On the tracking and interception of a moving object by a wheeled mobile robot

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Abstract—This paper deals with the problem of tracking and interception of an object moving with unknown maneuvers by a wheeled mobile robot. We design a closed loop control law based on a guidance strategy for this purpose. The guidance strategy uses geometrical rules combined with the kinematics equations, where the robot’s angular velocity is equal to the rate of turn of the line of sight angle. In some situations it is necessary to use a heading regulation phase in order to put the robot’s linear velocity on the line of sight and apply the guidance strategy. In the presence of obstacles, two navigation modes are used, namely tracking mode and obstacle avoidance mode. Simulation examples show the efficiency of the method.

I. INTRODUCTION

Wheeled mobile robots are extremely used for both domestic and industrial applications. These applications require efficient and low computational cost path planning algorithms. Despite the huge literature on the motion control of wheeled mobile robots; the problem of motion control is still theoretically a challenging problem. The main difficulty in mobile robot motion control and planning is due to the nonholonomic constraint that states that the robot cannot move parallel to its main axis. Brockett theorem ([1], [2]) states that these systems cannot be stabilized by time invariant smooth state feedback.

Navigation is one of the most important and elementary functions in mobile robotics. The literature on navigation methods is huge, where various techniques from artificial intelligence, artificial vision, etc. are used. Most navigation algorithms consider a stationary goal. The problem becomes more difficult when the target is moving. In this paper, we consider moving objects tracking and interception by a wheeled mobile robot of the unicycle type. Various techniques from control theory and artificial vision were suggested for this task. For example in [4], [11] and [12] a Lyapunov like approach was suggested for the robot motion control. This control strategy suffers from the classical problems encountered in nonlinear control theory such as the difficulty of the construction of Lyapunov functions. Algorithms based on artificial vision were also suggested to design a vision-based control for the robot ([9], [10]). However, these algorithms may suffer from expensive computational cost.

We consider that the motion of the target is unknown and unpredictable for the robot. This requires a real time control algorithm. Our control strategy is based on a guidance strategy which consists of a simple control law based on geometrical rules combined with the kinematics equations. Tracking and interception of a moving target is a global navigation problem. However, in the presence of obstacles, the problem becomes a combination between local and global navigation. Perhaps, the most obvious application of our control law is ball interception in soccer robotics. We use simulation to show that the robot reaches its goal successfully for different scenarios.

II. ROBOT MODEL

The robot is a simple wheeled mobile robot of the unicycle type. Figure 1 shows the geometry for the robot-moving target interception in the Cartesian frame of coordinates. The kinematics model for the robot is the following

\[
\begin{align*}
\dot{x}_r &= v_r \cos \theta_r \\
\dot{y}_r &= v_r \sin \theta_r \\
\dot{\theta}_r &= w_r
\end{align*}
\]  

(1)

where \((x_r, y_r)\) are the coordinates of the robot’s reference point in the Cartesian plane, the angle \(\theta_r\) is the orientation of the robot with respect to the positive x-axis. \(v_r\) and \(w_r\) are the linear and angular velocities, respectively. A configuration of the robot is given by \(X = (x_r, y_r, \theta_r)^T \in \mathbb{R}^2 \times S^1\), where \(S^1\) is the unit circle in the plane. The control variables for the robot are the linear velocity \(v_r\) and the time derivative of the orientation angle. It is well-known that wheeled mobile robots present a typical example
of the nonholonomic mechanism. From the robot kinematics model, the nonholonomic constraint is the following
\[ x_r \sin \theta_r - \dot{y}_r \cos \theta_r = 0 \]  
(2)
The physical meaning of this constraint is that the robot path is tangent to the robot main axis. The moving target is modeled as a geometrical point in the Cartesian plane. Thus there is no constraint on the motion of the moving target. The kinematics equations for the moving target are given by
\[ \dot{x}_b = v_b \cos \theta_b \\
\dot{y}_b = v_b \sin \theta_b \]  
(3)
where \( B = (x_b, y_b) \) is the moving target position in the Cartesian frame of coordinates and \( \theta_b \) is the moving target orientation with respect to the positive x-axis. We assume that the robot is moving with a constant linear velocity which is higher than the maximum linear velocity of the moving target, i.e.,
\[ v_r > v_{b \text{max}} > 0 \]  
(4)
Since the robot moves with a constant linear velocity, the only control variable is \( w_r \). We also assume that the minimum turning radius for the robot is smaller than the minimum turning radius for the moving target. It is assumed that the robot can detect the target and the obstacles in real time.

III. POSITION OF THE PROBLEM

Given a mobile robot and its model (1), Our task is to control the path of the mobile robot in order to navigate and reach the moving target. The geometry of the tracking problem is depicted in figure 1. Of course the moving target is moving in an unpredictable way. This means that the control strategy cannot be established off-line, and real time path planning is necessary.

As we mentioned previously our approach is based on geometric rules when taking into account the robot kinematics model and the moving target motion and maneuvers. The control strategy consists of a closed loop system. Closed loop systems offer more robustness to the external disturbance and uncertainties than open loop systems. With reference to figure 1, we define the line of sight as the line joining the robot reference point and the moving target. The line of sight angle denoted by \( \lambda \) is the angle from the positive x-axis to the line of sight. This angle is given by
\[ \tan \lambda (t) = \frac{y_b(t) - y_r(t)}{x_b(t) - x_r(t)} \]  
(5)
Note that the line of sight angle is not defined at the interception time. We also define the angles \( \delta_r \) and \( \delta_b \) for the robot and the moving target, respectively as follows
\[ \delta_r = \theta_r - \lambda \]  
(6)
and
\[ \delta_b = \theta_b - \lambda \]  
(7)
where \( \delta_r \) is the angle between the line of sight and the robot linear velocity vector. In a similar way, \( \delta_b \) is the angle between the line of sight and the moving target linear velocity vector. Both \( \delta_r \) and \( \delta_b \) are time-varying. At any time \( t \) it is possible to define the relative distance between the robot reference point and the moving target in the Cartesian frame of coordinates with respect to the x-, y-axes
\[ x_d = x_b - x_r \\
y_d = y_b - y_r \]  
(8)
The wheeled mobile robot reaches the moving target when both \( x_d \) and \( y_d \) are zero at a given time, thus the aim of the control law is to null both \( x_d \) and \( y_d \) at the same finite time \( t_f \). Note that the moving target and the robot reference point coordinates are related as follows
\[ x_b = x_r + r_d \cos \lambda \\
y_b = y_r + r_d \sin \lambda \]  
(9)
where \( r_d = \sqrt{x_d^2 + y_d^2} \) is the relative distance between the moving target and the robot reference point. By taking the derivative of (8) with respect to time, we get
\[ \dot{x}_d = \dot{x}_b - \dot{x}_r \\
\dot{y}_d = \dot{y}_b - \dot{y}_r \]  
(10)
This is equivalent to write
\[
\begin{align*}
\dot{x}_d &= v_b \cos \theta - v_r \cos \theta_r \\
\dot{y}_d &= v_b \sin \theta - v_r \sin \theta_r
\end{align*}
\] (11)

The initial distances with respect to the x-, y-axes are \(x_{d0}\) and \(y_{d0}\), with at least \(x_{d0} \neq 0\) or \(y_{d0} \neq 0\). In the next section, we describe our control strategy.

IV. CONTROL STRATEGY

The guidance strategy being used in this paper is used by some animals predators in order to intercept them preys [5]. The principle of the method is to make the robot linear velocity lies on the line joining the robot and the moving target. This principle is illustrated in figure 2, where at any time, the robot velocities \(v_{r1}, v_{r2}, v_{r3}, \ldots, v_{rn}\) coincide with the lines of sight \(R_1B_1, R_2B_2, R_3B_3, \ldots, R_nB_n\). As a result the robot orientation \(\theta_b(t)\) is equal to the line of sight angle \(\lambda(t)\) i.e.,
\[
\theta_r(t) = \lambda(t)
\] (12)

By taking the time derivative of equation (12), we get for the wheeled mobile robot angular velocity
\[
w_r = \dot{\lambda}(t)
\] (13)

So in our strategy, the robot angular velocity is equal to the rate of turn of the line of sight angle. By considering the equation (12), the relative velocities \(\dot{x}_d\) and \(\dot{y}_d\) become
\[
\begin{align*}
\dot{x}_d &= v_b \cos \theta - v_r \cos \lambda \\
\dot{y}_d &= v_b \sin \theta - v_r \sin \lambda
\end{align*}
\] (14)

By taking into account equation (7), we get the following nonlinear system of differential equations
\[
\begin{align*}
\dot{x}_d &= v_b (\delta_b + \lambda) - v_r \cos \lambda \\
\dot{y}_d &= v_b \sin (\delta_b + \lambda) - v_r \sin \lambda
\end{align*}
\] (15)

Recall that \(\lambda\) is a function of the robot and the moving target positions. By writing \(\lambda\) as a function of \(x_d\) and \(y_d\), we get the following differential equations
\[
\begin{align*}
\dot{x}_d &= v_b \cos (\delta_b + \frac{\pi}{2}) - v_r \cos (\delta_b + \frac{\pi}{2}) \\
\dot{y}_d &= v_b \sin (\delta_b + \frac{\pi}{2}) - v_r \sin (\delta_b + \frac{\pi}{2})
\end{align*}
\]

This system is highly nonlinear. The solution of equation (16) provides the trajectory of the robot with respect to the moving target in the Cartesian frame of coordinates. The wheeled mobile robot reaches the moving target successfully from any position when \(v_r > v_b\).

Under the pursuit control law, the robot moves in the Cartesian plane according to the following kinematics equations
\[
\begin{align*}
\dot{x}_r &= v_r \cos \lambda \\
\dot{y}_r &= v_r \sin \lambda \\
\dot{\theta}_r &= w_r = \dot{\lambda}
\end{align*}
\] (17)

By integration of the third equation in system (17), we get \(\theta_r(t) - \theta_{r0} = \lambda(t) - \lambda_0\), where \(\theta_{r0} = \theta_r(t_0)\) and \(\lambda_0 = \lambda(t_0)\). If at the initial time \(t_0\), we have
\[
\theta_{r0} = \lambda_0
\] (18)

then the application of the control law is straightforward. If \(\theta_{r0} \neq \lambda_0\) then it is necessary to drive \(\theta_r(t)\) to \(\lambda(t)\) at a given time \(t_1 < t_f\) (\(t_f\) is the interception time) in order to apply the control law. This will be discussed in the next paragraph.

A. Heading regulation

As we mentioned previously, if at the initial time, \(v_r\) lies on the line of sight, then the application of the equation (12) is straightforward; otherwise, a heading regulation in order to put \(v_r\) on the line of sight is necessary. The control strategy is divided into two phases, namely heading regulation and tracking. Various control strategies can be used for the heading regulation. For example it is possible for the robot to perform a circular motion that drives it from its initial orientation angle to the line of sight angle. In this case, the robot angular velocity is constant, and the robot moves in an arc of a circle of a given radius. At the end of this phase, the robot orientation angle is equal to the line of sight angle; and the robot velocity lies on the line of sight.
B. A particular case: Target moving in a straight line

The target is moving in a straight line: \( v_b = \text{constant} \) and \( \theta_b = \text{constant} \). We assume that initially \( \theta_{r0} = \lambda_0 \), and thus, the heading regulation phase is not necessary. For simplicity and without loss of generality, we assume that the moving target is moving parallel to the x-axis in the direction of the increasing \( x \). The moving target trajectory is given by

\[
\begin{align*}
x_b(t) &= v_b t + x_{b0} \\
y_b(t) &= y_{b0} \\
\theta_b(t) &= 0
\end{align*}
\]  

(19)

According to the tracking control strategy, the robot kinematics model is the following

\[
\begin{align*}
\dot{x}_r &= v_r \cos \lambda \\
\dot{y}_r &= v_r \sin \lambda \\
\dot{\lambda}_r &= \omega_r = \lambda
\end{align*}
\]

with

\[
\lambda(t) = \arctan2 \left( \frac{y_{b0} - y_r(t)}{v_b t + x_{b0} - x_r(t)} \right)
\]  

(20)

The aim of the pursuit in this case is to null the line of sight angle. Simulation for this scenario will be considered in the next section.

V. SIMULATION EXAMPLES

This section deals with the numerical implementation of our control algorithm. We consider two different scenarios.

1) The target is moving in a straight line with a constant linear velocity, without loss of generality, we consider that the moving target moves in a horizontal line.

2) The target is moving with a constant angular velocity.

We restrict ourselves to the case where the moving target is moving with constant linear velocity. Note that the control algorithm intercepts when \( v_b \) is time-varying also.

A. Target moving in a horizontal line

Let us assume that the target is moving in a horizontal line with a constant velocity. The moving target trajectory is given by equation (19), with \( v_b = 2 \text{m/s}, \) \( (x_{b0}, y_{b0}) = (1, 10) \). The robot is initially at \( (x_{r0}, y_{r0}) = (1, 11) \), with \( v_r = 8 \text{m/s} \). The initial line of sight angle is \( \lambda_0 = \frac{\pi}{2} \), and the initial distance is \( r_{d0} = 9 \text{m} \).

Figure 3 shows the trajectories for the moving target and the wheeled mobile robot, where the interception takes place at \( t_f = 1.311 \text{s} \), the interception point is \( (x, y) = (3.622, 10) \). Figure 4 shows the evolution of the normalized line of sight angle \( \frac{\lambda}{\lambda_0} \) as a function of time. It is clear that \( \frac{\lambda}{\lambda_0} \) goes from its initial value to zero. The pursuit in this case aims to null the line of sight angle.

B. Target moving with constant angular velocity

This is another simple case. Since \( w_b \) is constant, we have for the moving target orientation angle

\[
\theta_b(t) = w_b t + \theta_{b0}
\]

(21)

Simulation for this case is shown in figure 5, where it is clear that the robot intercepts the moving target successfully. The initial positions for the moving target and the robot are respectively as follows

\[
\begin{align*}
(x_{b0}, y_{b0}) &= (3, 10) \\
(x_{r0}, y_{r0}) &= (10, 10)
\end{align*}
\]  

(22)

The interception time is \( t_f = 1.18 \text{s} \), and the interception position is \( (x, y) = (1.63, 11.9) \).

VI. NAVIGATION IN THE PRESENCE OF OBSTACLES

In the presence of obstacles, two navigation modes are used in order to reach the moving target and avoid obstacles at the same time. Therefore, the combination between global and local navigation is necessary. The navigation modes are as follows

1) Tracking mode: under this mode, the guidance strategy described previously is used.

2) Obstacles avoidance mode: an algorithm based on approximate cell decomposition is used in this mode.
Here, we briefly describe an algorithm which can be used for the tracking problem in the presence of obstacles.

A. General description of the algorithm

The position of the robot is contained in a rectangular region $D \subset \mathbb{R}^2$, let $L = \text{int}(D) \times [0, 2\pi]$. The free space is represented by

$$C_{\text{free}} = L - C_B$$

where $C_B$ is the obstacle region. Let $\Omega$ be a rectangular decomposition $P$ of $\Omega$ as a finite collection of rectangles $\{e_i\}_{i=1}^k$ which satisfies

1) $\Omega = \bigcup_{i=1}^k e_i$.

2) The interiors of the $e_i$’s do not intersect, i.e.,

$$\text{int}(e_i) \cap \text{int}(e_j) = \emptyset, \forall i, j \in [1, k], i \neq j$$

(23)

Each $e_i$ is called a cell. A cell can be classified as follows

1) Empty, when $\text{int}(e_i) \cap C_B = \emptyset$.

2) Full, when $e_i \subseteq C_B$.

3) Mixed, otherwise.

Let $O_i$ be an arbitrary shaped obstacle in $C_B$. All obstacles $O_i$ are included in circles denoted by $C_i$. This is reasonable approach, when the sensors input is sonar or laser range reading. Circles $C_i$ are increased by the robot radius. Obstacle $C_i$ increased by the robot radius is denoted by $B_i$, we have $O_i \subseteq C_i \subseteq B_i$.

This is illustrated in figure 6. Under this formulation, the robot can be seen as a point-like vehicle. The discretization of equation (17) gives

$$\begin{align*}
x_r(k+1) &= v_r \cos(\lambda_k) h + x_r(k) \\
y_r(k+1) &= v_r \sin(\lambda_k) h + y_r(k) \\
\lambda_k &= \text{atan2}(y_r(k)-y,(x_r(k)-x_r(k))
\end{align*}$$

(24)

System (24) is obtained from system (17) by using Euler method, $h$ is the integration step. For simplicity, we take $h = 1$. Assume that the robot is at position $(x_r(k), y_r(k))$, the position $(x_r(k+1), y_r(k+1))$ is obtained from (24). Two cases are possible:

1) $(x_r(k+1), y_r(k+1))$ corresponds to a free cell.

2) $(x_r(k+1), y_r(k+1))$ corresponds to a full or mixed cell, which we denote $e_f$.

The obstacle avoidance mode is activated only in the second case, and instead of $\lambda_k$, we use another value for the line of sight angle denoted by $\hat{\lambda}_k$. $\hat{\lambda}_k$ is chosen in such a way that $(x_r(k+1), y_r(k+1))$ fall in the closest free cell to $e_f$. Unlike $\lambda_k$, $\hat{\lambda}_k$ is not calculated based on the target coordinates. An illustration of this
algorithm is considered, where the path of the moving target and the robot is shown in figure 7. The guidance strategy is applied for all robot states except states (5) and (6), where the obstacle avoidance mode is activated.

VII. CONCLUSION

In this paper, we applied a guidance strategy based on geometrical rules for the interception of a moving target by a robot. The robot is a simple wheeled mobile robot of the unicycle type.

The principle of the our guidance strategy is to make the robot heading towards the moving target at any time. As a result the robot linear velocity lies on the line of sight joining the robot reference point and the moving target. The interception of the moving target is accomplished successfully when the robot is faster than the moving target. By considering simulation examples, it is shown that the approach presents an efficient control law for moving target interception, furthermore the closed loop control is simple.

REFERENCES