On the control of a wheeled mobile robot goalkeeper

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Abstract—This paper deals with the control of a wheeled mobile robot goalkeeper, where the goalkeeper’s task is to intercept the ball before it goes inside the goal. The control law is based on the use of the kinematics equations and some geometrical rules. In our control strategy, the goalkeeper moves in a predefined path which corresponds to a rendezvous course. The goalkeeper is controlled in the linear velocity to intercept the ball. We consider various simulation examples.

I. INTRODUCTION

In the last decade, wheeled mobile robots have seen important practical and theoretical developments, where very sophisticated robots were designed for both industrial and domestic applications. Despite the important advances in motion planning and control, there exist many theoretical challenging problems. In fact, there exist important restrictions on the control space, for example it is not possible to rigorously control the robot using a linear controller. Furthermore an important theorem due to Brockett [1] states that wheeled mobile robots cannot be stabilized using a smooth static feedback law. To solve this problem various control algorithms that use unsmooth or time-varying feedback laws were suggested ([2], [3]).

Soccer robotics has attracted the attention of many researchers from various fields such as control theory and artificial vision. In fact motion control of robot soccer player includes difficult problems such as moving obstacles avoidance and ball tracking. Various techniques based on nonlinear and intelligent control theory (see for example [5]) and vision-based control systems were suggested for mobile robots soccer player.

In this paper, we design a control strategy for a wheeled mobile robot goalkeeper. Recall that the goalkeeper is a soccer player who has as a task to catch the ball before the ball goes inside the goal. Our strategy is based on the use of the goalkeeper-ball kinematics model and some geometric rules. In fact in [4], a control strategy based on geometrical rules was suggested, where the goalkeeper is controlled in the orientation and moves with constant velocity. This procedure is not well adapted to the nonholonomic constraint. To overcome this problem we use a different approach, we suggest that the robot moves in a predefined direction and is controlled in the linear velocity. Our approach is illustrated using several examples.

II. ROBOT MODEL

The kinematics model for a wheeled mobile robot of the unicycle type is given as follows

\begin{align}
\dot{x}_g &= u_g \cos \phi_g \\
\dot{y}_g &= u_g \sin \phi_g \\
\dot{\phi}_g &= \omega_g
\end{align}

where \(x_g\) and \(y_g\) are the coordinates of the robot reference point in the Cartesian frame of coordinates, \(\phi_g\) is the robot orientation with respect to the positive x-axis. A configuration of the robot is given by a position with a given orientation, i.e., by the triple \(q_g = (x_g, y_g, \phi_g)^T\). The linear and angular velocities are \(u_g\) and \(\omega_g\), respectively, \(u_g\) and \(\omega_g\) are also the control variables. Wheeled mobile robots are known to present a nonholonomic mechanism, i.e., there exists a non-integrable constraint on the velocities. From equation (1), the nonholonomic constraint states that

\[\dot{x}_g \sin \phi_g = \dot{y}_g \cos \phi_g\] (2)

The physical meaning of this constraint is that the path of the robot is tangent to the robot main axis. This constraint results in a restriction of the control space. Wheeled mobile robots are under-actuated, since the control space is smaller than the configuration space (the robot has three variables and two control inputs). The motion of the ball is unpredictable and completely unknown to the robot; hence the control algorithm must be established in real time. In this case closed loop control systems are recommended, since closed loop systems present more robustness to the external disturbances.
disturbance and uncertainties. Before we discuss the control strategy, we make the following assumptions

1. The robot can move forward and backward.
2. The robot can measure using sensors the ball’s linear and angular velocities, and the line of sight angle.

III. POSITION OF THE PROBLEM

With reference to figure 1, the ball which is denoted by $b$ moves from its initial position towards the goal. Our aim is to control the mobile robot goalkeeper in order to catch the ball before it goes inside the goal. As stated before, the path of the ball is not known a priori and the control algorithm must be elaborated in real time. In our formulation of the problem, the mobile robot goalkeeper moves in the line $AB$ as depicted in figure 1, which is the goal line. As we mentioned previously, we assume that the mobile robot can move forward and backward, this corresponds to positive and negative values of the linear velocity, respectively. Since the robot moves in a predefined path, the only control variable is the robot’s linear velocity. In this paper, we assume that the model for the ball is the following

\[ \begin{align*}
\dot{x}_b &= u_b \cos \phi_b \\
\dot{y}_b &= u_b \sin \phi_b \\
\phi_b &= \omega_b 
\end{align*} \]

The ball can perform two types of motions, namely accelerating and non-accelerating motions. We consider both cases in this paper.

With reference to figure 1, we define the following quantities

1. The line of sight $gb$ is the line joining the goalkeeper and the ball.

2. The relative distance goalkeeper-ball is $r_d$.
3. The line of sight angle $\sigma$ is the line between the positive x-axis and the line of sight.

We also define the angles $\delta_b$ (ball) and $\delta_g$ (goalkeeper) as follows

\[ \phi_b = \delta_b + \sigma \] (4)

and

\[ \phi_g = \delta_g + \sigma \] (5)

The projection of the relative distance goalkeeper-ball on the x-, y-axes gives

\[ \begin{align*}
x_d &= x_b - x_g \\
y_d &= y_b - y_g 
\end{align*} \] (6)

The aim of our control strategy is to null $x_d$ and $y_d$ at the same time. In this paper we formulate the problem in polar coordinates. Let us consider the following variable change

\[ x = r \cos \sigma \\
y = r \sin \sigma \\
r^2 = x^2 + y^2 \] (7)

This variable change is valid for both the ball and the goalkeeper. Consider the relative distance goalkeeper-ball given by

\[ r_d = r_b - r_g \] (8)

By taking the time derivative for $r_d$, we get

\[ \dot{r}_d = \dot{r}_b - \dot{r}_g \] (9)

By using the kinematics models for the robot and the ball, equation (7) and some trigonometric identities, we get

\[ \begin{align*}
\dot{r}_g &= u_g \cos \delta_g \\
\dot{r}_b &= u_b \cos \delta_b 
\end{align*} \] (10)

which gives

\[ \dot{r}_d = u_b \cos \delta_b - u_g \cos \delta_g \] (11)

If $\dot{r}_d < 0$, then the ball is approaching the goalkeeper, if $\dot{r}_d > 0$, then the distance between the goalkeeper and the ball is increasing. Our aim is to design a closed loop control algorithm capable to drive $r_d$ to zero before the ball goes inside the goal. Of course the interception time and point depend on many factors such as the robot and the ball velocities and directions. A possible control strategy [4] is to keep the line of sight angle $\sigma$ constant during the interception process and control the robot in the direction. However because of the nonholonomic constraint this approach
may fail in some cases. We introduce the following modification.

The robot goalkeeper is moving in a predefined straight line. In this case the robot orientation angle is constant i.e., \( \phi_g(t) = \phi_{g0} \) and the control variable is the linear velocity. The angle \( \phi_{g0} \) depends only on the position and the geometry of the goal. For example for the geometry of figure 1, we have \( \phi_{g0} = \frac{\pi}{2} \).

For a constant line of sight angle and a constant orientation angle, the relative distance from the goalkeeper reference point to the ball varies as follows

\[
\dot{r}_d = u_b \cos(\phi_b - \sigma_0) - u_g \cos(\phi_{g0} - \sigma_0)
\]

This equation is obtained by the combination of equations (4), (5) with equation (11). As we mentioned previously, the only control variable for the robot goalkeeper is the linear velocity, so \( u_g \) is used to control the robot in such a way that the robot and the ball arrive at the interception point at the same time.

1) Closed loop control algorithm: The line of sight angle varies according to the following differential equation [4]

\[
r_d \dot{\sigma} = v_b \sin(\phi_b - \sigma) - v_g \sin(\phi_{g0} - \sigma)
\]

For a constant line of sight angle, we get

\[
v_b \sin(\phi_b - \sigma) - v_g \sin(\phi_{g0} - \sigma) = 0
\]

Since we keep the line of sight angle equal to the initial line of sight angle, we get

\[
u_b \sin(\phi_b - \sigma_0) = u_g \sin(\phi_{g0} - \sigma_0)
\]

The closed loop control law for the goalkeeper linear velocity is derived from (13). We have in this case

\[
u_g = \frac{u_b \sin(\phi_b - \sigma_0)}{\sin(\phi_{g0} - \sigma_0)}
\]

with \( \phi_{g0} \) must be different from \( \sigma_0 \). The case where \( \phi_{g0} = \sigma_0 \) means that the line of sight coincide with the goal line. This case corresponds to the pure pursuit, which is the opposite extreme of our control law. The robot linear velocity depends on the following parameters

1. The ball linear and angular velocities.
2. The line of sight angle, which is constant and equal to the initial line of sight angle.

This means that the implementation of this control law requires the knowledge of the initial line of sight angle and a continuous measurement of the ball linear and angular velocities.

It is clear, since \( u_g \) is proportional to \( u_b \) that the robot will not move if the ball does not move (a stationary ball corresponds to stationary goalkeeper). According to the control law (14), the relative distance between the robot goalkeeper and the ball varies as follows

\[
\dot{r}_d = u_g \left[ \cos(\phi_b - \sigma_0) - \frac{\sin(\phi_b - \sigma_0)}{\tan(\phi_{g0} - \sigma_0)} \right]
\]

By considering some trigonometric identities, we get the distance between the goalkeeper and the ball

\[
\dot{r}_d = u_g \frac{\sin(\phi_{g0} - \phi_b)}{\sin(\phi_{g0} - \sigma_0)}
\]

The robot goalkeeper will catch the ball when \( r_d = 0 \), this corresponds to both \( x_d = 0 \) and \( y_d = 0 \) at the same time.

Under the control law (14), the kinematics equations for the robot goalkeeper are given by

\[
\begin{align*}
\dot{x}_g &= u_g \cos \phi_{g0} \\
\dot{y}_g &= u_g \sin \phi_{g0} \\
\dot{\phi}_g &= \omega_g = 0
\end{align*}
\]

with \( u_g \) as given by equation (14). For the particular case of figure 1, the kinematics equations for the mobile robot goalkeeper are as follows

\[
\begin{align*}
\dot{x}_g &= 0 \\
\dot{y}_g &= u_g(t) \\
\dot{\phi}_g &= \omega_g = 0
\end{align*}
\]

In this case the robot is moving in a straight line parallel to the y-axis. The trajectory for the wheeled robot goalkeeper in the Cartesian frame of coordinates can be obtained by the integration of (18) to obtain

\[
\begin{align*}
x_g(t) &= x_{g0} + \int_{t_0}^{t} u_g(t) \, dt + y_{g0} \\
y_g(t) &= \int_{t_0}^{t} u_g(t) \, dt + y_{g0}
\end{align*}
\]

If the ball is moving in a straight line with constant linear velocity (this case corresponds to \( \phi_b = \text{constant}, \ u_b = \text{constant} \)), then the control law states that the mobile robot goalkeeper moves with a constant velocity, and hence the robot is not accelerating and moves as follows

\[
\begin{align*}
x_g(t) &= x_{g0} \\
y_g(t) &= u_g t + y_{g0}
\end{align*}
\]

If the ball moves with time-varying linear velocity but with constant orientation angle, then the control law has the following form

\[
u_g = k u_b(t)
\]

where \( k \) is a constant depending on the constant orientations for the robot and the ball. It is clear that
Fig. 2. Particular case when the ball’s linear velocity lies on the line of sight

$u_g$ is proportional to the ball linear velocity, with a proportionality constant which can be either greater or smaller than one depending on the angles $\phi_{g0}$, $\phi_b$ and $\sigma_0$. It is also possible to see from the control law (14) that the goalkeeper does not move when the ball orientation is constant and lies on the line of sight ($\phi_b = \sigma_0$); this is shown in figure 2. Obviously in this case the goalkeeper does not need to move to catch the ball, since the ball is coming straight to the goalkeeper.

The case of non-accelerating ball (the ball is moving in a straight line with a constant velocity) is analytically simple. In fact in this case, it is possible to derive the exact solution for the interception time. The solution for $r_d$ is the following

$$r_d(t) = u_b \frac{\sin (\phi_b - \sigma_0)}{\sin (\phi_{g0} - \sigma_0)} t + r_{d0}$$

where $r_{d0}$ is the initial value for the Euclidean distance between the robot reference point and the ball. Of course the interception requires the quantity

$$a = u_b \frac{\sin (\phi_b - \sigma_0)}{\sin (\phi_{g0} - \sigma_0)}$$

to be negative. In this case the interception time is given by

$$t_f = \frac{r_{d0} \sin (\phi_{g0} - \sigma_0)}{u_b \sin (\phi_b - \sigma_0)}$$

Note that the relative coordinates of the robot with respect to the ball coordinates in the Cartesian frame can be found easily when $r_d$ is known. This can be accomplished by using

$$x_g = x_b - r_d \cos \sigma_0$$
$$y_g = y_b - r_d \sin \sigma_0$$

where $r_d$ can be obtained by solving equation (16) in the general case, or using equation (22) in the case of non-accelerating ball.

A. In which direction to move?

With reference to figure 3, the robot is initially at middle point of the distance between A and B. When the ball is launched, the robot goalkeeper has to make the decision in which direction to move, i.e., from the initial position towards point A or point B. The first case corresponds to positive values for $u_g$ and the second case corresponds to negative values. Equation (14) gives the directions for $u_g$. From which we get

1. $u_g < 0$, when $\pi + \sigma_0 < \phi_b < \sigma_0$.
2. $u_g > 0$, when $\sigma_0 < \phi_b < \pi + \sigma_0$.

These two cases are illustrated in figure 3. In some situations, it is possible for the robot to change its orientation suddenly when the ball changes its orientation.

To prove that the goalkeeper under our control law intercepts the ball successfully, (this is equivalent to $r_d = 0$ for a finite time) we consider the geometry of figure 1, note that the proof is similar for any other geometry, since any geometry can be obtained from figure 1 by a simple shift of coordinates or variable change.

For the scenario of figure 1, the intervals for the robot orientation and the line of sight angle are the following

$$\phi_b \in \left( \frac{\pi}{2}, \frac{3\pi}{2} \right)$$
$$\lambda_0 \in \left( -\frac{\pi}{2}, \frac{\pi}{2} \right)$$

Our aim is to prove that $r_d(t)$ is a decreasing function ($\dot{r}_d < 0$). Using (26), and the goalkeeper orientation angle we get

$$\phi_{g0} - \phi_b \in (-\pi, 0)$$
$$\phi_{g0} - \lambda_0 \in (0, \pi)$$

By using system (27) it is easy to see that the numerator in (16) is always negative while the denominator is always positive, hence $\dot{r}_d < 0$.

In the next section, we present our simulation examples.

IV. SIMULATION EXAMPLES

We consider three cases corresponding to non-accelerating and accelerating ball.
Fig. 3. Goalkeeper direction according to the ball orientation

A. Ball moving with constant velocity and constant orientation angle

This is the simplest case; we assume the following initial coordinates for the ball and the robot, respectively

\[
(x_b, y_b) = (16, 6) \\
(x_g, y_g) = (1, 5)
\]  

(28)

with the initial time \( t_0 = 0 \). The goal lies in \([y, 9]\) with \( x = 1 \). We assume the following trajectory for the ball

\[
x_b(t) = -3t + 16 \\
y_b(t) = \frac{2}{5}t + 6
\]  

(29)

with the following values for the ball velocity and orientation, respectively \( u_b = 3.025\) m/s, \( \phi_b = 3.009\) rd, \( \theta_0 = 0.067\) rd. Of course the goalkeeper does not require the knowledge of the ball motion model (29) but only the linear and angular velocities. Using to the control law (14), the goalkeeper linear velocity is given by \( u_g = 0.6\) m/s. This is a constant value. The robot goalkeeper motion is described by the following system

\[
x_g(t) = 1 \\
y_g(t) = 0.6t + 5
\]  

(30)

The interception time computed using equation (24) is \( t_f = 5\) s. This can be verified easily. In fact we have \( x_g - x_b = 0 \) and \( y_g - y_b = 0 \) at \( t = 5\) s. The ball interception is shown in figure 4.

B. Ball moving with constant orientation but time-varying velocity

Here we consider that the ball is moving with time-varying velocity and constant orientation angle. We consider two different scenarios in this situation.

1) 1st scenario: Let’s take the initial position for the ball and the goalkeeper as follows

\[
(x_b, y_b) = (20, 4) \\
(x_g, y_g) = (1, 5)
\]  

(31)

with an orientation angle for the ball \( \phi_b = 3.001\) rd. The initial line of sight angle is \( \theta_0 = 0.0526\) rd, and the initial distance is \( r_{d0} = 19.026\) m. We assume that the ball maneuvers with the following time-varying linear velocity

\[
u_b = 3.025(0.1 \cos t + 1)
\]  

(32)

The velocity profile for the ball is depicted in figure 5, where the velocity is decreasing in the first phase and increasing in the second phase. This maneuver maybe more complicated than the realistic case, since in the realistic case, the velocity of the ball is always decreasing. According to the control law, the robot’s linear velocity is given by

\[
u_g = 0.266(0.1 \cos t + 1)
\]  

(33)

Similarly to \( u_b, u_g \) is also time-varying, furthermore \( u_g \) has the same velocity profile as \( u_b \). Note that \( u_g \) is positive, hence the goalkeeper will move from its initial position towards point A. In order to prove that the interception of the ball is accomplished successfully, we consider equation (15) or (16). For the considered values of \( \phi_{g0}, \phi_b \) and \( \sigma_0 \), we get the following equation

\[
\dot{r} = -3(0.1 \cos t + 1)
\]  

(34)

Since \((0.1 \cos t + 1) \geq 0\) for all \( t \), the relative distance is decreasing with the following solution

\[
r(t) = -3(0.1 \sin t + t) + 19.026
\]  

(35)
The interception time can be found by setting $r(t) = 0$ in (35), and solving the nonlinear equation in time. By using a graphical technique, we get $t_f = 6.3367 s$. The interception for this scenario is shown in figure 6.

2) 2\textsuperscript{nd} scenario: This scenario is similar to the previous one, with a small difference in the initial state of the ball, here we take

$$\left( x_{b0}, y_{b0} \right) = (3, 4)$$

(36)

The ball velocity and orientation angle are the same as in the previous scenario. The line of sight angle is negative and given by $\sigma_0 = -0.464\text{rd}$. After some calculations, we get for $u_g$ in this case

$$u_g = -3.3487(0.1 \cos t + 1)$$

There are two main differences with the 1\textsuperscript{st} scenario:

1. Here $u_g$ is negative for all $t$, this means that the goalkeeper will move from its initial position towards point $B$.
2. $|u_g(t)| > |u_b(t)|$. The robot must be faster in order to catch the ball.

V. CONCLUSION

In this paper, we have presented a real time control strategy for a wheeled mobile robot goalkeeper. The task of the goalkeeper is to catch the ball before the ball goes inside the goal. In our formulation of the problem the goalkeeper moves forward and backward within the goal line. The control strategy is derived based on geometrical rules and the robot goalkeeper is controlled in the linear velocity according to the ball velocity and direction. In our control strategy, the goalkeeper predicts the interception point, where the ball and the goalkeeper will arrive at the same time, and adjusts its linear velocity in order to keep the lines of sight parallel to the initial line of sight. The control strategy was illustrated using several examples.

REFERENCES