_0$ for all values of t . By considering (10) for the relative distance and (16), we get:

$$\dot{r} = v_b \left[\cos \alpha_b - \frac{\sin \alpha_b \cos(\theta_{gdef}(t) - \sigma_0)}{\sin(\theta_{gdef}(t) - \sigma_0)} \right]. \quad (26)$$

This equation can be simplified using some trigonometric identities, which allows us to get the following equation for the ball–goalkeeper distance:

$$\dot{r} = v_b \left[\frac{\sin(\theta_{gdef}(t) - \theta_b)}{\sin(\theta_{gdef}(t) - \sigma_0)} \right]. \quad (27)$$

The interception of the ball corresponds to $r(t) = 0$ for a given time $t < +\infty$. In general, proving that the interception takes place is equivalent to proving that $\dot{r} < 0$, which means that the range is a decreasing function of time.

5.5. Relative motion in the Cartesian plane

The ball motion in the Cartesian frame of coordinates is given by the following system:

$$\begin{aligned} \dot{x}_b &= v_b \cos \theta_b, \\ \dot{y}_b &= v_b \sin \theta_b, \end{aligned} \quad (28)$$

where x_b and y_b are the coordinates of the ball in the Cartesian frame. Let us define the following relative velocities:

$$\begin{aligned} \dot{x}_d &= \dot{x}_b - \dot{x}_g, \\ \dot{y}_d &= \dot{y}_b - \dot{y}_g. \end{aligned} \quad (29)$$

This system is equivalent to (10) which gives the time derivative of the relative distance. By considering the robot motion under the control law, we get:

$$\begin{aligned} \dot{x}_d &= v_b [\cos \theta_b - k(t) \cos \theta_{gdef}], \\ \dot{y}_d &= v_b [\sin \theta_b - k(t) \sin \theta_{gdef}]. \end{aligned} \quad (30)$$

By replacing $k(t)$ by its value and considering some trigonometric identities we get:

$$\begin{aligned} \dot{x}_d &= v_b \cos \sigma_0 \frac{\sin(\theta_{gdef} - \theta_b)}{\sin(\theta_{gdef} - \sigma_0)}, \\ \dot{y}_d &= v_b \sin \sigma_0 \frac{\sin(\theta_{gdef} - \theta_b)}{\sin(\theta_{gdef} - \sigma_0)}. \end{aligned} \quad (31)$$

The interception corresponds to $x_d = 0$ and $y_d = 0$ at the same time.

In the next section, we provide an analysis to the case where the goalkeeper moves on the goal line. The analysis for the case where the goalkeeper moves in an arc of a circle is quite similar.

6. GOALKEEPER MOVING ON THE GOAL LINE

This case corresponds to a constant orientation angle. We denote $\theta_{g\text{def}} = \theta_{g0}$. For example, for the configuration of Fig. 1, we have $\theta_{g0} = \frac{\pi}{2}$. The control law (16) can be rewritten as follows:

$$v_g = \frac{v_b \sin(\theta_b - \sigma_0)}{\sin(\theta_{g0} - \sigma_0)}. \quad (32)$$

The denominator in (32) is constant and the numerator is a function of the ball parameters. In this case we have $\theta_{g0} \neq \sigma_0$, except in a very special limit case when the ball is at the limit of the soccer field in the same line as the goal line (the line of sight coincides with the goal line). This case can be seen easily in Fig. 1 where $\theta_{g0} = \pi/2$, and the limit values of σ_0 are $\pi/2$ and $-\pi/2$, which correspond to the ball in the same line as the goal line. These limit values are excluded since they are not of practical importance.

Equation (27) for the relative distance between the robot and the ball becomes:

$$\dot{r} = v_b \frac{\sin(\theta_{g0} - \theta_b)}{\sin(\theta_{g0} - \sigma_0)}. \quad (33)$$

Depending on whether the ball is accelerating, we have two cases which we consider in the following section.

6.1. Non-accelerating ball

In this case, from (32), the goalkeeper moves with a constant linear velocity, which means that the goalkeeper is not accelerating and \dot{r} is constant. Here, it is easy to find the solution for the relative distance, and the interception time and position. In fact, the relative distance varies with time according to the following equation:

$$r(t) = \int_0^t \dot{r}(\tau) d\tau. \quad (34)$$

Since \dot{r} is constant, we have:

$$r(t) = r_0 + \dot{r}t. \quad (35)$$

By replacing \dot{r} by its value we get:

$$r(t) = r_0 + v_b \frac{\sin(\theta_{g0} - \theta_b)}{\sin(\theta_{g0} - \sigma_0)} t. \quad (36)$$

In order to have an interception, i.e. $r(t) = 0$ for $t < +\infty$, the following inequality must be satisfied:

$$\frac{\sin(\theta_{g0} - \sigma_0)}{\sin(\theta_{g0} - \theta_b)} < 0. \quad (37)$$

Of course, θ_{g0} has a fixed value, and θ_b and σ_0 lie in given intervals for which inequality (37) is satisfied, as we will see.

6.2. Accelerating ball

If the ball's linear velocity or orientation angle is time varying, then (32) states that the goalkeeper's linear velocity is also time varying and \dot{r} is not constant anymore. In other words, if the ball is accelerating then the goalkeeper is also accelerating. Equation (34) is valid for this case also, but since \dot{r} is time dependent, the derivation of the solution requires the knowledge of the ball's maneuvers and the use of numerical techniques. If the ball is moving in a constant direction, but with a time-varying linear velocity, then v_g is proportional to v_b with a constant proportionality factor. An important realistic case is when the ball moves in a constant direction with the following linear velocity:

$$v_b = v_{b0}a^t, \quad (38)$$

where $v_{b0} = v_b(t_0 = 0)$, and $a < 1$. Equation (38) interprets a physical law which states that the ball becomes slower the longer it rolls. The robot linear velocity varies as follows:

$$v_g = \frac{\sin(\theta_b - \sigma_0)}{\sin(\theta_{g0} - \sigma_0)} v_{b0}a^t \quad (39)$$

$$= kv_{b0}a^t. \quad (40)$$

It turns out that v_g is time decreasing similarly to v_b .

The discussion of the interception of the ball is stated as follows.

PROPOSITION 1. *When the wheeled mobile robot is moving in the goal line with a constant orientation angle that satisfies $\theta_{g0} \neq \sigma_0$, the control law (16) results in a successful interception of the ball by the goalkeeper.*

Proof. We consider the configuration of Fig. 1 for the goal geometry. As we mentioned previously, any other configuration can be obtained from Fig. 1 by a simple rotation.

From the configuration of Fig. 1, the predefined direction of the robot is given by

$$\theta_{g0} = \frac{\pi}{2}, \quad (41)$$

and the ball is always to the right side of the goalkeeper ($x_b > x_g$). This means that the line of sight angle is:

$$\sigma_0 \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right). \quad (42)$$

In the same way for the ball to reach the goal, the ball's orientation angle is restricted to

$$\theta_b \in \left(\frac{\pi}{2}, \frac{3\pi}{2} \right). \quad (43)$$

From (41) and (42), and (41) and (43), respectively, we get

$$\theta_{g0} - \sigma_0 \in (0, \pi), \quad (44)$$

and

$$\theta_{g0} - \theta_b \in (-\pi, 0). \quad (45)$$

By considering the intervals for $\theta_{g0} - \sigma_0$ and $\theta_{g0} - \theta_b$ in (44) and (45), it is easy to see that the right-hand side term in (33) has a positive numerator and a negative denominator, which means that \dot{r} is negative and hence the relative ball-goalkeeper distance is decreasing. \square

PROPOSITION 2. *The interception time of a non-accelerating ball by the goalkeeper moving under the control law (16) is given by:*

$$t_f = -\frac{r_0 \sin(\theta_{g0} - \sigma_0)}{v_b \sin(\theta_{g0} - \theta_b)}. \quad (46)$$

Proof. Let us rewrite the equation for the relative distance in the case of a non-accelerating ball:

$$r(t) = r_0 + v_b \frac{\sin(\theta_{g0} - \theta_b)}{\sin(\theta_{g0} - \sigma_0)} t. \quad (47)$$

From Proposition 1 it is stated that inequality (37) is satisfied. The interception time corresponds to $r(t_f) = 0$ which gives:

$$t_f = -\frac{r_0 \sin(\theta_{g0} - \sigma_0)}{v_b \sin(\theta_{g0} - \theta_b)}. \quad (48)$$

Obviously, t_f is proportional to the initial distance and inversely proportional to the ball's velocity. \square

It is simple to verify (46) by considering the solution for system (31) for a non-accelerating ball with $\theta_{\text{gdef}} = \theta_{\text{g0}} = \text{constant}$. The solution is in this case:

$$\begin{aligned} x_d(t) &= v_b \cos \sigma_0 \frac{\sin(\theta_{\text{g0}} - \theta_b)}{\sin(\theta_{\text{g0}} - \sigma_0)} t + x_{d0}, \\ y_d(t) &= v_b \sin \sigma_0 \frac{\sin(\theta_{\text{g0}} - \theta_b)}{\sin(\theta_{\text{g0}} - \sigma_0)} t + y_{d0}, \end{aligned} \quad (49)$$

where x_{d0} and y_{d0} are the initial states for x_d and y_d , respectively. It is easy to see that by replacing t by t_f , we get:

$$\begin{aligned} x_d(t_f) &= -r_0 \cos \sigma_0 + x_{d0}, \\ y_d(t_f) &= -r_0 \sin \sigma_0 + y_{d0}, \end{aligned} \quad (50)$$

which means that $x_d(t_f) = 0 = y_d(t_f)$.

PROPOSITION 3. *The interception position of a non-accelerating ball by the goalkeeper moving under the control law (16) is given by*

$$\begin{aligned} x_g &= x_{g0} - r_0 \cos \theta_{g0} \frac{\sin(\theta_b - \sigma_0)}{\sin(\theta_{g0} - \theta_b)}, \\ y_g &= y_{g0} - r_0 \sin \theta_{g0} \frac{\sin(\theta_b - \sigma_0)}{\sin(\theta_{g0} - \theta_b)}, \end{aligned} \quad (51)$$

where x_{g0} and y_{g0} represent the initial position of the goalkeeper.

Proof. Of course the interception takes place on the goal line. The solution for x_g and y_g for a non-accelerating ball is given by:

$$\begin{aligned} x_g(t) &= [v_b k \cos \theta_{g0}] t + x_{g0}, \\ y_g(t) &= [v_b k \sin \theta_{g0}] t + y_{g0}. \end{aligned} \quad (52)$$

By replacing t by t_f and k by its value, we get:

$$\begin{aligned} x_g &= x_{g0} - r_0 \cos \theta_{g0} \frac{\sin(\theta_b - \sigma_0)}{\sin(\theta_{g0} - \theta_b)}, \\ y_g &= y_{g0} - r_0 \sin \theta_{g0} \frac{\sin(\theta_b - \sigma_0)}{\sin(\theta_{g0} - \theta_b)}. \end{aligned} \quad (53)$$

□

We have considered here the interception time and position when the ball is not accelerating. These quantities are easy to compute in this case. However, they can be numerically calculated or estimated for the case of an accelerating ball. It is worth noting that (46) and (51) give the exact solutions. In the next section, we discuss the second approach.

7. ROBOT CONTROLLED IN THE ORIENTATION ANGLE

In this formulation the goalkeeper moves with constant linear velocity and is controlled in the orientation angle. The robot control input can be obtained from (13) as follows:

$$\theta_g = \sin^{-1}\left(\frac{v_b}{v_g} \sin(\theta_b - \sigma_0)\right) + \sigma_0. \quad (54)$$

This approach provides more flexibility to the robot in terms of the path. The path here depends on the ball path, and other factors such as the robot and the ball velocities and initial positions. Since the sine function vibrates within the region $[-1, 1]$, in order for (54) to make sense without dependence on the values of θ_b and σ_0 , it is required that $v_g > v_b$. However, when the difference in the angle $\theta_b - \sigma_0$ is small enough, the robot can move slower than the ball and still intercepts it. In all cases the robot linear velocity must be chosen such that:

$$\frac{1}{k} \sin(\theta_R - \sigma_0) < 1. \quad (55)$$

In the limit case when $\theta_b = \sigma_0$ (the ball is moving in the line of sight), it turns out that the goalkeeper orientation angle is given by $\theta_R = \sigma_0$ [from (54)], which means that the goalkeeper also moves in the line of sight, but in the opposite direction to the ball. In this particular case, the robot can move with any value for $v_g > 0$ and intercept the ball.

In general, when the ball is kicked towards the goal, we have $(\theta_b - \sigma_0) \in (\pi/2, 3\pi/2)$, which means that the ball is approaching from the goal. This corresponds to a negative value for the ball radial velocity $v_{b\parallel}$, which can be written as:

$$v_{b\parallel} = -v_b \sqrt{1 - \sin^2(\theta_b - \sigma_0)} < 0. \quad (56)$$

The proof that the goalkeeper intercepts the ball successfully can be stated as follows:

PROPOSITION 4. *Under the control law given by (54), the robot intercepts the ball successfully.*

Proof. The aim is to prove that the relative goalkeeper–ball distance is decreasing under the control law (54); thus $\dot{r} < 0$. When (55) is satisfied, we have:

$$\sin^{-1}\left(\frac{1}{k} \sin(\theta_R - \sigma_0)\right) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right). \quad (57)$$

The robot radial velocity under the control law (54) is the following:

$$v_{g\parallel} = v_g \cos\left(\sin^{-1}\left(\frac{1}{k} \sin(\theta_R - \sigma_0)\right)\right). \quad (58)$$

By considering (57), it turns out that $v_{g\parallel} > 0$. Thus, we can write:

$$v_{g\parallel} = v_g \sqrt{1 - \frac{1}{k^2} \sin^2(\theta_R - \sigma_0)}. \quad (59)$$

The relative distance between the goalkeeper and the robot varies as follows:

$$\dot{r} = -v_b \sqrt{1 - \sin^2(\theta_b - \sigma_0)} - v_g \sqrt{1 - \frac{1}{k^2} \sin^2(\theta_b - \sigma_0)}. \quad (60)$$

Clearly, when k is chosen such that $(1/k) \sin(\theta_b - \sigma_0) < 1$, $\dot{r} < 0$, $\forall \theta_b, \forall \sigma_0$. Thus, the robot intercepts the ball successfully. \square

Visual occlusions occur frequently in real soccer robotics. Short-time occlusions do not affect the control loop in general. However, long-time occlusions can affect the control loop. Assumption (iii) in Section 2 states that only short-time occlusions are allowed. The implementation of parallel navigation in both cases requires the following measurements

- Ball velocity and orientation angle.
- Initial value of the line of sight angle.

The real-time measurement of the ball's quantities can be established using various types of sensors. For the implementation of parallel navigation it is better to use sensors that are less sensitive to visual occlusions.

8. SIMULATION

This section simulates the interception of the ball by the goalkeeper in various scenarios. We consider the following cases:

- Goalkeeper moving on the goal line, non-accelerating ball.
- Goalkeeper moving on the goal line, accelerating ball.
- Goalkeeper moving on the goal line, ball performing a piece-wise linear motion.
- Goalkeeper controlled in the orientation angle, non-accelerating ball.
- Goalkeeper controlled in the orientation angle, accelerating ball.

8.1. Goalkeeper moving on the goal line, non-accelerating ball

We take the configuration of Fig. 1; the goal position is $x_{\text{goal}} = 2$, $y_{\text{goal}} \in (1, 9)$. The ball moves according to the following motion equations:

$$\begin{aligned} x_b &= -2t + 12, \\ y_b &= 1.1547t + 3, \end{aligned} \quad (61)$$

with $\theta_b = 150^\circ$, $v_b = 2.309$ (for simplicity, we assume that the velocities, the positions and the time are without units). The initial position of the ball is (12, 3).

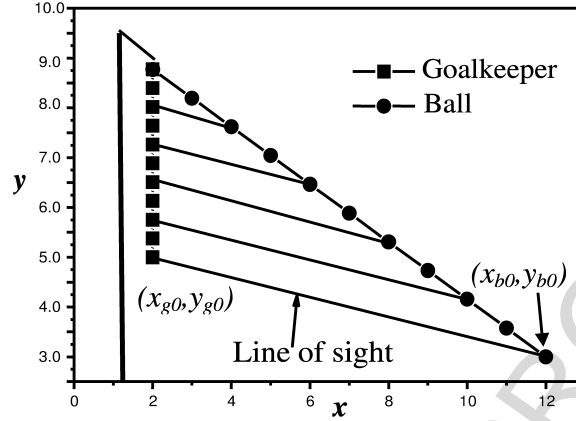


Figure 7. Interception for the scenario of Section 8.1.

At the initial time the goalkeeper is at $x_{g0} = 2$, $y_{g0} = 5$, which is the middle of the goal.

The ball is moving straight to the goal. Of course the goalkeeper does not know the ball's motion, but can measure in real-time θ_b and v_b , and compute v_g . The line of sight angle is $\sigma_0 = -11^\circ.31$. It turns out that $v_g = 0.7546$, which is constant since θ_b and v_b do not change. Simulation for this scenario is shown in Fig. 7. The interception point is $(x_i, y_i) = (2, 8.77)$ and the interception time is $t_f = 5$. These values are obtained from simulation. The exact values can be obtained from equations (46) and (51).

8.2. Goalkeeper moving on the goal line, accelerating ball

The ball is accelerating by changing its orientation angle, and performs a curved trajectory. The initial line of sight angle, and the ball and goalkeeper initial positions are the same as the previous example. The interception for this scenario is shown in Fig. 8. It is clear that the linear velocity for the goalkeeper is decreasing. The interception point is $(2, 7.03)$.

8.3. Goalkeeper moving on the goal line, ball performing a piece-wise linear motion

The ball may hit another player and change its orientation suddenly. This case poses problems even for professional goalkeepers. The ball performs a piece-wise linear motion as shown in Fig. 9. During the first phase, the ball moves according to:

$$\begin{aligned} x_b &= -1.3t + 11, \\ y_b &= -2t + 15, \end{aligned} \quad (62)$$

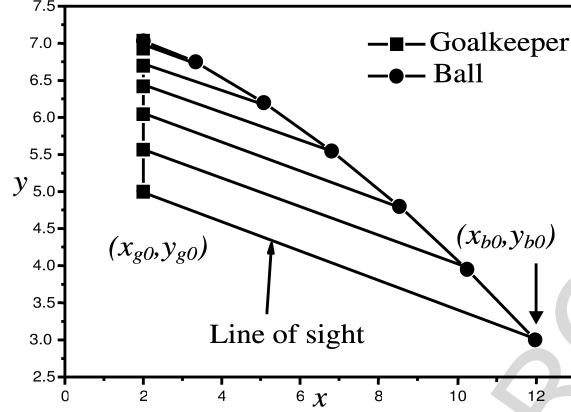


Figure 8. Interception for the scenario of Section 8.2.

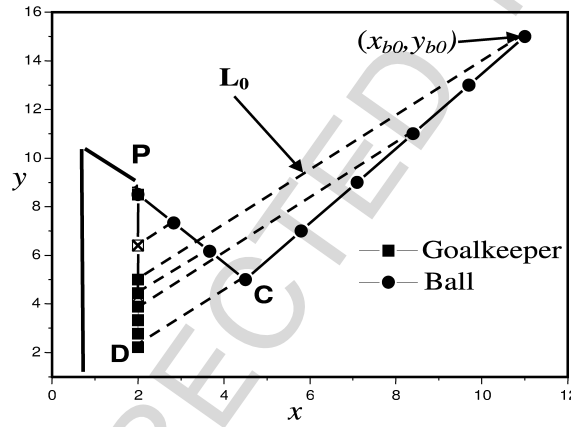


Figure 9. Interception for the scenario of Section 8.3.

with $v_b = 2.38$, $\theta_b = 237^\circ$, $\sigma_0 = 48^\circ.01$ and $(11, 15)$ is the ball's initial position. During the second phase the ball moves according to:

$$\begin{aligned} x_b &= -0.833t + 4.5, \\ y_b &= 1.167t + 5, \end{aligned} \quad (63)$$

with $v_b = 1.43$, $\theta_b = 127^\circ.48$, σ is kept the same of course and $(4.5, 5)$ are the coordinates of point C where the ball changes its orientation.

The computation of the linear velocity gives for the first phase $v_g = -0.5556$. This negative value means that the goalkeeper will move towards point P_2 . The second phase starts when the ball arrives at point C. At this time the goalkeeper is at position D and the linear velocity for the goalkeeper becomes $v_g = 2.19$. Thus, the goalkeeper changes direction and starts moving towards point P_1 to intercept the ball at point $(2, 8.5)$.

Table 1.Robot and ball coordinates for $t = 0$ to $t = 5$

Time t_k	Goalkeeper position	Ball position
0	(2, 5)	(11, 15)
1	(2, 4.444)	(9.7, 13)
2	(2, 3.889)	(8.4, 11)
3	(2, 3.333)	(7.1, 9)
4	(2, 2.778)	(5.8, 7)
5	(2, 2.222)	(4.5, 5)

Table 2.Robot and ball coordinates for $t = 5$ to $t = 8$

Time t_k	Goalkeeper position	Ball position
5	(2, 2.222)	(4.5, 5)
6	(2, 4.313)	(3.667, 6.167)
7	(2, 6.407)	(2.834, 7.334)
8	(2, 8.5)	(2, 8.5)

Tables 1 and 2 shows the positions for the ball and the goalkeeper at discrete values of time. The interception for this scenario is shown in Fig. 9. The second slide in the PowerPoint attachment shows the interception for a similar scenario.

8.4. Goalkeeper controlled in the orientation angle, non-accelerating ball

The goalkeeper starts from the initial position (2, 5). The ball moves in a straight line according to the following equations:

$$\begin{aligned} x_b &= -0.866t + 12, \\ y_b &= 0.5t + 3, \end{aligned} \quad (64)$$

with $\theta_b = 150^\circ$, $v_b = 1$. The initial line of sight angle is $\sigma_0 = -16^\circ.7$. It turns out that the goalkeeper also moves in a straight line (θ_g is constant). This scenario is shown in Fig. 10, where two different speeds are considered ($v_g = 0.25$ and $v_g = 0.5$). The goalkeeper's path is different for different values of v_g as shown in Fig. 10. In both cases, the goalkeeper reaches the ball successfully.

8.5. Goalkeeper controlled in the orientation angle, accelerating ball

We consider two scenarios, where the ball starts from two different initial positions (12, 2) and (12, 8) with $\sigma_0 = -163^\circ.3$ and $\sigma_0 = 163^\circ.3$, respectively. The two scenarios are shown in Figs 11 and 12. The trajectory of the ball presents a slight curvature. In this case the robot path is also curved. In both scenarios the robot reaches the ball successfully. In Fig. 12, the robot moves with two different speeds ($v_g = 0.5$ and $v_g = 1/3$). Clearly, the goalkeeper path depends on the speed.

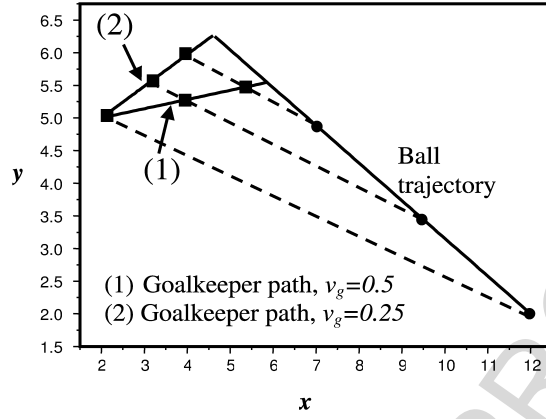


Figure 10. Interception for the scenario of Section 8.4.

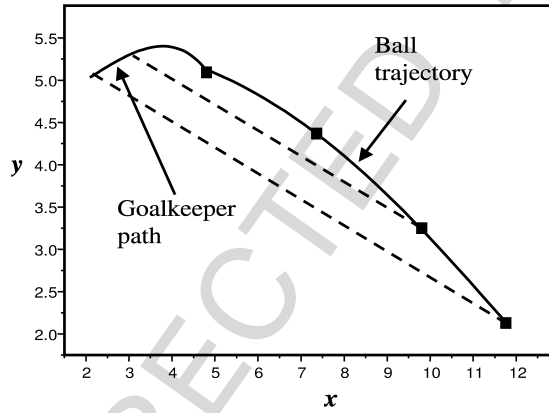


Figure 11. Interception for the scenario of Section 8.5, ball starting from (12, 2).

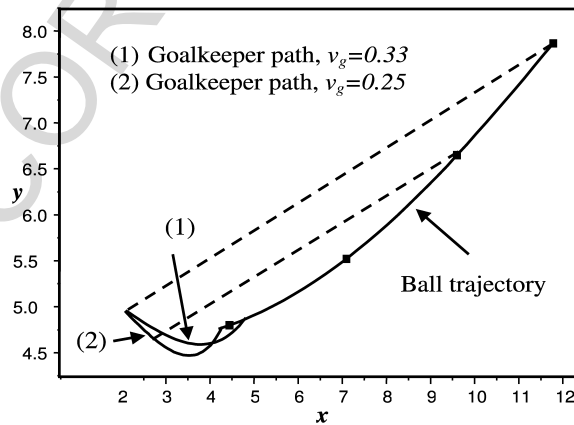


Figure 12. Interception for the scenario of Section 8.5, ball starting from (12, 8).

9. CONCLUSIONS

In this paper, we presented a control strategy for a wheeled mobile robot goalkeeper whose task is to intercept the ball before it goes inside the goal. Our control strategy is designed based on the parallel navigation guidance law, where the robot moves on lines which are parallel to the initial line of sight that joins the robot and the ball. This strategy puts the robot on the interception course. We first derive a relative kinematics model, which models the motion of the ball and the goalkeeper in polar coordinates. The equation for the interception course is then derived based on the kinematics model. Two different versions of the proportional navigation guidance law are considered. Because of the particularity of the goalkeeper problem, a new version of parallel navigation adapted to this case is suggested and used. Here, the goalkeeper moves in a predefined path that covers the goal and is controlled in the linear velocity. The particular case where the robot moves in the goal line is considered in more detail where it turns out that the goalkeeper is not accelerating when the ball is not accelerating and accelerates when the ball accelerates in the linear velocity or orientation angle. In the second approach, the robot is controlled in the orientation angle to stay in the intersection course. This approach gives more flexibility to the path of the goalkeeper. In this case, the path of the goalkeeper depends on the ball path, and the robot and the ball linear velocities and initial positions. It is proven that the method allows a zero miss distance, i.e. the interception is guaranteed under certain conditions. The control strategy is illustrated by considering an extensive simulation for different scenarios, which confirms our theoretical results.

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